

Deutsche
Geophysikalische
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Elektromagnetische Tiefenforschung

22. Kolloquium

Hotel Maxičky, Děčín, Tschechische Republik

1.-5. Oktober 2007

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Protokoll
über das Kolloquium
Elektromagnetische Tiefenforschung

22. Kolloquium, Hotel Maxičky, Děčín, Tschechische Republik,
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I

Vorwort



Das 22. Kolloquium Elektromagnetische Tiefenforschung fand vom 1. bis 5. Oktober 2007 im Hotel Maxičky in Děčín in der Tschechischen Republik statt. Josef Pek mit seinen Kolleginnen und Kollegen von der Akademie der Wissenschaften in Prag hatte diesmal die Organisation des ersten außerdeutschen Kolloquiums übernommen.

Fast 90 Teilnehmern zeigten wieder ein sehr reges Interesse an unserem Kolloquium und machten es zum bisher „internationalsten“ Treffen. Insgesamt waren 34 verschiedene Einrichtungen vertreten, 21 aus dem Ausland und 13 aus Deutschland. Vor dem Kolloquium fand der von der Bergakademie Freiberg organisierte, internationale 3D Induction Workshop statt, was vielen KollegInnen aus dem Ausland die Teilnahme an beiden Workshops ermöglichte. Besonders für Teilnehmer aus den osteuropäischen Ländern vereinfachte sich die Anfahrt zum Austragungsort des Kolloquiums in der Tschechischen Republik.

Die Internationalisierung des Kolloquiums wird auch in den Beiträgen für den Kolloquiumsband deutlich. Von den insgesamt 38 Beiträgen sind diesmal nur drei auf Deutsch verfasst. Beiträge auf Englisch sind im Hinblick auf die weltweite Verfügbarkeit des Protokolls im Internet sicher sehr sinnvoll. Außerdem arbeiten in den verschiedenen Gruppen in Deutschland immer öfter Mitarbeiter aus aller Welt, für die es einfach leichter ist auf Englisch zu publizieren.

Während des Kolloquiums wurden viele der hier publizierten Themen intensiv in kleinen Gruppen diskutiert. Leider haben diesmal die abendlichen Diskussionen nicht stattgefunden, die bei den vorherigen Treffen immer auf großes Interesse gestoßen sind. Obwohl während der Sitzungen etliche attraktive Themen von allgemeinen Interesse, wie z.B. die deutliche Zunahme von multidisziplinären Ansätzen in der elektromagnetischen Tiefenforschung, angesprochen wurden, sollten wir in Zukunft auch wieder vermehrt „elektromagnetische Rätsel“ vorstellen, die die Diskussion besonders anfeuern.

In diesem Kolloquiumsband erscheinen insgesamt 38 Beiträge, die die gesamte Breite des Spektrums der elektromagnetischen Induktion in die Erde repräsentieren. Nach wie vor gibt es noch viel Bedarf

an methodischen Arbeiten, die analytische Modelle (Weidelt, Hvoždara, Frömmel), neue Analyseverfahren (Lilley, Virgil, Ziekur) und fortgeschrittene numerische Methoden zur Lösung elektromagnetischer Probleme in komplexen Strukturen (Franke 2x, Schmucker, Baranwal, Afanasjew) umfassen. Weitere Beiträge befassen sich mit neuen Erfahrungen aus der Datenverarbeitung (Chen, Krings), was in Anbetracht der oftmals schwierigen Messbedingungen in Europa wichtig ist. Ein weiterer Beitrag behandelt Konzepte der Datenorganisation und -verwaltung (Friedrichs). Der größte Block von Beiträgen sind 23 „case studies“, bzw. „case studies“ in Kombination mit neuer Methodik, die wir hier nicht alle namentlich nennen können. Dabei werden angewandte wie geodynamische Fragestellungen auf verschiedensten Skalen behandelt, von der Erdoberfläche bis tief in den Mantel hinein und hoch zu den Körpern im Sonnensystem. Allerdings zeigt sich auch hier die Internationalität unserer Forschung, da nur zwei der Fallstudien aus Deutschland stammen, der Rest über die ganze Welt verteilt ist. Neue Instrumente, bzw. Ideen für neuartige Instrumente werden von Hördt und Roßberg vorgestellt.

Unser Dank geht an alle, die sich für den glatten Verlauf des 22. Kolloquiums eingesetzt haben. Leider hat einer der größten Förderer der Idee das deutsche EM Kolloquium auf tschechischem Boden zu organisieren, Dr. Oldřich Praus, die Verwirklichung dieser Idee nicht mehr erlebt. Einer der Pioniere der Magnetotellurik weltweit und Gründer der tiefen induktiven Geoelektrik in der ehemaligen Tschechoslowakei, Polarforscher und Mitentdecker der karpathischen Leitfähigkeitsanomalie starb im Jahr vor der Tagung im Alter von 77 Jahren.

Für die finanzielle Hilfe danken wir auch unseren Sponsoren: der Firma Metronix Braunschweig, dem Geophysikalischen Institut Prag und der Grantagentur der AdW der Tschechischen Republik. Frau Marta Tučková, Marcela Švamberková und Herr Josef Telecký sei für ihre unschätzbare Hilfe in administrativen und technischen Dingen des Kolloquiums gedankt. Der Bibliothek des GeoForschungsZentrum Potsdam danken wir für die online Veröffentlichung unseres Kolloquiumsbands.

Oliver Ritter, Heinrich Brasse und Josef Pek

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Guided waves in marine CSEM and the adjustment distance in MT: a synopsis

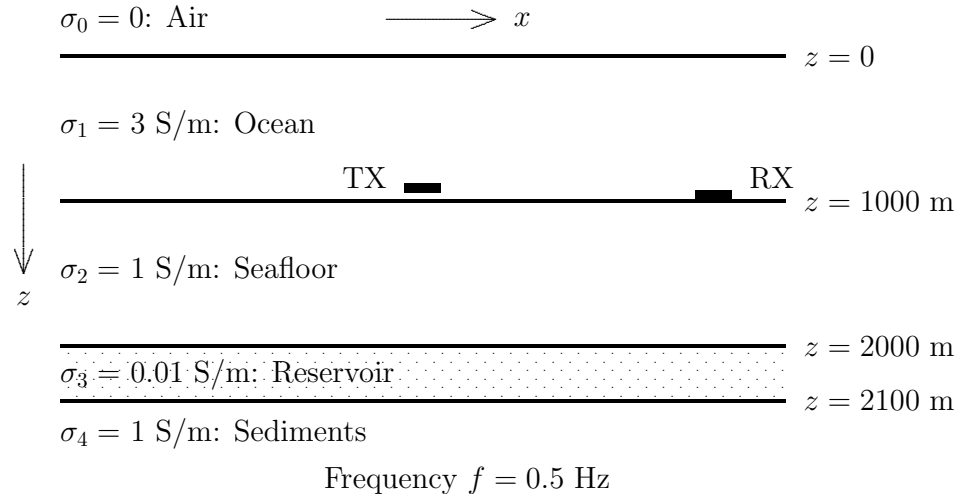
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1. Introduction

The Controlled Source Electromagnetic Method (CSEM) has recently found some interest in off-shore hydrocarbon exploration: An electric dipole with a transient or low-frequency continuous wave excitation is towed over an array of seafloor receivers measuring the electric and/or magnetic field. The target is a deep resistive layer as possible indicator for the presence of a hydrocarbon reservoir. The present study is confined to continuous wave excitation (with a typical frequency of 0.5 Hz) and to electric field data. A more detailed presentation of the CSEM results is given by Weidelt (2007).

Standard model of marine CSEM



Ways of energy propagation between TX and RX:

- direct
- along air-earth interface ('airwave')
- resistive layer mode

Figure 1: Experimental setup and standard conductivity model for CSEM. The receiver TX measures the electric field. Used is both the inline configuration (as shown) and the broadside configuration with RX parallel to TX, but off the plane on the y -axis.

It turns out that the model shown in Fig. 1 has much in common with the resistive-layer distortion of the electric field in magnetotellurics (MT) due to the presence of a conducting

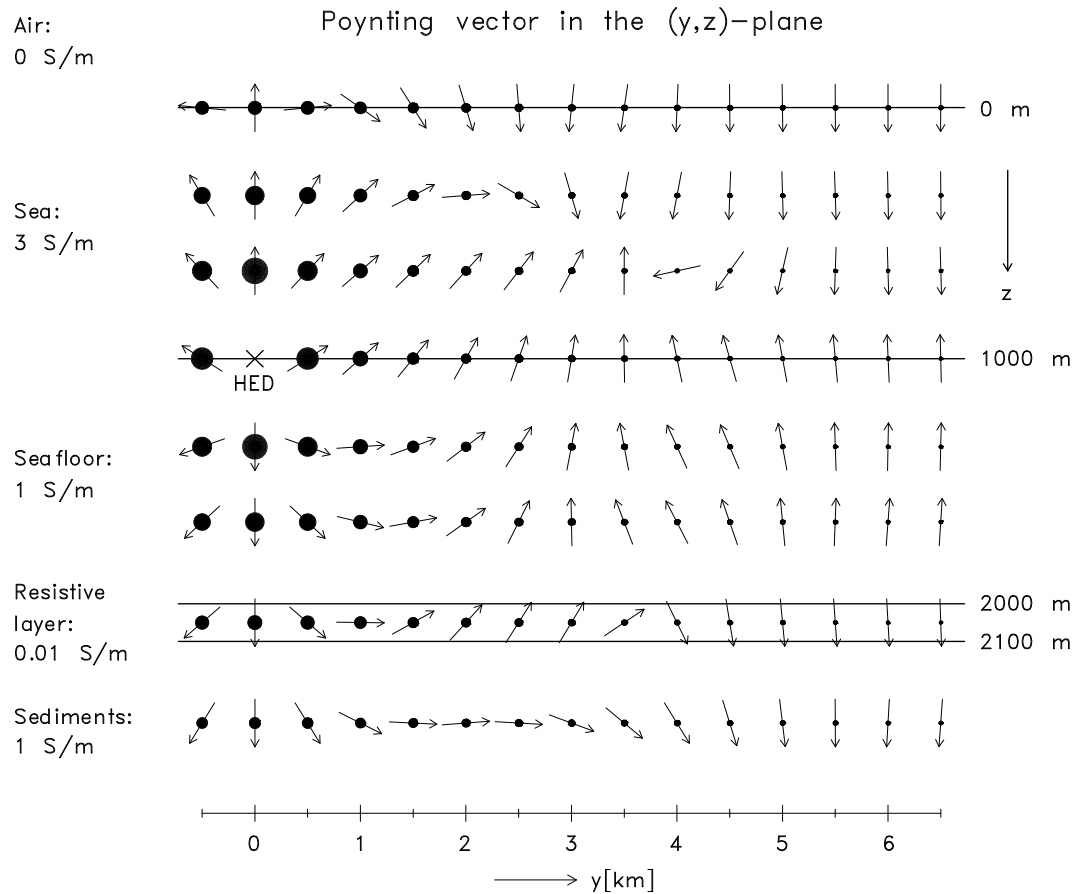
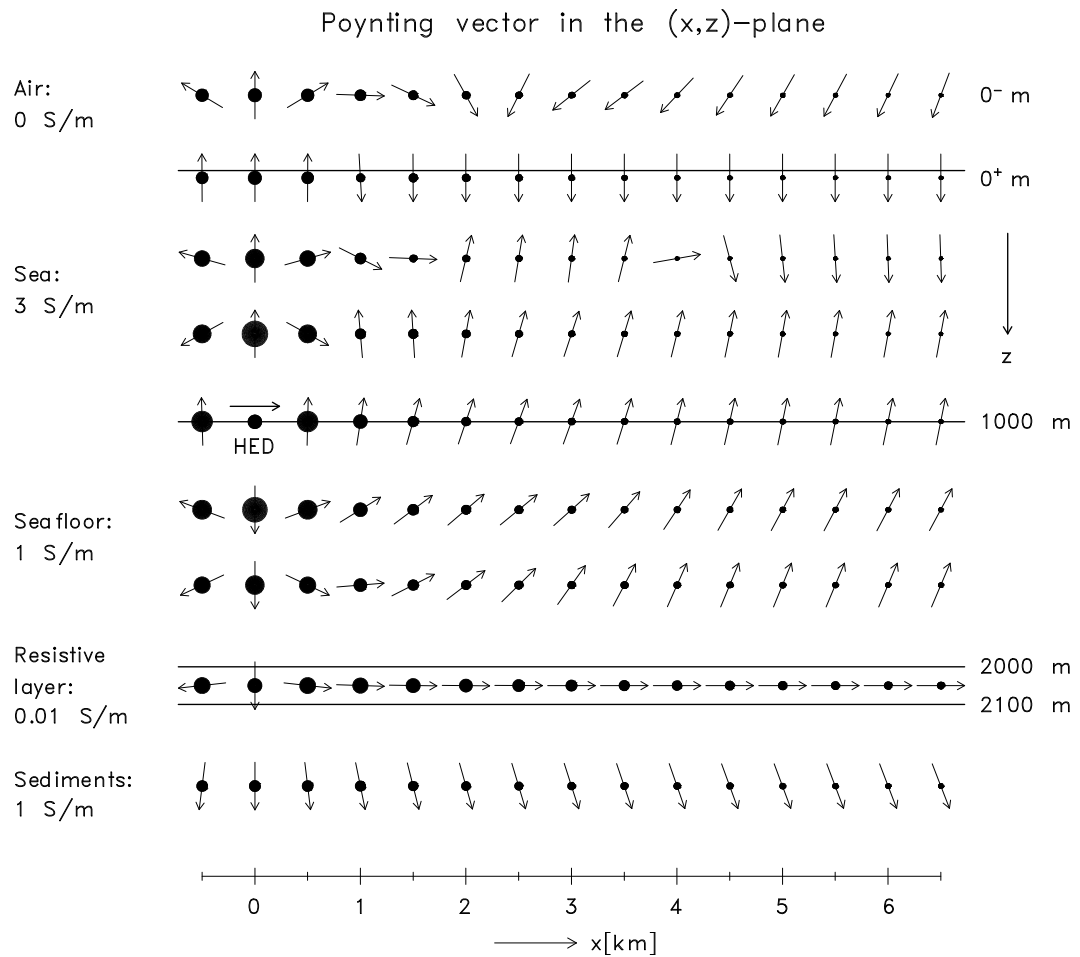


Figure 2: Energy flow density (Poynting vector) in two orthogonal planes

laterally nonuniform conductivity anomaly. Here the electric field can attain its asymptotic layered-earth values only at a very great distance from the anomaly. This distance is the so-called *adjustment distance*. It will be briefly discussed in Sect. 5.

For the representative model of Fig. 1, the time-averaged energy flow density (= Poynting vector) is shown in Fig. 2, both in the plane containing the horizontal electric dipole (top) and in the plane orthogonal to it (bottom) in Fig. 2.

The time averaged real Poynting vector

$$\bar{\mathbf{S}} := \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$$

reads in the (x, z) -plane at $y = 0$ (top of Fig. 2):

$$\bar{S}_x = -\frac{1}{2} \text{Re}(E_z H_y^*), \quad \bar{S}_z = +\frac{1}{2} \text{Re}(E_x H_y^*)$$

and in the (y, z) -plane at $x = 0$ (bottom of Fig. 2):

$$\bar{S}_y = -\frac{1}{2} \text{Re}(E_x H_z^*), \quad \bar{S}_z = +\frac{1}{2} \text{Re}(E_x H_y^*).$$

Of relevance for the physics of CSEM and the interpretation of seafloor data are two guided waves, namely the airwave and the resistive-layer mode:

- **Airwave**

The airwave is guided at the air-ocean interface with a decay $\sim 1/r^3$ for great TX-RX separations r . In Fig. 2 the airwave is dominant, where close to the interface the flow of energy is vertical. For shallow water depth the strong airwave masks the signal from the target layer. The airwave is a TE-mode. In marine CSEM it has to be considered as noise.

- **Resistive-layer mode**

This exponentially decaying mode (typical decay length 1700 m) is seen only in the plane containing the dipole (Fig. 2, top). It is associated with a strong horizontal energy flow, carried in the resistive layer by H_y and the strong component E_z . It is a TM-mode and contains the useful signal.

2. The airwave

In the sequel cylindrical coordinates (r, φ, z) are used. Attention is confined to the ‘inline configuration’, i.e. to the measurement of the radial component E_r in direction of the electric dipole. Moreover, the field is normalized with the current moment p of the dipole. Fig. 3 displays the modulus of $E_r(r)/p$ as a function of the TX-RX separation r for various depths d_1 of the sea water. In this log-log plot the airwave with its $1/r^3$ -decay is easily visible as a straight line section in the farfield. The resistive-layer mode is clearly visible only at the great water depth $d_1 = 1000$ m as the concave feature in the range $1 \text{ km} < r < 10 \text{ km}$; for shallower depth it is partly masked by the airwave.

For a 1D conductivity distribution $\sigma(z) > 0$ in $z > 0$ with source at depth z_0 , receiver at depth z , angular frequency $\omega = 2\pi f$ and current moment p , the leading term of the airwave is given by

$$E_r^{air}(\mathbf{r}) = \frac{i\omega\mu_0 p \cos \varphi}{2\pi r^3} \cdot \frac{e(z) e(z_0)}{[e'(0)]^2}, \quad E_\varphi^{air}(\mathbf{r}) = \frac{i\omega\mu_0 p \sin \varphi}{\pi r^3} \cdot \frac{e(z) e(z_0)}{[e'(0)]^2} \quad (1)$$

where $e(z)$ is the downward diffusing solution of

$$e''(z) = i\omega\mu_0\sigma(z)e(z). \quad (2)$$

Physically, $e(z)$ is the electric field in 1D magnetotellurics.

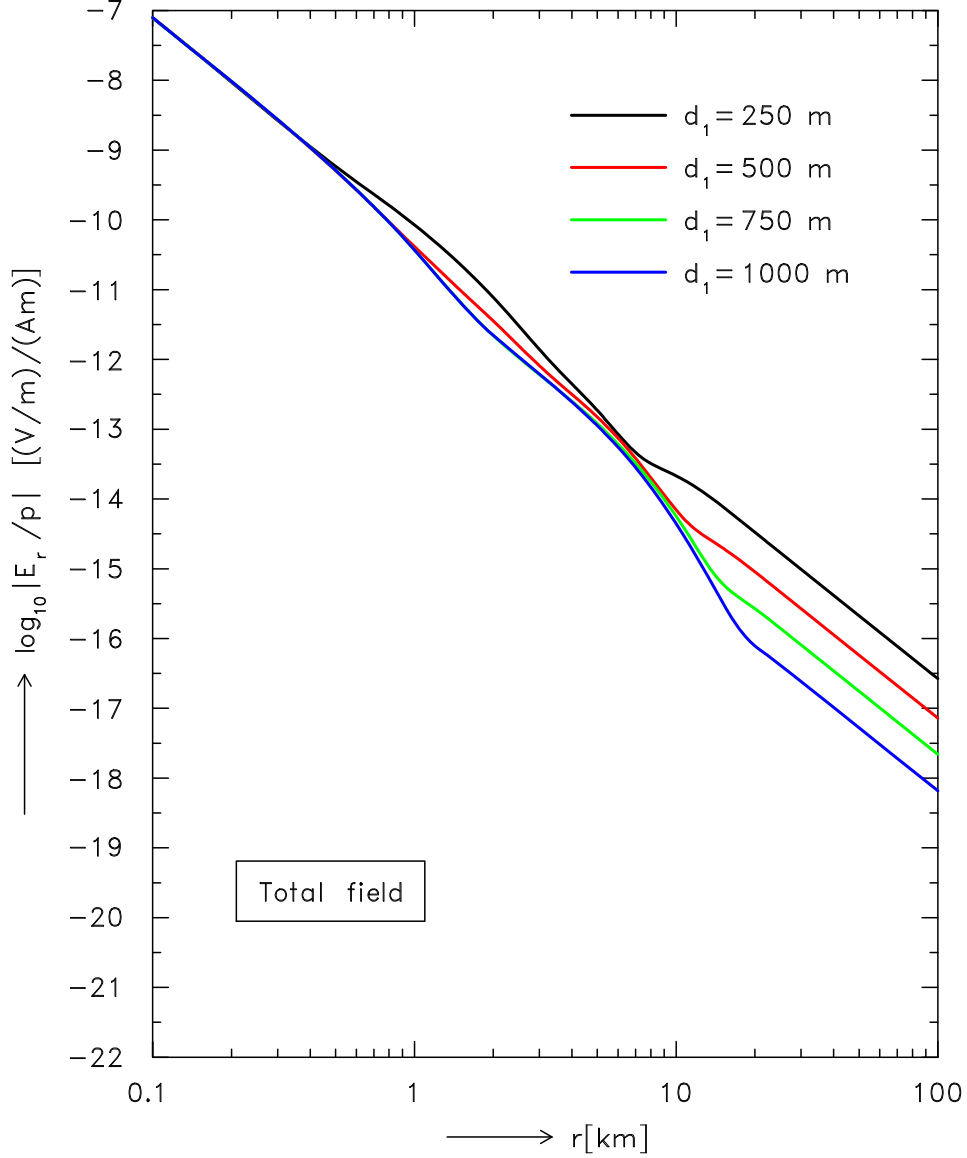


Figure 3: Radial component E_r of the electric field, normalized with the current moment p of the electric dipole for various water depths d_1 . Conductivity model of Fig. 1 with frequency $f = 0.5$ Hz.

The complete representation of the radial component E_r of a grounded horizontal electric dipole with TX at $\mathbf{r}_0$ and RX at $\mathbf{r}$ in cylindrical coordinates (r, φ, z) is

$$E_r(\mathbf{r}) = \int_0^\infty [Q_e(z|z_0, \kappa) (1/r) + Q_m(z|z_0, \kappa) \partial_r] J_1(\kappa r) d\kappa \cos \varphi, \quad (3)$$

where Q_e and Q_m describe, respectively, the contributions from TE- and the TM-mode. It is shown in Weidelt (2007)

- that due to the presence of the air-halfspace the TE-mode decays in powers of $1/r$ and
- that the TM-mode decays exponentially.

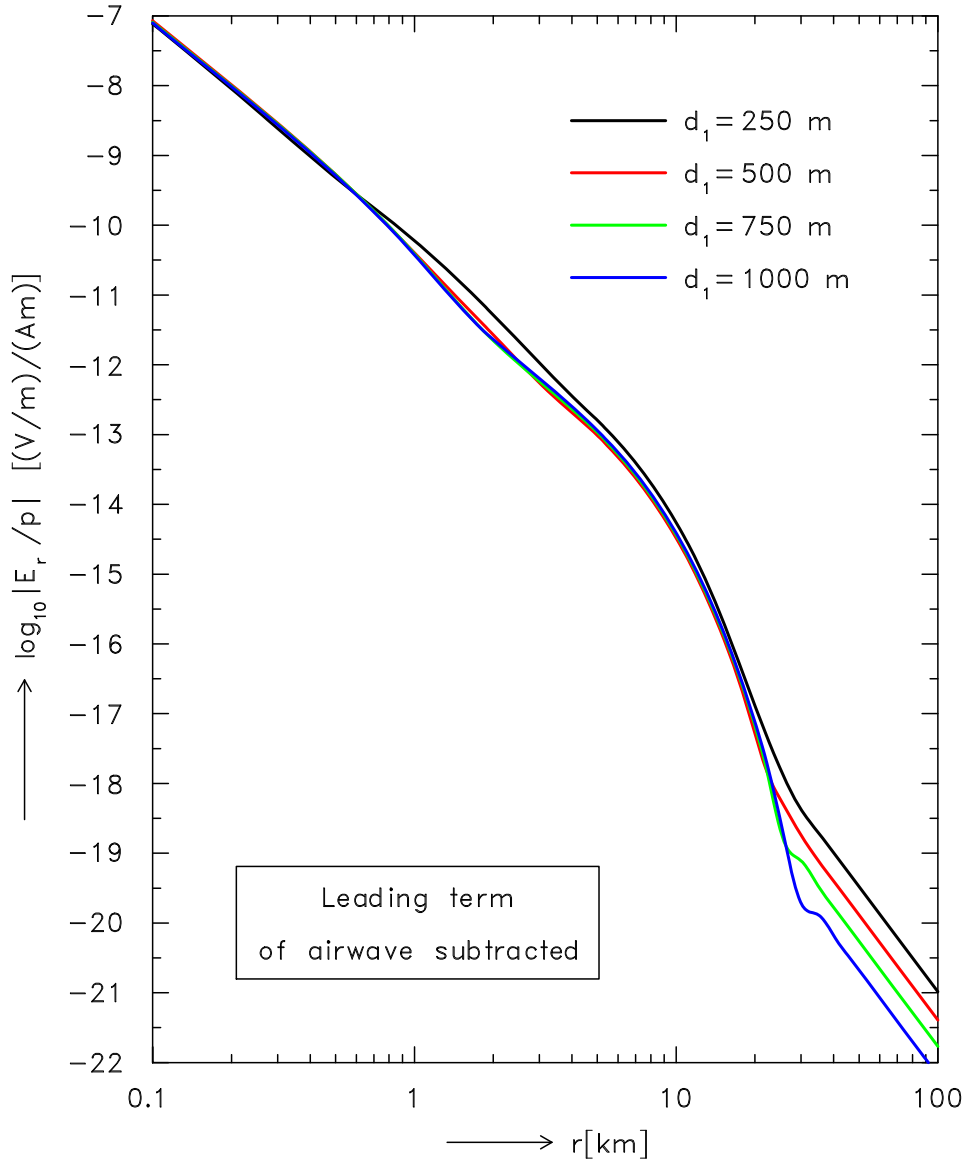


Figure 4: Subtraction of the leading airwave term from the data shown in Fig. 3. Now the resistive-layer mode becomes visible for all water depths d_1 . The subtraction removes only the farfield with its $1/r^3$ -decay, higher order terms $\sim 1/r^5$ evolve for $r > 20$ km. Air-ocean reflections with exponential decay, occurring at intermediate separations, are not eliminated.

The leading $1/r^3$ TE-mode term was given above. After the removal of the $1/r^3$ term, an asymptotic $1/r^5$ -term appears, etc. The *complete airwave* removes all algebraic asymptotic terms $\sim 1/r^{2n+1}$ (see Fig. 5) and can be described in terms of the TE-mode: If the TE-mode part of E_r is given by

$$E_{er}(\mathbf{r}) = \int_0^\infty Q_e(z|z_0, \kappa) J_1(\kappa r) d\kappa,$$

then the complete airwave reads

$$E_r^{air}(\mathbf{r}) = \frac{1}{i\pi} \int_0^\infty [Q_e(z|z_0, +it) - Q_e(z|z_0, -it)] K_1(tr) dt, \quad (4)$$

with $J_1(\cdot)$ and $K_1(\cdot)$ as Bessel functions in conventional notation.

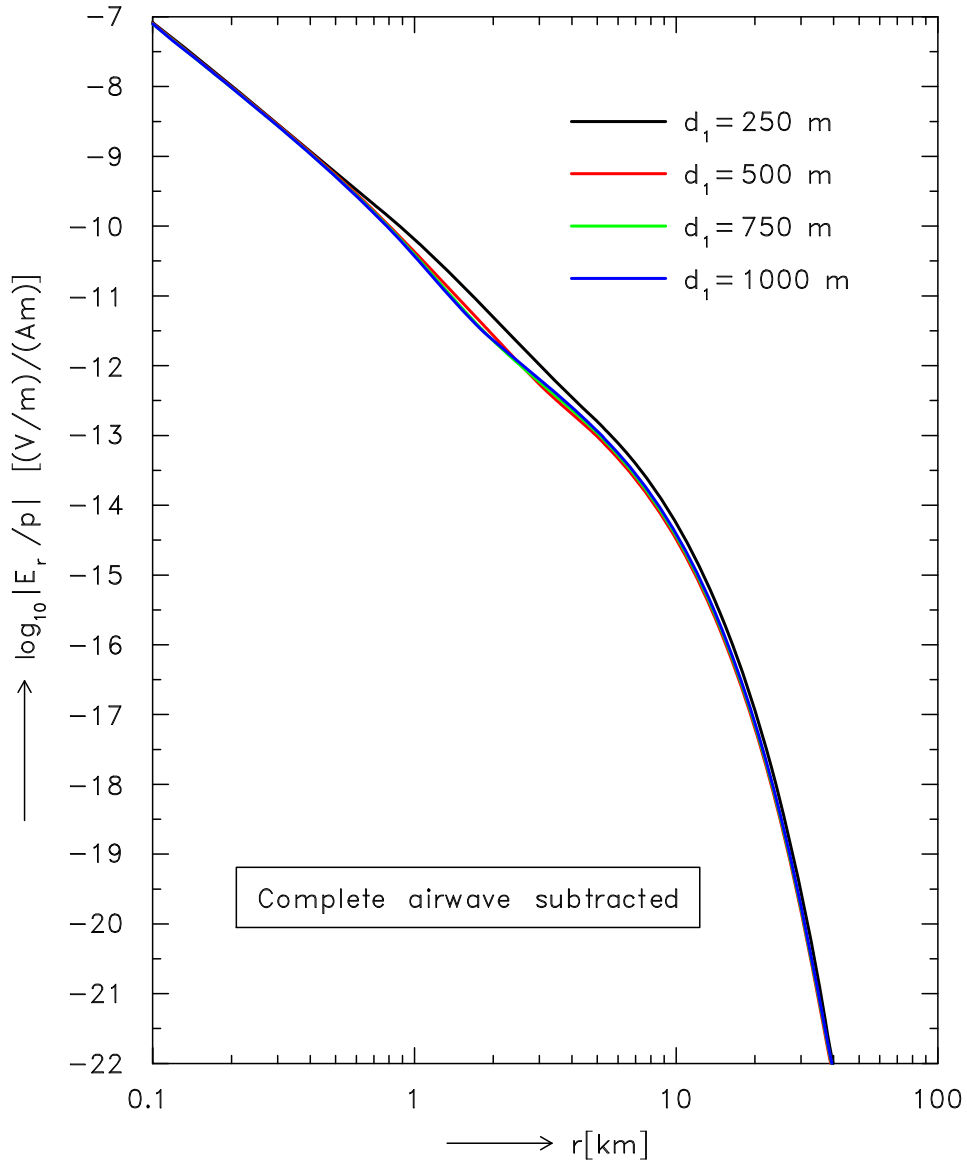


Figure 5: Subtraction of the complete airwave. The remainder approaches the exponentially decaying resistive-layer mode, see the blue lines in Figs. 6 and 12 (in the latter figure mostly coincident with the red line).

The successful removal of the airwave in Fig. 5 was possible only because we have assumed in Eqs. (1) and (4) a knowledge of the conductivity $\sigma(z)$. An approximate removal without recourse to $\sigma(z)$ is possible by observing in (1) that the leading term of the broadside configuration (E_φ at $\varphi = 90^\circ$) is twice as large as that of the inline configuration (E_r at $\varphi = 0^\circ$). Therefore, the quantity

$$E_r^{rem} := E_r(r, \varphi = 0^\circ) - (1/2)E_\varphi(r, \varphi = 90^\circ) \quad (5)$$

is free of the leading term of the airwave. Since the signal from the resistive layer is not present in the broadside component (see the bottom of Fig. 2), the difference field (5) is controlled in the farfield by the resistive layer mode and therefore presents a good approximation to the field obtained by removing the exact airwave (see Fig. 6).

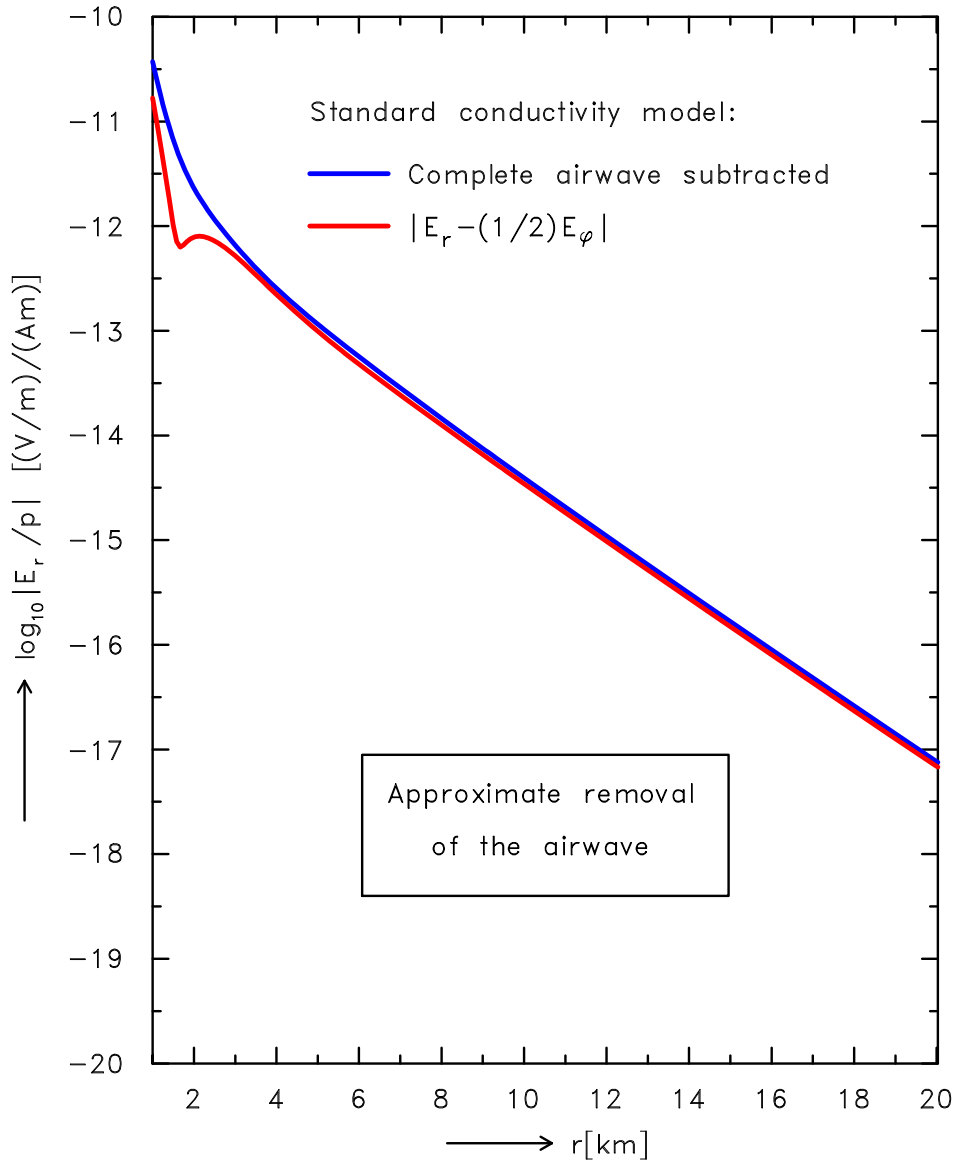


Figure 6: Approximate removal of the airwave according to Eq. (5). In the linear r -scale, the asymptotic exponential decay is evident in the blue line (corresponding to the blue line in Fig. 5).

3. The complex wavenumber plane

Usually, the field is obtained by superposing spectral terms with a real wavenumber κ in a Bessel function integral [as in (3)]. However, the extension of this superposition to complex wavenumbers clearly illuminates the nature of airwave and resistive-layer mode. If $f(\kappa)$ satisfies for real κ the symmetry $f(\kappa) = f(-\kappa)$, the extension from the positive κ -halfline to the complex upper κ -halfplane is performed via

$$\int_0^\infty f(\kappa) J_1(\kappa r) d\kappa = \frac{f(0)}{r} + \frac{1}{2} \int_{-\infty}^{+\infty} f(\kappa) H_1^{(1)}(\kappa r) d\kappa,$$

where $H_1^{(1)}(\cdot)$ is the Hankel function of first kind and first order. Above the indented real-line contour in Fig. 7 this function is analytic and decays exponentially.

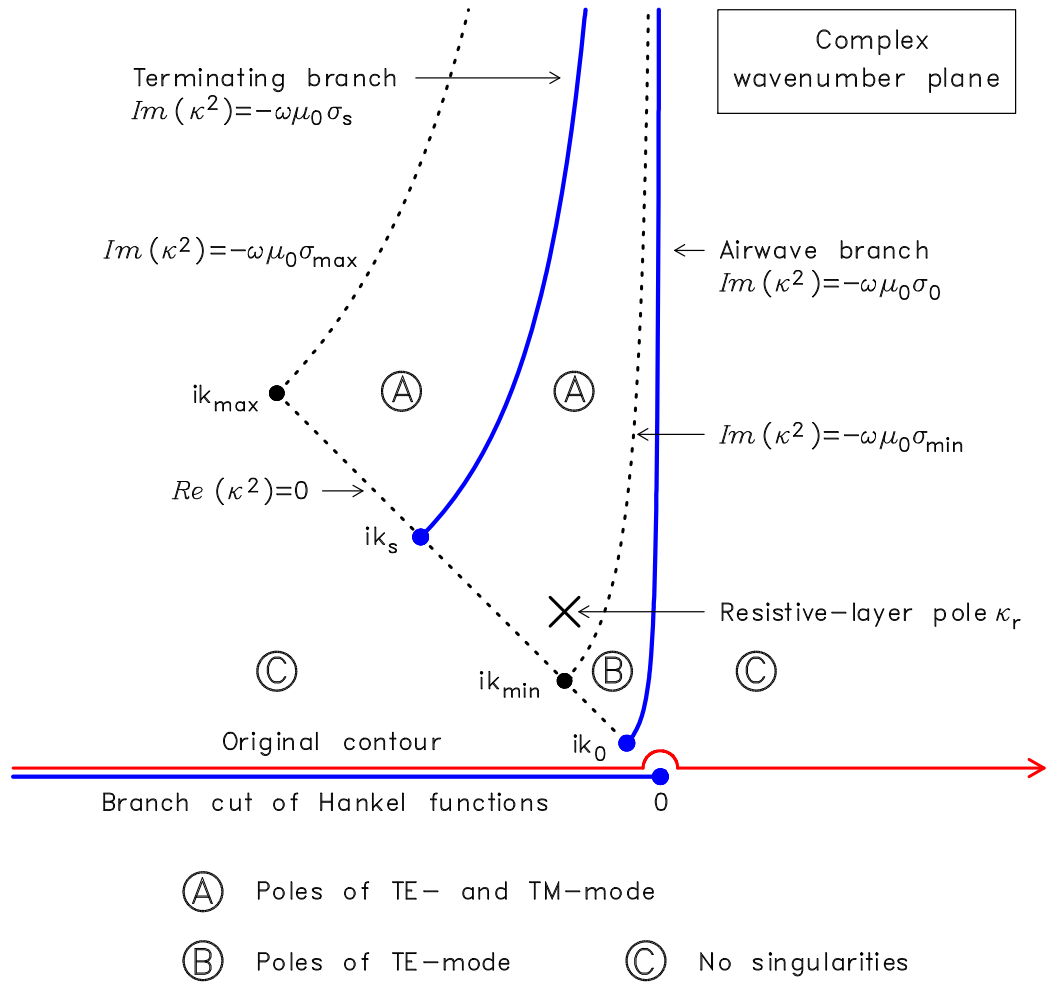


Figure 7: Analytical properties of the Bessel function integral kernel in the complex wavenumber plane. The full lines mark branch cuts. Inside the Earth the conductivity is assumed to vary in the limits $0 < \sigma_{\min} \leq \sigma(z) \leq \sigma_{\max} < \infty$. The branch points ik_0 and ik_s (referring to the limiting uniform halfspaces) and the endpoints $ik_{\min}$ and $ik_{\max}$ (referring to the limiting conductivities) are given by $\kappa = ik := (-1 + i)\sqrt{\omega\mu_0\sigma/2}$. The situation near $\kappa = 0$ is disentangled by assigning to the air a small positive conductivity σ_0 and considering the limit $\sigma_0 \rightarrow 0^+$.

The analytical properties of the integration kernel allow to deform the original contour of Fig. 7 to the contour shown in Fig. 8. This contour is of advantage, if highly precise farfield data are required, since – apart from the airwave – only exponentially decaying terms are summed, whereas the contour along the real axis obtains the farfield by the unstable process of destructive interference: For $r \rightarrow \infty$ each path element gives a contribution decaying exponentially $\sim \exp[-r\text{Im}(\kappa)]$. The TM-mode contour can be closed already along the path $\text{Im}(\kappa^2) = -\omega\mu_0\sigma_{\min}$ (see Fig. 7) and therefore, the TM-mode is a superposition of contributions with a strict exponential decay. The critical point $\kappa = 0$ contributes only to the TE-mode, where the resulting decay in powers of $1/r$ forms the airwave.

First principles of the computation in the complex wavenumber domain for TE-mode sources are given by Kaufman & Keller (1983, p. 432-445) and Goldman (1990, Ch. 2).

The actual position of the poles in the complex κ -domain for the standard conductivity model of Fig. 1 is shown in Fig. 9 for $f = 0.5$ Hz. The *position* of the poles is an intrinsic feature of $\sigma(z)$ and f and is independent of the position of source and receiver. The *residual*, however, depends on the position.

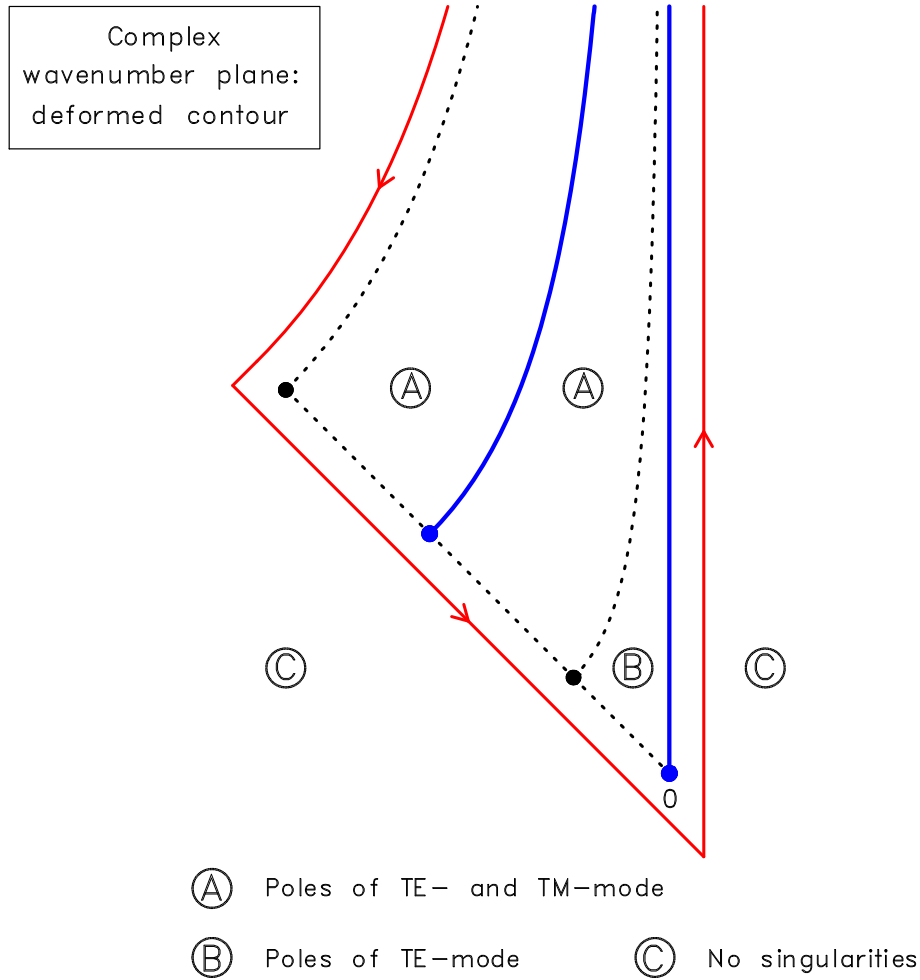


Figure 8: Deformation of the contour, originally confined to the real κ -axis (see Fig. 7), to a contour surrounding the region of singularities in the complex κ -plane. Contrary to Fig. 7, the air has again the conductivity $\sigma_0 = 0$. For $r \rightarrow \infty$ each path element gives a contribution decaying exponentially $\sim \exp[-r\text{Im}(\kappa)]$. The TM-mode contour can be closed already along the path $\text{Im}(\kappa^2) = -\omega\mu_0\sigma_{min}$ (see Fig. 7), and therefore the TM-mode is a superposition of contributions with a strict exponential decay. The critical point $\kappa = 0$ contributes only to the TE-mode, where the resulting algebraic decay in powers of $1/r$ forms the airwave.

Signatures of guided waves in the complex wavenumber plane are

- The ‘airwave branch’ (see Fig. 7) along the positive imaginary axis: The integral along both banks of the branch gives the complete pure airwave (4), considered as noise in marine CSEM.
- The resistive-layer pole κ_r : the residual at this point gives the resistive-layer mode, which is the signal of the target.

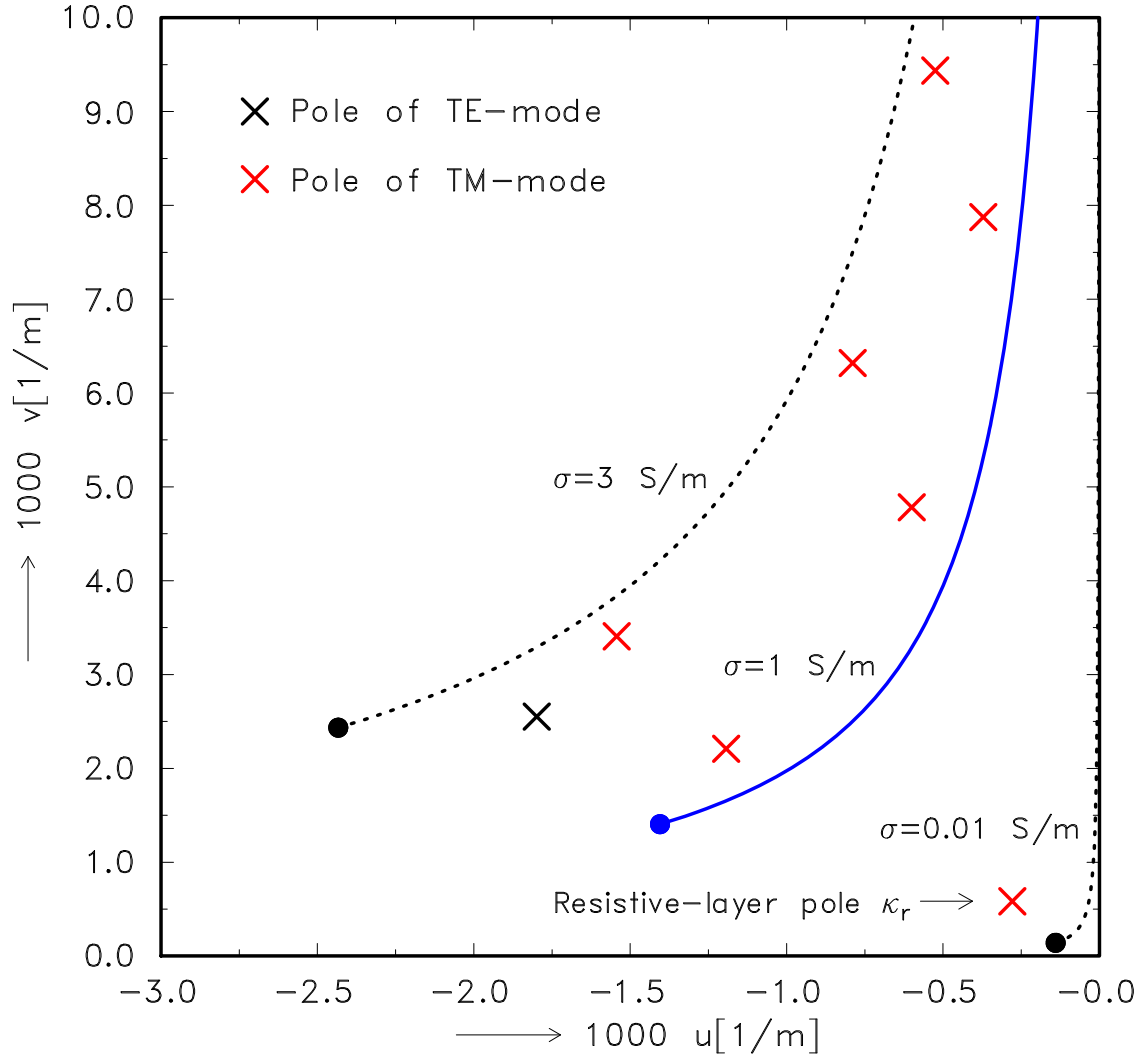


Figure 9: Actual position of the poles for the standard conductivity model of Fig. 1 in the complex plane $\kappa = u + iv$. The frequency is $f = 0.5$ Hz. Only the first six of an infinite number of TM-mode poles are shown. They describe highly damped reflections between air-ocean interface and the resistive layer. Also shown is the resistive-layer pole κ_r , an isolated TM-mode pole (close to the origin), which is responsible for the resistive-layer mode. – Note the different scales in u - and v -direction.

4. The resistive-layer mode

For general κ , the differential equation for the spectral TM-mode potential $f_m(z, \kappa)$,

$$\sigma(z)[f'_m(z)/\sigma(z)]' = [\kappa^2 + i\omega\mu_0\sigma(z)]f_m(z)$$

has as two *linearly independent* solutions an upward propagating solution $f_m(z) =: f_{ma}(z)$ vanishing for $z \rightarrow 0$ and a downward propagating solution $f_m(z) =: f_{mb}(z)$ vanishing for $z \rightarrow \infty$. At the poles of the TM-mode integral kernels, e.g. at $\kappa = \kappa_r$, the solutions become *linearly dependent*, such that

$$f_{ma}(z, \kappa_r) = f_{mb}(z, \kappa_r) =: f_{mr}(z)$$

is an eigensolution, which decays for $z \rightarrow 0$ and for $z \rightarrow \infty$. This particular eigensolution, with its peak amplitude in the resistive layer, is the *resistive-layer mode*. It is displayed in Fig. 10. Physically, $f_m(z, k)$ is proportional to the spectral vertical current density $J_z(z, \kappa)$ [therefore $f_{ma}(0, \kappa) = 0$].

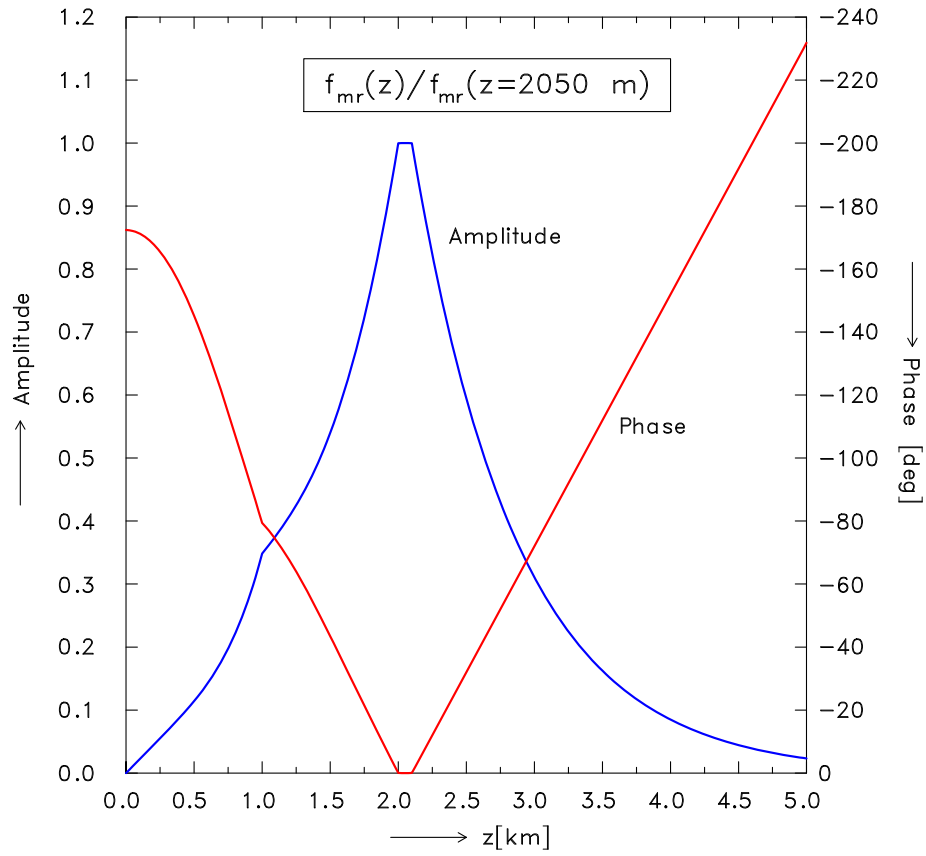


Figure 10: The eigensolution $f_{mr}(z)$ of the resistive-layer mode for the standard conductivity model of Fig. 1. The slope discontinuities at $z = 1$ km mark the seafloor.

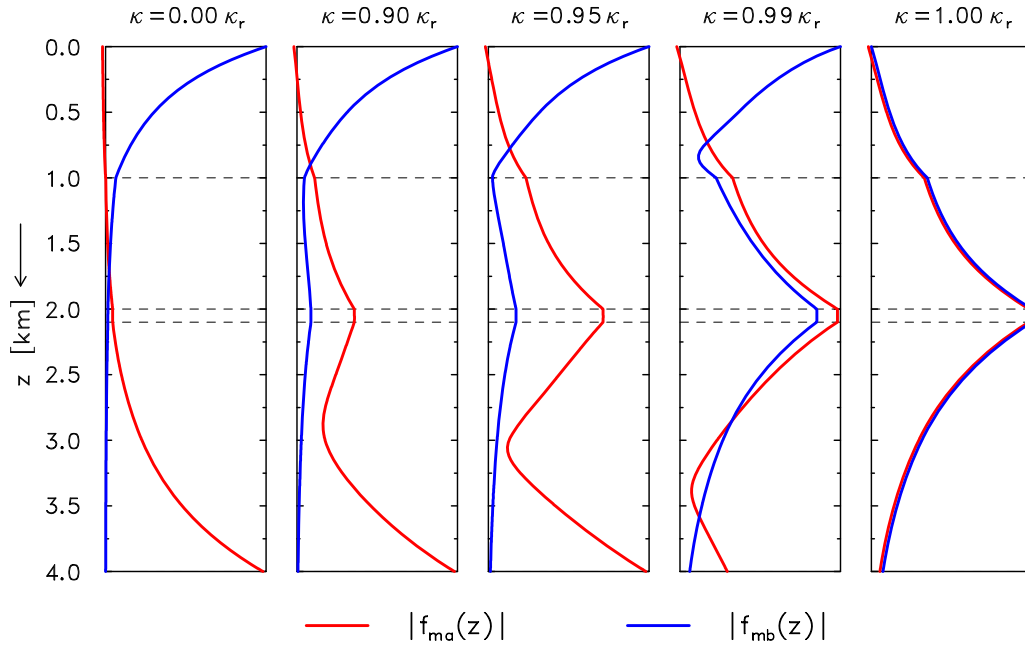


Figure 11: The transition from linear independence to linear dependence when approaching $\kappa = \kappa_r$ along a straight line from the origin $\kappa = 0$. The dotted lines mark the layer boundaries.

The resistive-layer pole lies at

$$\kappa_r = (-2.788726 + i 5.831761) \cdot 10^{-4} \text{m}^{-1},$$

corresponding to a scale length of

$$L_r = 1/\text{Im}(\kappa_r) = 1714.75 \text{ m}.$$

The resistive-layer mode $E_r^{rlm}(\mathbf{r})$, which is the residual at κ_r , is shown in Fig. 12 as the red line. Apart from a factor, independent of the position of TX and RX, it is given by

$$E_r^{rlm}(\mathbf{r}) \sim \frac{f'_{mr}(z)f'_{mr}(z_0)}{\sigma(z)\sigma(z_0)} \cdot \partial_r H_1^{(1)}(\kappa_r r) \cos \varphi.$$

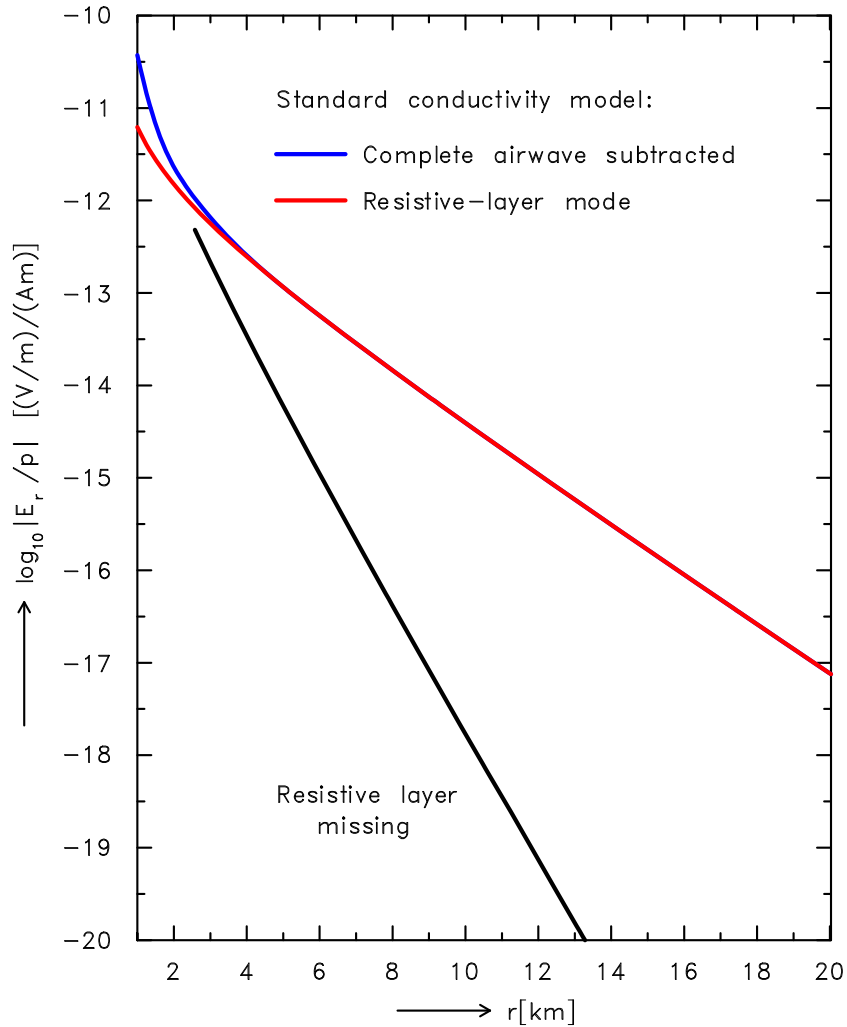


Figure 12: The blue line (coinciding in the farfield with the red line) represents the same information as the blue lines in Figs. 5 and 6. In the linear r -scale the asymptotic exponential character becomes obvious. The farfield agreement with the resistive-layer mode (red line) shows that the latter provides an excellent farfield approximation of E_r freed from the airwave. After the removal of the airwave, the distinction from a model without the resistive layer becomes easily possible.

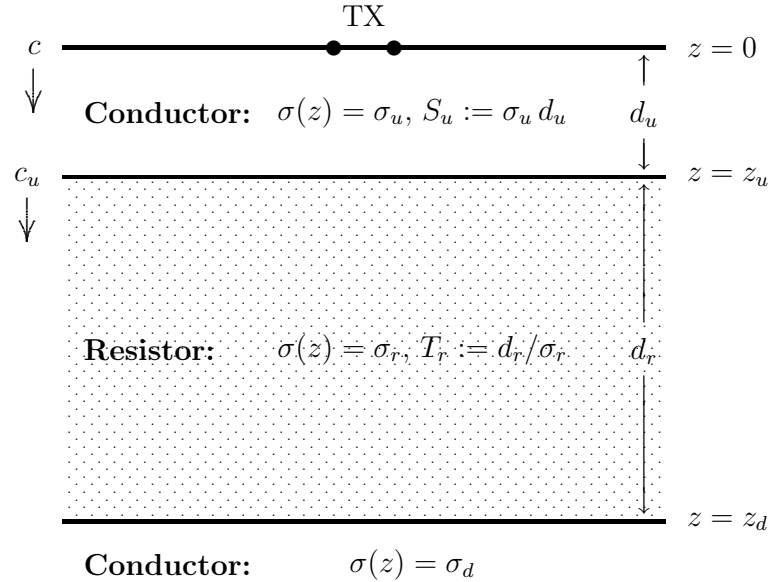
5. The adjustment distance in MT

If $\sigma(z) > 0$ in $z > 0$, all horizontal components of the electric and magnetic field decay at great separations r between TX and RX for magnetic dipoles (HMD and VMD) and for horizontal electric dipoles $\sim 1/r^3$. Therefore the surface impedance as ratio between orthogonal components of $\mathbf{E}$ and $\mathbf{H}$ tends at great separations to the plane-wave limit. The scale length for reaching this limit is the adjustment distance L_a . In general, L_a is in the order of the modulus Schmucker's complex inductive scale length $c(\omega)$, defined for a layered earth as

$$c(\omega) := c(0, \omega) \quad \text{and} \quad c(z, \omega) := -E_h(z, \omega)/E'_h(z, \omega).$$

Here E_h is a horizontal electric field component. There is a notable exception, however, where the farfield will start at considerably greater separations. This is – in the simplest case – a three-layered earth with a (thick) resistive layer sandwiched between a (thin) conductive overburden and a conductive substratum. The corresponding conductivity model is shown in Fig. 13.

Adjustment distance: Typical conductivity model



DC-estimate of adjustment distance (Ranganayaki & Madden 1980):

$$L_a \approx \sqrt{S_u T_r}$$

Inductive estimate of adjustment distance (Fainberg & Singer 1987):

$$L_a = \frac{1}{\text{Im}(\kappa_a)} \approx \frac{\sqrt{S_u T_r}}{\text{Re}\sqrt{c_u/c}} \leq \sqrt{S_u T_r}$$

$$\kappa_a^2 \approx -\frac{c_u/c}{S_u T_r}$$

Figure 13: Conductivity model and experimental setup, in which the plane-wave impedance is reached only at great separations, essentially due to 2D propagation at small and moderate separations.

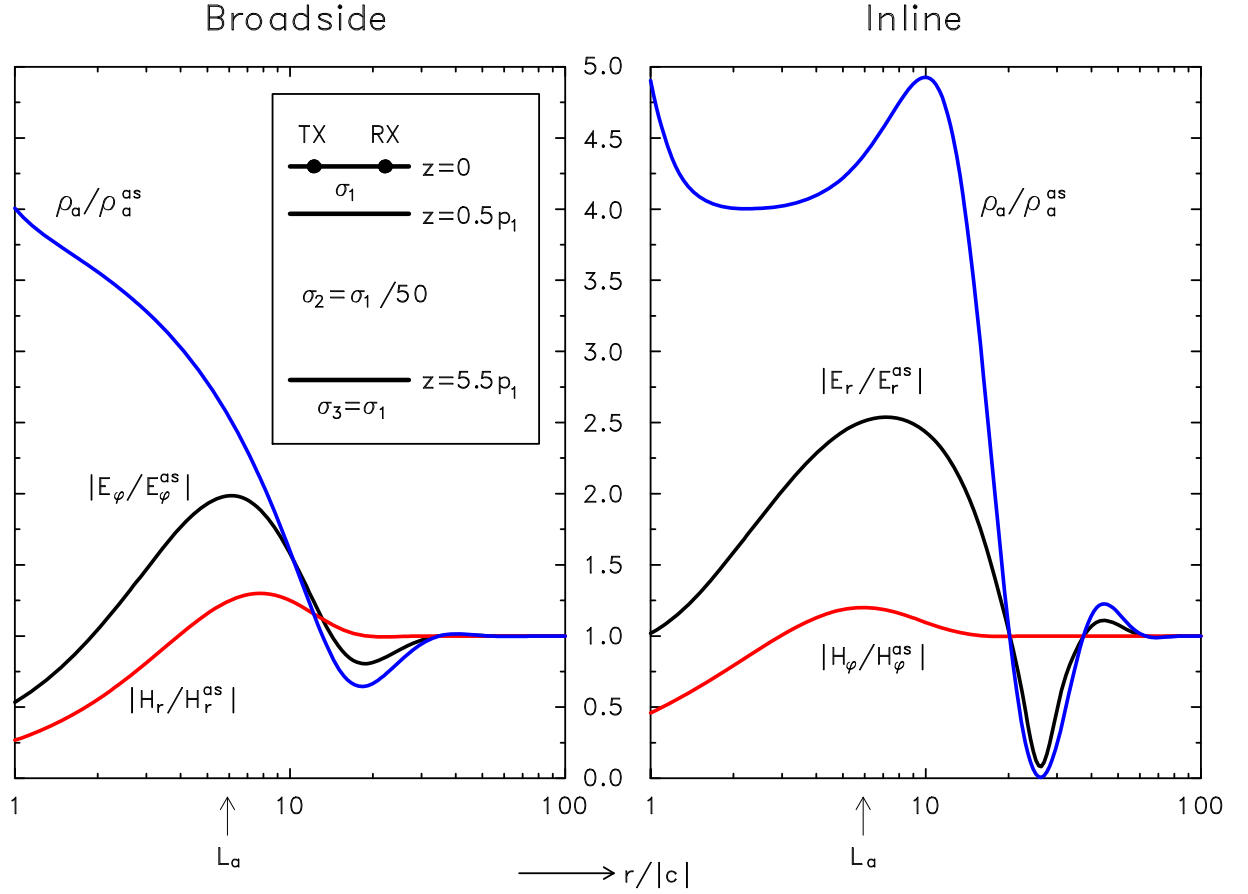


Figure 14: Horizontal electric and magnetic field components of a HED, normalised with their asymptotic values (superscript *as*) decaying $\sim 1/r^3$ (= airwave). Because of the $1/r^2$ -decay of E_ϕ and E_r at small and intermediate separations, the *normalized* values of the electric field components increase with r up to $r \approx L_a$ and then sharply decrease because of the exponential decay of the TM-mode part.

The source is a HED in or on the conductive overburden. Due to the galvanic contact, the currents are at small and moderate distances confined to the conductive overburden. The reduced dimensionality leads to TM-mode electric dipole fields decaying first only $\sim 1/r^2$. At greater separations the vertical electric currents of the source dipole will penetrate the resistive central layer. When sensing the conductive substratum, the TM-mode currents decay exponentially rather than $\sim 1/r^2$. This exponential TM-mode decay is valid under the assumption that $\sigma(z) > 0$ in $z > 0$. If this condition is violated by a perfectly *insulating* intermediate layer, the $1/r^2$ -decay of the TM-mode field would perpetuate for all separations. If this condition is satisfied only marginally by a (highly) resistive intermediate layer, the exponential decay would be present, but would start at great separation, which is quantified by the *adjustment distance* L_a . The galvanic currents will stay the longer in the conductive overburden the better the overburden conductivity σ_u and the smaller the resistive-layer conductivity σ_r . At separations $r \gg L_a$, the TM-mode electric field has disappeared and TE-mode part, decaying always $\sim 1/r^3$, will prevail.

The consideration of the adjustment distance will certainly be relevant for Controlled Source Audio Magnetotellurics (CSAMT) over a conductivity structure of the type sketched in Fig. 13. Moreover, the adjustment distance plays also a role in ordinary MT, since the anomalous electric fields caused by lateral conductivity heterogeneities can be represented as a superposition of HED fields with sources in the anomalous domain and with amplitudes controlled by the conductivity contrast.

For MT the first estimate of L_a was given by Ranganayaki & Madden (1980), who showed that

$$L_a \approx \sqrt{S_u T_r}, \quad (6)$$

where $S_u := \sigma_u z_u$ is the conductance of the overburden and $T_r := d_r/\sigma_r$ is the integrated resistivity of the intermediate layer (see Fig. 13). The simple and useful estimate (6) reflects already correctly the dependence on σ_u and σ_r as anticipated above. It is a direct current approximation, which does not yet take induction into account. This was first achieved in the approximation of Fainberg & Singer (1987), who relate L_a to the TM-mode pole κ_a with the smallest imaginary part. This pole is approximately positioned at

$$\kappa_a^2 \approx -\frac{c_u/c}{S_u T_r}, \quad (7)$$

such the exponential radial decay $\sim \exp[-r\text{Im}(\kappa_a)]$ yields

$$L_a = \frac{1}{\text{Im}(\kappa_a)} \approx \frac{\sqrt{S_u T_r}}{\text{Re}\sqrt{c_u/c}}. \quad (8)$$

Here c and c_u are the inductive scale lengths at $z = 0$ and $z = z_u$, respectively. Approximately we have

$$c_u/c \approx 1 + i\omega\mu_0 S_u c_u. \quad (9)$$

Since $\text{Im}(c_u) < 0$, Eq. (9) implies $\text{Re}\sqrt{c_u/c} \geq 1$. Therefore the consideration of induction according to (8) and (9) leads to a reduction of the DC estimate (6). A numerical example will be given below.

The adjustment distance is illustrated in the simple example of Fig. 14. The source is a HED at $z = 0$. The left panel shows the *broadside* configuration (with bipoles of TX and RX pointing into the figure) and the right panel shows the *inline* configuration (with TX and RX bipoles in the plane of the figure). The field components are normalized by their asymptotic farfield values, denoted by the superscript *as*. Up to separations $r \approx L_a$, the normalized electric field increases with r because E is dominated by the TM-mode part decreasing $\sim 1/r^2$, whereas $E^{as} \sim 1/r^3$. For $r > L_a$, the exponential decrease of the TM-mode leads to a sharp decrease of E , which passes through a pronounced minimum before it is well presented by its asymptotic values. In the present model, E_ϕ and E_r reach the asymptotic regime only for $r \approx 40|c|$ and $r \approx 50|c|$, respectively. Shown are also the corresponding normalized apparent resistivities ϱ_a/ϱ_a^{as} . The reason for the pronounced minimum will be discussed below.

In the present example, the DC estimate (6) yields $L_a/|c| = 11.31$. The reduction of this estimate due to induction is remarkable: The Fainberg-Singer estimate gives $L_a/|c| = 5.61$, which is not too different from the true value $L_a/|c| = 5.92$, obtained by exactly determining the κ_a as the pole with the smallest imaginary part (see Fig. 15).

Taking again the model of Fig. 14 as an example, Fig. 15 shows both the exact locations of the first six TM-mode poles and the line $\text{Im}(\kappa^2) = -\omega\mu_0\sigma_r = -\omega\mu_0\sigma_2$, on which the poles are located for the case that the resistive layer is sandwiched between two *perfect* conductors. The separation of poles decreases for increasing thickness d_r of the resistive layer and increases for decreasing thickness. Therefore, only the single pole κ_r is present in the marine CSEM model with a *thin* resistive layer (see Fig. 9). Fig. 15 shows also the first six poles of an infinite sequence of TE-mode poles. In particular the TE-mode pole with the smallest imaginary part plays a role for the decay of the E-field at intermediate distances, but finally the leading TE-mode term from the airwave branch will dominate the pole contributions.

Adjustment distance model: Pole positions

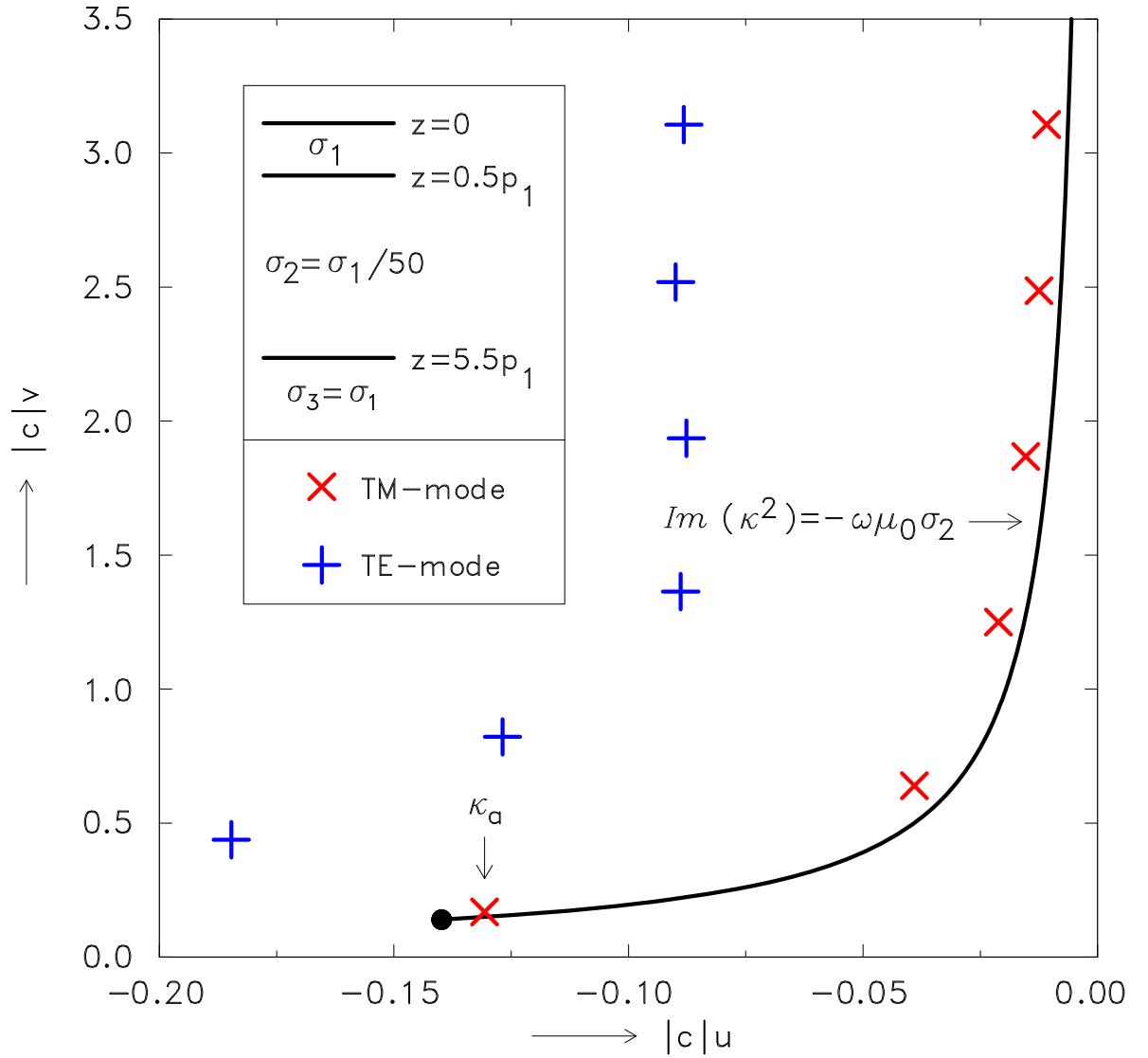


Figure 15: TM- and TE-mode poles for the conductivity model assumed in Fig. 14 and repeated here for convenience. The TM-mode pole κ_a gives rise to the adjustment distance $L_a = 1/\text{Im}\kappa_a$. The line $\text{Im}(\kappa^2) = -\omega\mu_0\sigma_2$ corresponds to the line $\text{Im}(\kappa^2) = -\omega\mu_0\sigma_{min}$ in Fig. 7. Note the different scales in u - and v -direction.

At the pronounced $|E_r|$ -minimum at $r/|c| \approx 26$, the contributions from TE- and TM-mode are approximately of the same size, but of different sign, $E_r^{(e)}/E_r^{(m)} = -1.053 + i 0.068$, such that the contributions almost annihilate.

The *tertium comparationis* between the resistive-layer mode in CSEM and the adjustment distance in MT is a resistive layer sandwiched between two conductive layers, which in both cases gives rise to pronounced energy propagation in the resistor with well-defined long decay length L_r and L_a . Both scale lengths are associated with a TM-mode pole close to the origin $\kappa = 0$. The airwave in CSEM corresponds to the asymptotic behaviour of the MT fields. Whereas unwanted noise CSEM, the asymptotic field provides a safe reference in MT.

Facit

- The contribution discusses the relevant guided waves in marine CSEM with restriction to frequency sources and electric fields for 1D conductivity models. The emphasis lies on fundamental principles rather than on practice orientation. The results are given without an attempt of an explicit derivation.
- A simple general expression is given for the leading term of the airwave for an arbitrary 1D conductivity distribution.
- Working in the complex wavenumber domain and locating poles and branch points allows
 - to isolate the complete airwave and the resistive-layer mode;
 - to compute highly precise farfield data;
 - to infer immediately that TM-mode contributions will decay exponentially, whereas TE-mode contributions may show a decay in powers of $1/r$;
 - to quantify decay lengths of individual field components.
- Fig. 12 shows that – for a representative model – the superposition of airwave and resistive-layer mode provides an excellent description of the electric field over a wide range of separations.
- For the model with a sandwiched resistor between conductors, an intimate connection is established between CSEM and MT, where the TM-mode pole with the smallest imaginary part defines the appropriate scale length of radial decay. This is strictly valid only for CSAMT, but is of relevance also for the *anomalous* fields of lateral conductivity anomalies generated by plane-wave sources (as assumed in MT).

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Invariants of rotation of axes and indicators of dimensionality in magnetotellurics

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SUMMARY

A magnetotelluric tensor from a particular site is taken as an example and analysed in terms of invariants of rotation of the measuring axes. The invariants presented range from the results of principal value decompositions of the real and quadrature parts taken separately to the results of phase tensor analysis. Attention is paid especially to those invariants which indicate dimensionality. Estimates are also obtained for 2D strike angle.

Key words: magnetotellurics, dimensionality, invariants, Mohr diagrams

1 INTRODUCTION

Central to a magnetotelluric study of Earth structure is the determination, from field observations at an array of sites, of values of the magnetotelluric impedance tensors for those sites. Often the interpretation of such observed tensors is straight-forward, enabling the magnetotelluric study to proceed to completion.

Sometimes, however, individual sites may appear anomalous, and need extra attention before their interpretation can proceed. In such cases, calculating and displaying invariants

of rotation may be helpful in understanding perplexing characteristics. The present paper gives graphical analyses of a selected example. While the techniques may be most useful in complicated cases, the example presented is relatively simple, as a simple example makes a good introductory case.

The example is presented against a wider background. The procedure for 1D inversion is always based on an invariant (the observed 1D impedance). Similarly 2D inversion is commonly based on the TE (E-pol) and TM (B-pol) impedances, which in this paper are emphasized as invariants of rotation. As the subject of magnetotelluric interpretation advances further into 3D inversion and modelling, the question of which parameters to invert, from a wide range of possible candidates including notably invariants, may be expected to need frequent re-visiting.

2 INVARIANTS OF ROTATION

The significance of invariants of rotation in magnetotelluric interpretation has been recognized for some time (Ingham 1988; Park & Livelybrooks 1989; Fischer & Masero 1994; Lilley 1998). In recent years Szarka & Menvielle (1997) and Weaver et al. (2000) have investigated sets of seven invariants which, together with an eighth value in the form of a geographic bearing, have been needed to fully describe a complex magnetotelluric tensor of eight elements.

The development of phase tensor analysis by Caldwell et al. (2004), and see also Bibby et al. (2005), led Weaver et al. (2003) to present and discuss three invariants (J1, J2 and J3) which arise in phase tensor analysis. Weaver et al. (2003) was reprinted as Weaver et al. (2006).

The recognition that just three invariants carry much important information in many practical situations is now explored in the example of this paper. The three invariants are calculated and displayed as functions of period. For comparison, a variety of other invariants are also displayed, notably the principal decomposition values of Lilley (1998), and the seven invariants of Weaver et al. (2000).

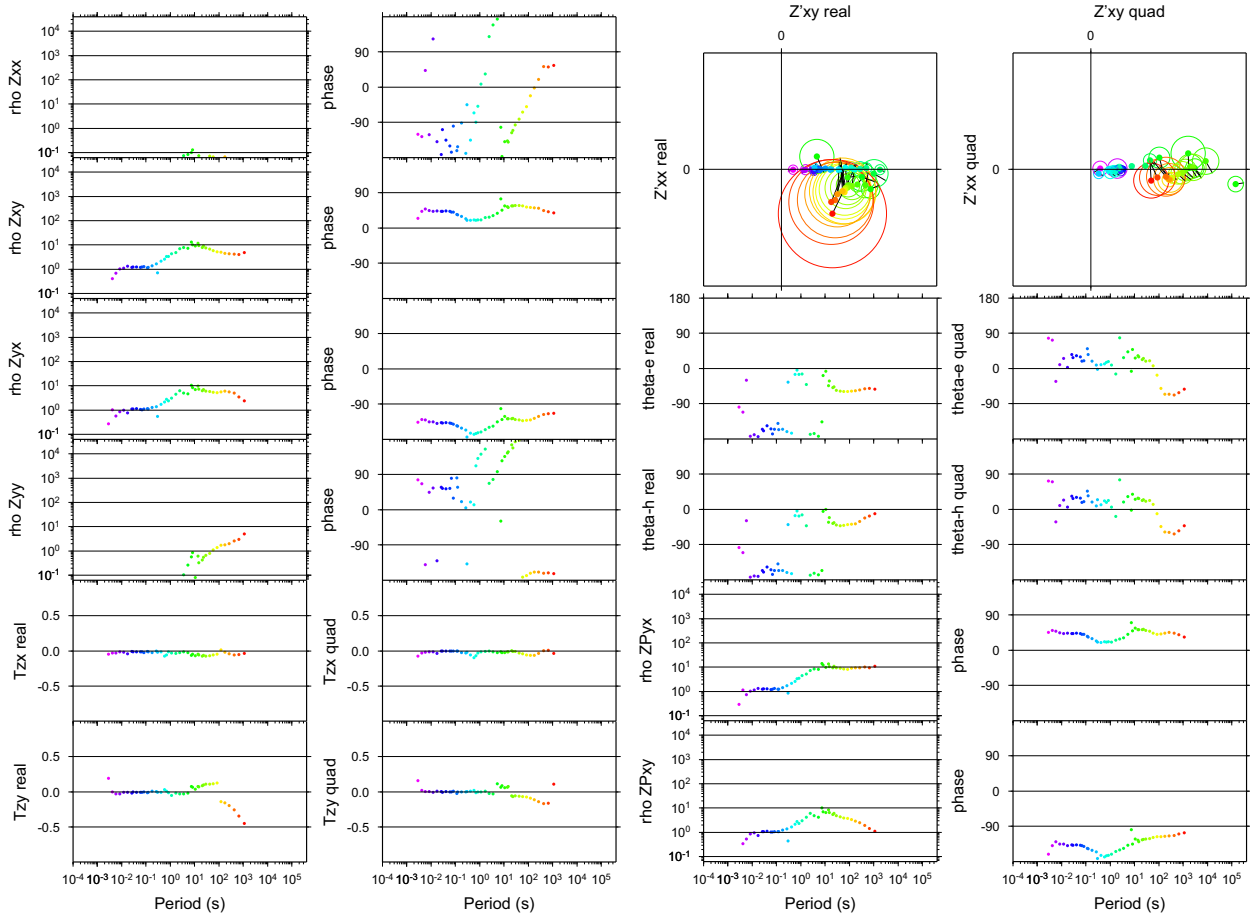


Figure 1. The mimic003 data and their basic decomposition. The units of apparent resistivity (ρ_{Zxx} , ρ_{Zxy} , ρ_{Zyx} , ρ_{Zyy} , ρ_{ZPyx} and ρ_{ZPxy}) are ohm m, and angles of phase and direction are given in degrees. For the Mohr circles, each tensor element value has been scaled by multiplication by the square root of the period, to make the plot of a set of circles more compact.

3 THE EXAMPLE FROM AUSTRALIA

3.1 Data and basic decomposition

The example is from a sedimentary basin in Australia. It is site mimic003 in the Magnetotelluric Investigation of the Mt Isa Crust (MIMIC) experiment of 1997 (Wang 1998; Lilley et al. 2003).

There are two figures presenting results for this site. The first (Figure 1) shows, in its far-left-hand panel, apparent resistivity and phase values calculated from the Z_{xx} , Z_{xy} , Z_{yx}

and Z_{yy} tensor elements as observed. The adjoining graphs in the centre-left panel show corresponding phase values (the T_{zx} and T_{zy} graphs are not part of the present discussion). The ρ_{Zxy} and ρ_{Zyx} amplitudes are appropriate for a sedimentary basin, being similar and showing an increase with period (and depth) from a conductive surface layer; they diverge at the longest periods. The Z_{xy} and Z_{yx} phases are in the appropriate quadrants, and again differ from each other only at long periods, consistent with the 1D sedimentary basin where they were recorded. The ρ_{Zxx} and ρ_{Zyy} amplitudes are generally small, and the phase values generally scattered, again consistent with 1-D behaviour (except at the long-period end of the spectrum, where departure from one-dimensionality below the sedimentary basin is detected).

The centre-right and far-right panels for this figure show, at the top, Mohr circles for the magnetotelluric data. These circles are generally of small diameter (scaled by the distance of the circle centre from the origin of the plot), and are centred on or near the horizontal axes, again indicating one-dimensionality of electrical conductivity structure. In the circles, invariants of rotation of the measuring axes become evident. For example the centres of the circles are fixed by the observed data, and would not change even were the measuring axes to be rotated and so aligned differently.

In the case of an ideal 2D structure, the E-pol and B-pol impedances are given by the two points where the circle (itself now centred on the horizontal axis) cuts the horizontal axis. Once determined, these points of intersection which indicate E-pol and B-pol values do not change with measuring axis rotation, and are invariants of rotation.

The panels below the circles show the results (centre-right : real; far-right : quad) of principal value decompositions of the magnetotelluric tensor, taking real and quad parts separately (Lilley 1998). The situation is envisaged where an ideal two-dimensional tensor is measured using axes (say aligned north and east) relative to which the geologic strike has bearing θ_h , and the electric field is distorted from its 2D direction by an angle (θ_e minus θ_h). Then rotating the magnetic axes by angle θ_h and the electric axes by angle θ_e recovers the original two-dimensional tensor, and its E-pol and B-pol values.

Using equations (107) - (114) of Lilley (1998), the panels show the theta-e and theta-h values for the mimic003 example, and below them, values of rhoZPyx and rhoZPxy respectively (centre-right : amplitude; far-right : phase). In the calculation of the rhoZPyx values, the real and quad parts of ZPyx have first been combined in the usual way to give ZPyx amplitudes, even though the real and quad parts of ZPyx may have resulted from different values of theta-h and theta-e. The same applies to ZPxy and rhoZPxy. For the simple 2D distortion case described, such rhoZPyx and rhoZPxy results are the undistorted E-Pol and B-pol values (possibly interchanged).

Examining the results plotted for the mimic003 example, it is evident that neither theta-e nor theta-h are well determined for most of the period range (stable values start to become evident for $T > 10$ s). This behaviour is consistent with 1D structure.

The plots for rhoZPyx and rhoZPxy are well behaved in both amplitude and phase. They agree for most of the spectrum, again indicating one-dimensionality; however at periods above 10 s they begin to diverge, showing the conductivity structure becoming more complicated, as already noted.

3.2 Invariants as a function of period

The second figure (Figure 2) moves to the calculation and presentation of the seven invariants of rotation of Weaver et al. (2000), denoted (in the far-right and centre-right panels) as I1 to I7. Two supplementary invariants are also included, I and I0 (Weaver et al. 2003, 2006). The seven invariants (I1 to I7) monitor the dimensionality of the impedance tensor as a function of period. Specifically, I1 and I2 gauge the scale of the tensor: see equation (24) of Weaver et al. (2000). The quantities I3 and I4 are dimensionless, vanish for 1D, and otherwise gauge the extent of two-dimensional anisotropy: see equations (26) and (27) of Weaver et al. (2000). The quantities I5, I6 and I7 (also dimensionless) gauge three-dimensionality, see equations (51) and (52) of Weaver et al. (2000). Of the supplementary invariants in Figure 2, I is related to I1 and I2. The supplementary invariant I0 is related to I7, in that the vanishing of I0 implies that I7 is undefined.

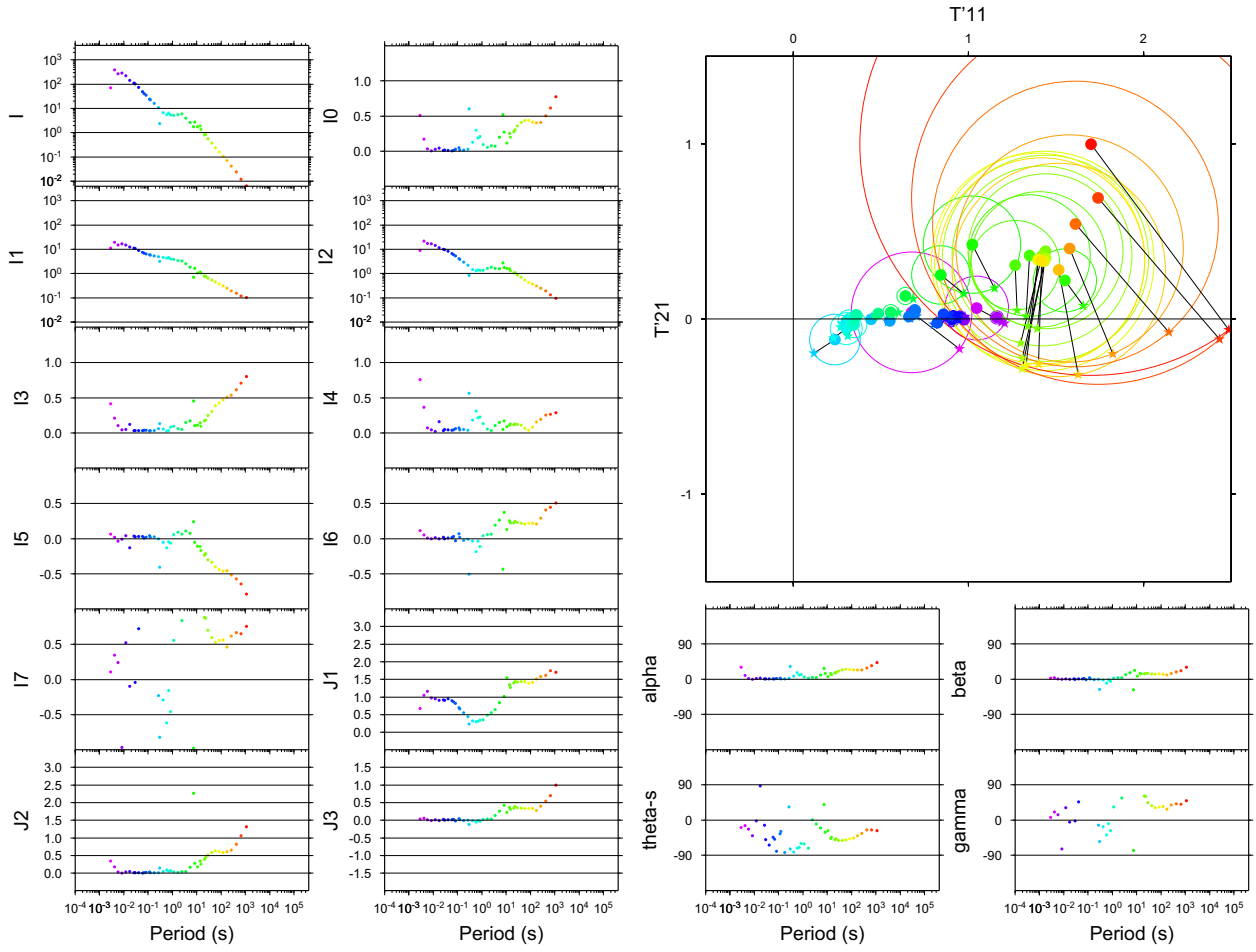


Figure 2. Invariants as a function of period for the mimic003 data. The units of I1 and I2 are those of the observed tensor elements, and I has those units squared. The invariants I0, I1 to I7, and J1 to J3 are dimensionless, as are the quantities T'_{11} and T'_{21} , plotted to give the Mohr circles. For 1D data (points on the horizontal axis) T'_{11} is the trigonometric tangent of the phase value of the magnetotelluric impedance.

Thus in Figure 2, invariants I1 and I2 show a common, well-behaved smooth decrease with increasing period (which is seen also in I). Invariants I3 and I4 show values near zero for most of their period range, a clear indication of one-dimensionality in the observed data. Further I5 and I6 also show values near-zero for most of their period range, indicating absence of three-dimensionality, and I7 is correspondingly indeterminate and unstable.

Three related independent invariants, J1, J2 and J3, as introduced by Weaver et al. (2003, 2006) and which can be expressed in terms of I, I0, I1, I3 and I7, are next considered. They

are closely related to the phase tensor analysis of Caldwell et al. (2004) and summarise neatly the extent to which the dimensionality of the data is 1D, 2D or 3D. Their values are plotted at the bottom of the far-left and centre-left panels. The values of J2 and J3 of zero for periods less than 10 s demonstrate very clearly where the data are one-dimensional.

These phase tensor invariants may also be displayed in Mohr circles, as demonstrated by Weaver et al. (2003, 2006). Such Mohr circles are presented at the top of the right-hand side of Figure 2. The horizontal distance from the origin to the centre of a circle is J1, and is a basic scale for the phase tensor. The radius of a circle is J2, and is a measure of the two-dimensionality of the data. The offset of the circle centre from the horizontal axis is J3, and is a measure of the three-dimensionality of the data.

Inspection of Figure 2 shows that indeed at short periods the circles are points close to the horizontal axes, and so are 1D in character. Two-dimensionality and three-dimensionality enter the data together as period increases to greater than 10 s. These characteristics, evident in the circle plots, are consistent with the numerical values of J1, J2 and J3 plotted at the bottom of the left-hand panels.

The angles alpha, beta and gamma, presented below the Mohr circles in Figure 2, are auxiliary to the analysis (Weaver et al. 2003, 2006). Alpha is closely related to J2, and is zero when J2 is zero. Beta is the arctangent of $(J3/J1)$, and is zero when J3 is zero. Gamma is the arcsine of $(J3/J2)$, and is unstable for a one-dimensional situation, when both J3 and J2 are small.

The angle theta-s, plotted below alpha, is presented by Weaver et al. (2003, 2006) as the angle of 2D strike, when such a strike exists. It is equivalent to the Bahr angle (Bahr 1988). In the present example only for periods greater than 10 s is this quantity at all well determined, and then, as has been seen, three-dimensional characteristics enter the data at the same time as two-dimensional characteristics. However, taken as the strike angle for a 2D model, the values plotted for theta-s may be compared with the values for theta-h (real and quad) in Figure 1. It can be seen there is consistency between these different determinations of 2D strike direction.

4 CONCLUSIONS

The graphical analysis of rotationally invariant quantities of a magnetotelluric tensor may help the interpreter to understand particularly some 3D behaviour. Such understanding may be crucial in deciding when simpler 2D or even 1D modelling and interpretation may be appropriate. Various estimates of 2D strike angle also arise in invariant analysis, and may be useful in the modelling and inversion of observed data.

Of the many invariants which may be calculated and examined, three which arise from phase tensor analysis have great appeal. In a direct way they scale and demonstrate 1D, 2D and 3D behaviour, respectively.

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Three-dimensional finite element simulation of magnetotelluric fields using unstructured grids

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1 Summary

The interpretation of an increasing number of three-dimensional data sets requires the simulation of the electromagnetic fields in three directions in space. Basing on Maxwell's equations different boundary value problems can be formulated in terms of electromagnetic potentials or fields including homogeneous or inhomogeneous boundary conditions.

The formulation of the equation of induction using vector and scalar potentials reduces the number of unknowns to four instead of six field components. Applying a secondary potential approach allows for the implementation of simple homogeneous Dirichlet boundary conditions. The boundary value problem is solved by means of the finite element method. We use quadratic Nédélec elements on unstructured tetrahedral grids that are well suited for incorporating arbitrary model geometries including surface and seafloor topography.

To expand the classic magnetotelluric frequency range towards lower periods ($T < 10^{-4}$ s) used by the Radio and Audio MT method for studying shallow conductivity structures displacement currents may need consideration. Beside the electric conductivity and permittivity, the presented finite element approach incorporates the magnetic permeability as model parameter that can be advantageous e.g. in the case of ore exploration and for studies of the earth's crust where basaltic rocks occur.

Keywords: Magnetotellurics, finite element method, numerical simulation

2 Introduction

The propagation of electromagnetic fields is governed by Maxwell's equations. They can be combined to yield the equation of induction in terms of the vector potential $\mathbf{A}$. The simulation of the secondary potential $\mathbf{A}_s$ minimizes the computational effort due to its local occurrence and the validity of homogeneous Dirichlet boundary conditions.

To solve the boundary value problem, we apply a finite element (FE) approach that allows for the efficient parametrization of arbitrary model geometries on unstructured tetrahedral grids. So-called vector or Nédélec elements are well suited to approximate the vector field $\mathbf{A}_s$ whose tangential components are continuous at element interfaces that represent possible jumps in the model parameters. Within one element, the vector potential $\mathbf{A}_s$ is described by polynomials.

The exploration of shallow conductivity structures requires the expansion of the classic magnetotelluric (MT) frequency range to shorter periods. Depending on the conductivity distribution the quasistatic approximation of Maxwell's equations might not be valid for periods shorter than 10^{-4} s. Hence, displacement currents need consideration. Furthermore, the presented FE algorithm incorporates the relative magnetic permeability μ_r as model parameter. By means of model studies we show the influence of displacement currents and the relative magnetic permeability μ_r on the apparent resistivity and the phase computed analytically for a homogeneous halfspace.

3 Equation of Induction

3.1 Maxwell's Equations

Assuming the harmonic time dependency $e^{i\omega t}$, the behaviour of the electric and the magnetic fields $\mathbf{E}$ and $\mathbf{H}$ is governed by Maxwell's equations of the form

$$\nabla \times \mathbf{H} = \mathbf{j} + i\omega \mathbf{D}, \quad (1)$$

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}, \quad (2)$$

$$\nabla \cdot \mathbf{D} = \rho,$$

$$\nabla \cdot \mathbf{B} = 0.$$

The eddy current density $\mathbf{j}$, the displacement current density $\mathbf{D}$, and the magnetic flux density $\mathbf{B}$ are combined with the electromagnetic fields by Ohm's law and the constitutive relations, respectively,

$$\mathbf{j} = \sigma \mathbf{E}, \quad \mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E}, \quad \text{and} \quad \mathbf{B} = \mu_0 \mu_r \mathbf{H}, \quad (3)$$

with the electric conductivity σ , the electric field constant $\epsilon_0 = 8.854 \cdot 10^{-12} \text{As/Vm}$, the relative electric permittivity ϵ_r , the magnetic field constant $\mu_0 = 4\pi \cdot 10^{-7} \text{Vs/Am}$, and the relative magnetic permeability μ_r .

3.2 Electromagnetic Potentials

The divergence-free field $\mathbf{B}$ can be expressed as curl of the vector potential $\mathbf{A}$

$$\mathbf{B} = \nabla \times \mathbf{A}. \quad (4)$$

Since

$$\nabla \times (\mathbf{E} + i\omega \mathbf{A}) = 0, \quad (5)$$

we can introduce the scalar potential V so that

$$\mathbf{E} = -\nabla V - i\omega \mathbf{A}. \quad (6)$$

Applying $\nabla \times$ on eq. (1), inserting Ohm's law and the constitutive relations (eqs. 3) yields

$$\nabla \times \mu^{-1} \nabla \times \mathbf{A} + (i\omega\sigma - \omega^2\epsilon)\mathbf{A} + (\sigma + i\omega\epsilon)\nabla V = 0. \quad (7)$$

Choosing

$$\tilde{\mathbf{A}} = \mathbf{A} - \nabla \Psi \quad \text{and} \quad \tilde{V} = V - \dot{\Psi}$$

with the gauge condition $\Psi = -iV/\omega$, we obtain

$$\tilde{\mathbf{A}} = \mathbf{A} - \frac{i}{\omega} \nabla V \quad \text{and} \quad \tilde{V} = 0 \quad (8)$$

that determine the same electromagnetic fields as $\mathbf{A}$ and V (cf. eqs 4 and 6). Using eq. (8), eq. (7) can be rearranged into an elliptic second-order partial differential equation for $\tilde{\mathbf{A}}$

$$\nabla \times \mu^{-1} \nabla \times \tilde{\mathbf{A}} + (i\omega\sigma - \omega^2\epsilon)\tilde{\mathbf{A}} = 0.$$

3.3 Secondary Potential Approach

The separation of the potential $\tilde{\mathbf{A}}$ (in the following: $\mathbf{A}$) into a normal ($\mathbf{A}_n$) and an anomalous ($\mathbf{A}_s$) contribution $\mathbf{A} = \mathbf{A}_n + \mathbf{A}_s$ results in a differential equation for $\mathbf{A}_s$

$$\nabla \times \mu^{-1}(\nabla \times \mathbf{A}_s - \mu_s \mathbf{H}_n) + (i\omega\sigma - \omega^2\epsilon)\mathbf{A}_s = (\sigma_s + i\omega\epsilon_s)\mathbf{E}_n \quad (9)$$

with

$$\epsilon = \epsilon_n + \epsilon_s, \quad \sigma = \sigma_n + \sigma_s, \quad \mu = \mu_n + \mu_s$$

and

$$\nabla \times \mathbf{E}_n = -i\omega\mu_n \mathbf{H}_n, \quad \nabla \times \mathbf{H}_n = (\sigma_n + i\omega\epsilon_n)\mathbf{E}_n.$$

The normal electromagnetic fields $\mathbf{E}_n$ and $\mathbf{H}_n$ are computed for a one-dimensional layered halfspace with parameter distributions σ_n , μ_n , and ϵ_n by Wait's algorithm (Wait, 1953).

3.4 Boundary Value Problem

Considering eq. (9) in the domain Ω with the outer boundary Γ_D and all internal boundaries Γ_{int} for which the conditions of continuity for the magnetic field are valid yields the boundary value problem: Find $\mathbf{A}_s$, so that

$$\nabla \times \mu^{-1}(\nabla \times \mathbf{A}_s - \mu_s \mathbf{H}_n) + (i\omega\sigma - \omega^2\epsilon)\mathbf{A}_s = (\sigma_s + i\omega\epsilon_s)\mathbf{E}_n \quad \text{in } \Omega, \quad (10)$$

$$\mathbf{A}_s = 0 \quad \text{on } \Gamma_D, \quad (11)$$

$$\mathbf{n}_1 \times \mathbf{H}_1 - \mathbf{n}_2 \times \mathbf{H}_2 = 0 \quad \text{on } \Gamma_{int} \quad (12)$$

with the outward unit normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$.

4 Finite Element Method

4.1 Weak Form

An equivalent formulation of eq. (10) as an inner product $(\mathbf{v}, \mathbf{u}) = \int_{\Omega} \bar{\mathbf{v}} \cdot \mathbf{u} dV$ with a complex vector-valued test function $\mathbf{v}$ from the function space V yields the so-called weak form of the boundary value problem: Find $\mathbf{A}_s \in U$, so that

$$\begin{aligned} \int_{\Omega} (\mu^{-1}(\nabla \times \mathbf{A}_s - \mu_s \mathbf{H}_n) \cdot \nabla \times \bar{\mathbf{v}} + (i\omega\sigma - \omega^2\epsilon)\mathbf{A}_s \cdot \bar{\mathbf{v}}) dV + \underbrace{\int_{\partial\Omega} \mathbf{n} \times (\mu^{-1}(\nabla \times \mathbf{A}_s - \mu_s \mathbf{H}_n) \cdot \bar{\mathbf{v}}) dS}_0 \\ = \int_{\Omega} (\sigma_s + i\omega\epsilon_s)\mathbf{E}_n \cdot \bar{\mathbf{v}} dV \quad \forall \mathbf{v} \in V, \end{aligned} \quad (13)$$

with

$$U := \{\mathbf{A}_s \in H(\text{curl}, \Omega) : \mathbf{A}_s \equiv 0 \text{ on } \Gamma_D\},$$

$$V := \{\mathbf{v} \in H(\text{curl}, \Omega) : \mathbf{v} \equiv 0 \text{ on } \Gamma_D\} \quad (14)$$

and

$$H(\text{curl}, \Omega) := \{\mathbf{u} \in (L^2(\Omega))^3, \nabla \times \mathbf{u} \in (L^2(\Omega))^3\}.$$

Applying the boundary conditions in eqs. (11) and (12), the boundary integral in eq. (13) vanishes.

4.2 Finite Element Analysis

A discrete approximation $\mathbf{A}_s^h \in U_h$ of $\mathbf{A}_s \in U$ arises from the linear combination of N real vector-valued basis functions $\phi_i \in U_h$ ($i = 1, \dots, N$) with the complex coefficients a_i ($i = 1, \dots, N$):

$$\mathbf{A}_s^h = \sum_{i=1}^N a_i \phi_i. \quad (15)$$

Using the discrete test functions $\mathbf{v}_i = \phi_i$ the boundary value problem can be written as matrix-vector-equation

$$\tilde{\mathbf{K}} \mathbf{A}_s = \mathbf{L} \quad (16)$$

whereas

$$\begin{aligned} K_{i,j} &= \int_{\Omega} (\mu^{-1} (\nabla \times \phi_i - \mu_s \mathbf{H}_n) \cdot \nabla \times \bar{\phi}_j + (i\omega\sigma - \omega^2\epsilon) \phi_i \cdot \bar{\phi}_j) dV, \\ L_i &= \int_{\Omega} (\sigma_s + i\omega\epsilon_s) \mathbf{E}_n \cdot \bar{\phi}_i dV. \end{aligned} \quad (17)$$

The domain Ω is decomposed into tetrahedra. On each tetrahedron piecewise quadratic basis functions ϕ_i are assumed. Their degrees of freedom are associated with the edges and the faces of the tetrahedral finite element (Fig. 1). This type of finite elements, the so-called vector or Nédélec elements, is especially suitable for the discretization of vector fields that show continuity in their tangential components.

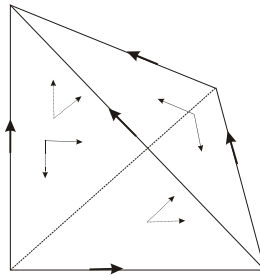


Fig. 1: Graphical representation of the degrees of freedom for quadratic Nédélec elements on a tetrahedron.

5 Influence of Permittivity and Permeability

5.1 Displacement Currents

Usually, in the simulation of the MT method displacement currents are neglected due to the classic frequency range not exceeding 1 kHz. To study shallow conductivity structures, the Radio and Audio MT method applies frequencies up to several MHz. Fig. 2 presents the apparent resistivities and phases for a homogeneous halfspace of $1000 \Omega \text{ m}$ (left) and $10000 \Omega \text{ m}$ (right), respectively, computed including ($\epsilon_r = 1$, 'o') and without (quasistatic, '+') displacement currents. Significant deviations are obvious for frequencies higher than 10 kHz especially for the more resistive halfspace.

5.2 Magnetic Rock Properties

So far, the magnetic rock properties have not been incorporated in electromagnetic applications. However, in addition to the electric conductivity σ , the relative magnetic permeability μ_r might yield geological information especially in the field of ore exploration and studies of regions where basaltic rocks occur. Fig. 3 displays the dependency of the apparent resistivity and phase on μ_r . $\mu_r > 1$ results in a higher induced current density (cf. eqs.

(2) and (3)) and therefore yields a lower apparent resistivity. The phase features no anomalous behaviour. Note, that there is no frequency dependency at all.

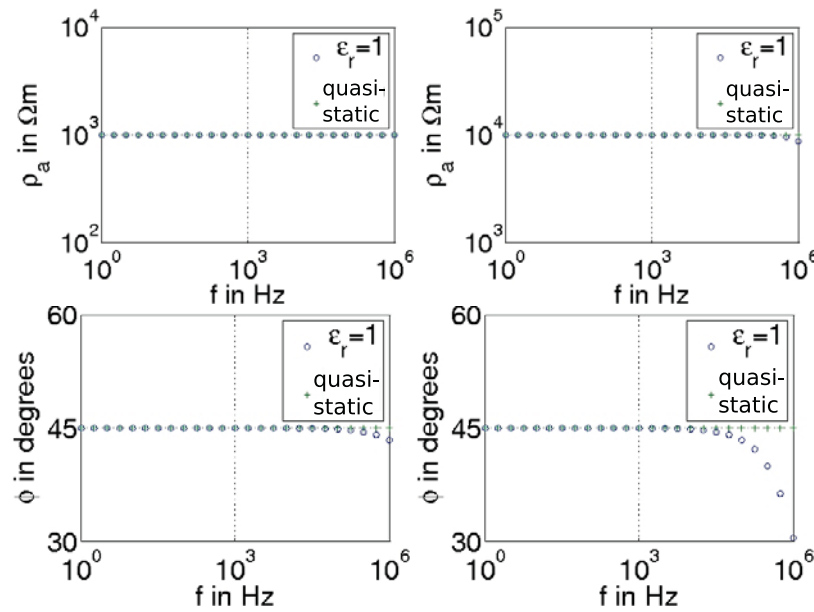


Fig. 2: Apparent resistivities and phases for a homogeneous halfspace of $1000 \Omega\text{m}$ (left) and $10000 \Omega\text{m}$ (right).

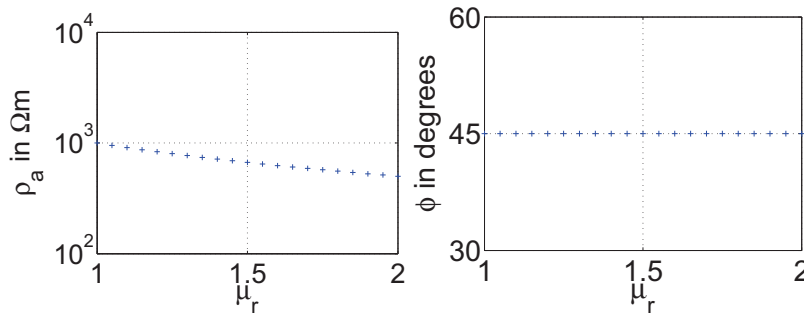


Fig. 3: Apparent resistivities and phases over μ_r varying in the range of $1 \leq \mu_r \leq 2$ for a homogeneous halfspace of $\rho = 1000 \Omega\text{m}$.

6 Comparison with an FD approach

We present the simulation results for the 3D-2 COMMEMI model (Weaver and Zhdanov, 1997) shown in Fig. 4. The apparent resistivities ρ_{xy} and ρ_{yx} as well as the phases ϕ_{xy} and ϕ_{yx} on the earth's surface at $x = 0$ computed by our FE algorithm and Mackie's finite difference (FD) code (Mackie, Madden and Wannamaker, 1993) are displayed in Fig. 5.

The anomalous resistivities of the two bodies are obvious. The steep slopes in ρ_{yx} and ϕ_{yx} (fig. 5, right) are due to the discontinuity of the normal component of the electric field whereas the tangential component used to determine ρ_{xy} and ϕ_{xy} (fig. 5, left) is continuous. The results for both of the approaches are in good agreement. However, we expect our FE code to yield the more accurate solution concerning the advantages of the FE method approximating electromagnetic fields especially at high parameter contrasts that we have experienced for two-dimensional simulations.

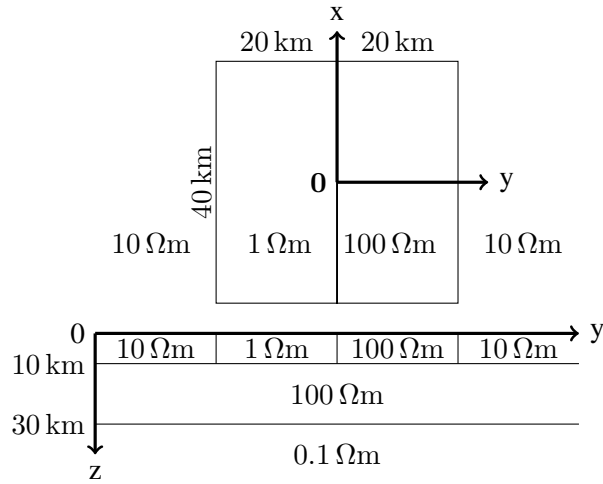


Fig. 4: COMMEMI model 3D-2.

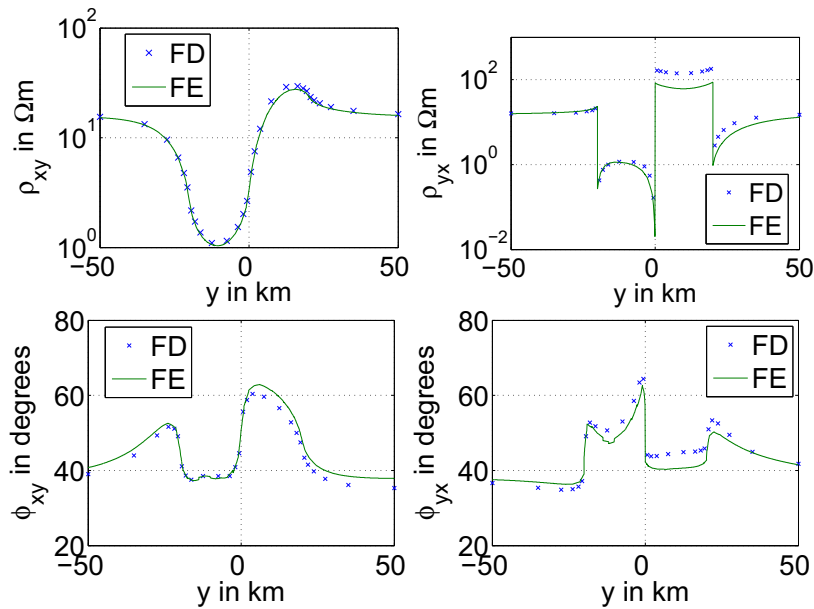


Fig. 5: Apparent resistivities and phases for the COMMEMI model 3D-2 at a period of $T = 100$ s.

7 Conclusions

The presented secondary potential approach has proven to be well suited for the three-dimensional simulation of electromagnetic fields. The local occurrence of the anomalous vector potential allows for the implementation of homogeneous Dirichlet boundary conditions for relatively small models hence minimizing the computational effort in terms of memory. However, future efficiency studies will include formulations of the boundary value problem for the total potential and the electromagnetic fields.

The approximation of the magnetic vector potential by quadratic Nédélec elements on unstructured tetrahedral grids achieves a satisfying accuracy in comparison with other numerical approaches. We intend to improve the efficiency and accuracy of our algorithm by employing an adaptive mesh refinement in connection with an error estimator function.

Model studies clearly show that the consideration of displacement currents is essential for frequencies higher than 10 kHz. Additionally, magnetic rock properties, namely $\mu_r > 1$, can influence the apparent resistivity and phase significantly. This might be of interest with regard to the hypothesis of the second-order magnetic phase transition taking place at medium depths of the earth's crust (Kiss, Szarka and Prácer, 2005).

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Numerical simulation of magnetotelluric fields at Stromboli

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1 Summary

Stromboli is a small volcanic island in the Mediterranean Sea off the west coast of Italy. It is famous for its characteristic Strombolian eruptions. To get a better understanding of these processes further explorations of the inner structure of the volcano are essential. By carrying out numerical simulations, we aim at showing that the magnetotelluric method using a wide frequency range, e.g. $10^{-4} \dots 10^4$ Hz, is applicable to this task.

To compute accurate electromagnetic fields the geometry of Stromboli volcano and the surrounding bathymetry need to be considered as detailed as possible. This becomes feasible using 2D and 3D finite element techniques on unstructured triangular and tetrahedral grids. First numerical simulations of MT measurements are computed applying a generalized geometry: a frustum as the volcano, an underlying halfspace and a layer of sea water surrounding the volcano.

Keywords: magnetotellurics, volcano, numerical simulation

2 Introduction

Stromboli volcano is 926 m high and extends down beneath the sea level to a depth of 2000 m. The first activities of the Palaeostromboli took place during the younger Pleistocene about 40,000 years ago. The characteristic Strombolian eruptions have proceeded in approximately the same manner for at least the last two thousand years. To get a better understanding of the processes that lead to eruptions further investigations of the inner structure of the volcano are essential. As shown by Müller and Haack (2004) the magnetotelluric (MT) method might be applicable to this task. In order to calculate highly accurate results, a detailed description of the geometry of Stromboli volcano and the surrounding bathymetry is necessary. We apply 2D and 3D finite element techniques on unstructured triangular and tetrahedral grids to incorporate arbitrary surface and seafloor topography (Franke, Börner and Spitzer, 2007).

First numerical simulations of MT measurements are computed applying a generalized geometry: a frustum of 3000 m height as the volcano, an underlying halfspace with a thickness of 5000 m and a layer of sea water surrounding the volcano. The electromagnetic fields, apparent resistivities and phases are calculated numerically at the seafloor, the slopes, and on top of the volcano. To resolve the upper structure of the volcano including the chimney as well as the layers underneath the volcano and the magma chamber, the computations are carried out for a wide frequency range.

3 Magnetotelluric method

The behaviour of electromagnetic fields is governed by Maxwell's equations. Assuming a harmonic time dependency $e^{i\omega t}$ as well as the magnetic and electric fields $\mathbf{H}$ and $\mathbf{E}$ as

$$\mathbf{H} = \mu^{-1}(\nabla \times \mathbf{A}) \quad \text{and} \quad \mathbf{E} = -i\omega \mathbf{A} - \nabla V,$$

the equation of induction for the magnetic vector potential $\mathbf{A}$ reads

$$\nabla \times \mu^{-1}(\nabla \times \mathbf{A}) + (i\omega\sigma - \omega^2\varepsilon)\mathbf{A} = 0$$

where the electric scalar potential V has been eliminated by a gauge condition.

To solve the boundary value problem in the bounded domain Ω , electric and magnetic insulation are required for boundaries parallel ($\Gamma_{\parallel}$) and perpendicular ($\Gamma_{\perp}$) to the current flow, respectively:

$$\mathbf{n} \times \mathbf{H} = 0 \quad \text{on} \quad \Gamma_{\parallel} \quad \text{and} \quad \mathbf{n} \times \mathbf{A} = 0 \quad \text{on} \quad \Gamma_{\perp}.$$

Furthermore, the magnetic field values for the top and bottom boundaries are calculated analytically for a 1D layered halfspace:

$$H_{\perp} = 1 \text{ Am}^{-1} \quad \text{on} \quad \Gamma_{top} \quad \text{and} \quad H_{\perp} = H(z) \quad \text{on} \quad \Gamma_{bottom}.$$

The conditions of continuity for the magnetic fields apply to at all interior boundaries representing possible jumps in the conductivity:

$$\mathbf{n}_1 \times \mathbf{H}_1 + \mathbf{n}_2 \times \mathbf{H}_2 = 0 \quad \text{on} \quad \Gamma_{int}.$$

To interpret MT measurements, apparent resistivity and phase are computed from the electromagnetic fields

$$\rho_{ij} = \frac{1}{\omega\mu} |Z_{ij}|^2, \quad \phi_{ij} = \arg(Z_{ij}) \quad \text{with} \quad Z_{ij} = \frac{E_i}{H_j} \quad \text{and} \quad i, j = x, y$$

where Z_{ij} is the impedance for different directions of polarisation.

4 The model

The geometry of Stromboli volcano and the surrounding bathymetry have to be considered as detailed as possible to obtain results that are close to reality. First 2D simulations of MT measurements imply a generalized geometry depicted in fig. 1 with the following parameters: a frustum of 3 km height as the volcano, an underlying halfspace with a thickness of 100 km, a 2 km thick layer of sea water and an air layer of 100 km. The electrical conductivities are assigned according to Friedel and Jacobs (1997) and UNESCO (1983). Preliminary 3D calculations use an axially symmetric model (cf. fig. 2) with a 50 km $\times$ 50 km $\times$ 17 km sized rectangular prism surrounding the volcano.

To the 2D simulations, we apply the finite element method using unstructured triangular grids and quadratic Lagrange elements. In the 3D case, tetrahedral grids and quadratic Nèdèlec elements are employed to compute the electromagnetic fields. These approaches are very well suited to take into account the steep topography and bathymetry.

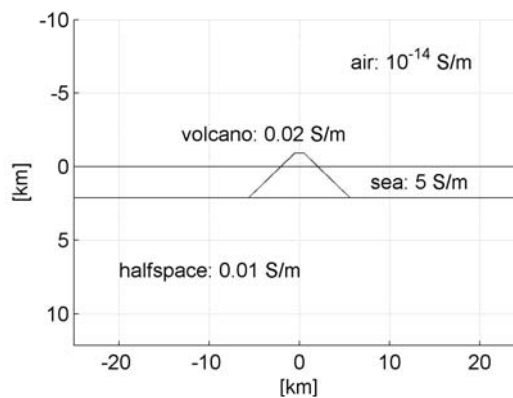


Fig. 1: Section of the 2D model including the electrical conductivity distribution.

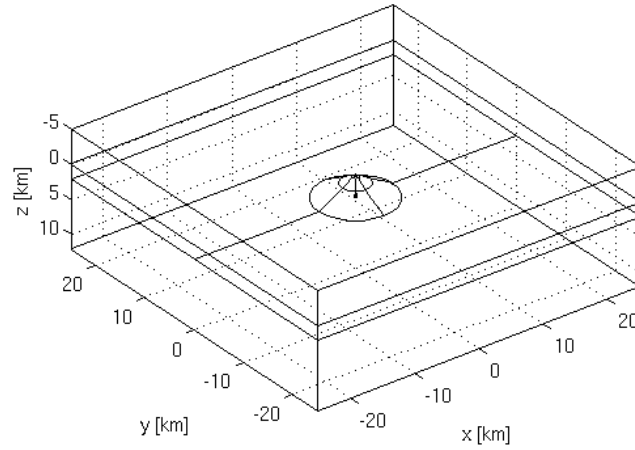


Fig. 2: 3D model.

5 Model studies

For the first analysis of the behaviour of MT data we have carried out 2D computations. Fig. 3 displays sounding curves of the apparent resistivity ρ_a and phase ϕ for the E-polarisation case on top of the volcano at $x = 0$ (left) and on the seafloor at $x = 50$ km (right), respectively.

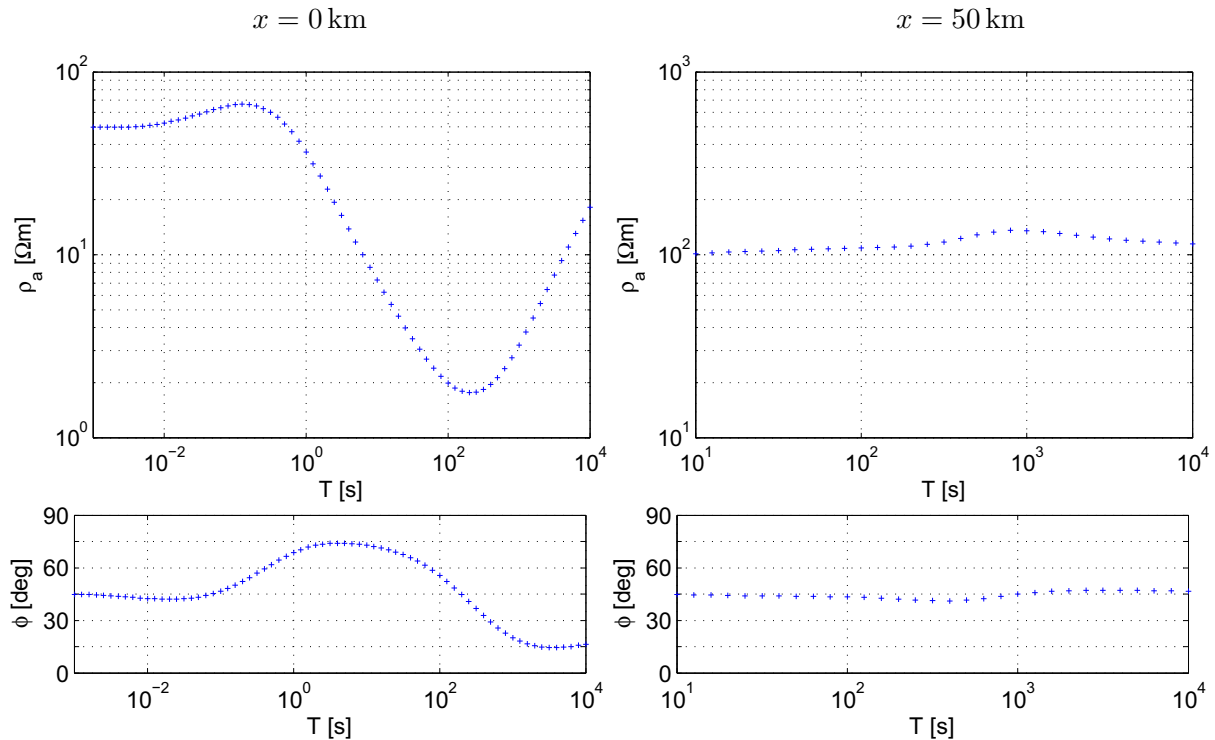


Fig. 3: 2D sounding curves of apparent resistivity (top) and phase (bottom) for E-polarisation on top of the volcano (left) and on the seafloor at $x = 50$ km (right).

On the seafloor, the effect of the volcano i.e. the deviations from the halfspace resistivity of $100 \Omega \text{ m}$ and phase of 45° are small and limited to the period range of $10^2 \dots 10^4 \text{ s}$ (cf. fig. 3, right). These periods yield a skin depth that is larger than the thickness of the sea layer and they are suited to register a lateral effect of the resistive volcano. On

top of the volcano, however, as shown in fig. 3 (left) the apparent resistivity and phase show variations for periods between 10^{-2} and 10^4 s due to the conductive sea water and the underlying halfspace. Hence, the challenge is to simulate the electromagnetic fields for a wide frequency range that is suited to yield information about the conductivity distribution of the halfspace and the volcano itself.

For the H-polarisation case, sounding curves of apparent resistivity ρ_a and phase ϕ on top of the volcano $x = 0$ and on the seafloor $x = 300$ km are depicted in fig. 4. On top of the volcano, the data show a strong static shift effect for periods longer than 10 s (cf. fig. 4, left). Due to the charge accumulation at the high conductivity contrast between the volcano and the highly conductive sea water the electric field on top of the volcano increases and, hence, yields higher apparent resistivities. The phase is not affected by the in-phase oscillating charge accumulation.

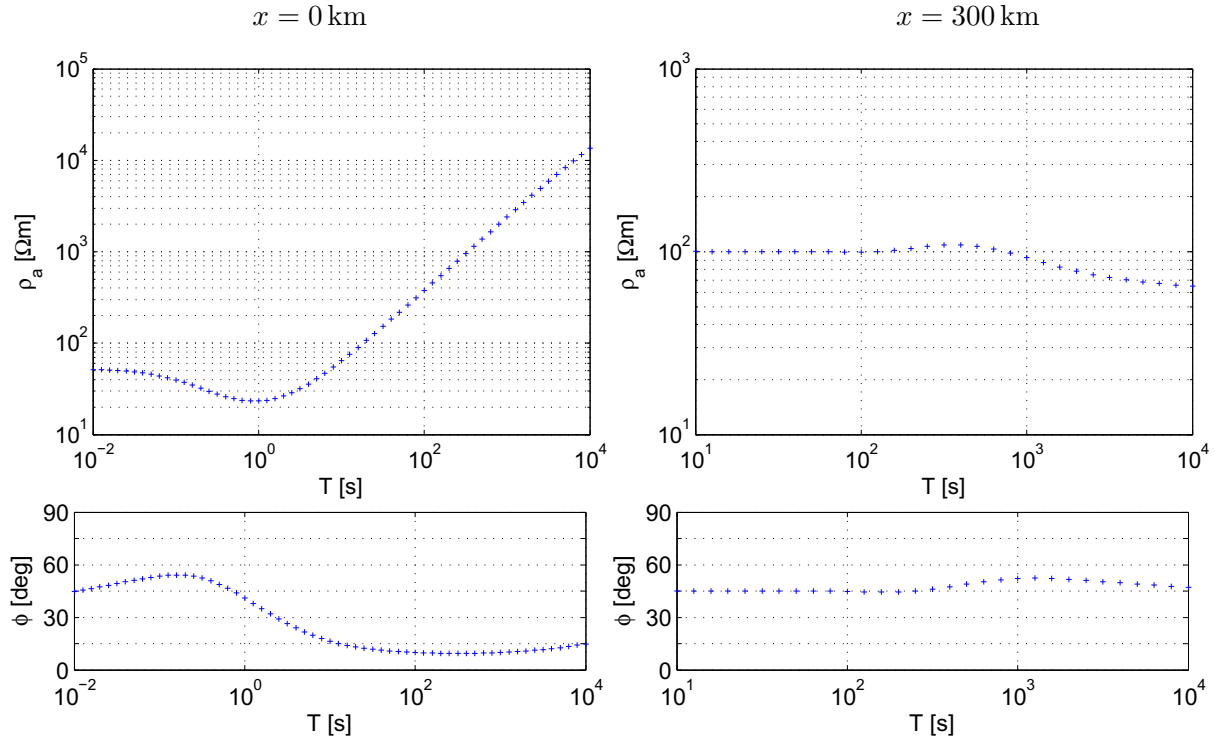


Fig. 4: 2D sounding curves of apparent resistivity (top) and phase (bottom) for H-polarisation on top of the volcano (left) and on the seafloor at $x=300$ km (right).

The 3D simulations show a very complex behaviour of the apparent resistivity and the phase. Since we do not have any analytical solution for the volcano model to compare with and convergence studies are not feasible because the 3D computations are still very time consuming and memory demanding we invoke the symmetry of the model to validate our simulation results. Fig. 5 displays the location of the data points 1 to 5 with respect to the midpoint of the bottom face of the frustum representing the volcano that is situated in the origin of the coordinate system. Considering xy-polarisation, i.e. H_y is the incident field, the appropriate electric field component E_x is parallel to the profile for the data points 4 and 5, however, it is perpendicular to the profile for the data points 2 and 3. By contrast, in the yx-polarisation case, i.e. considering H_x as the incident field, the electric field component E_y is parallel to the profile for the data points 1 and 2 but it is perpendicular to the profile for the data points 4 and 5. Since the model is axially symmetric regarding the z-axis, we expect the same results e.g. for ρ_{xy} at data point 2 and ρ_{yx} at point 4 ($E_{\perp}$, 10 km, cf. fig. 6, left). Similarly, congruent sounding curves e.g. for ρ_{yx} at point 3 and ρ_{xy} at point 5 ($E_{\parallel}$, 15 km) are shown in fig 7 and in fig. 8 for data point 1. As expected, beside small errors due to the numerical approach the sounding curves are congruent for symmetry reasons.

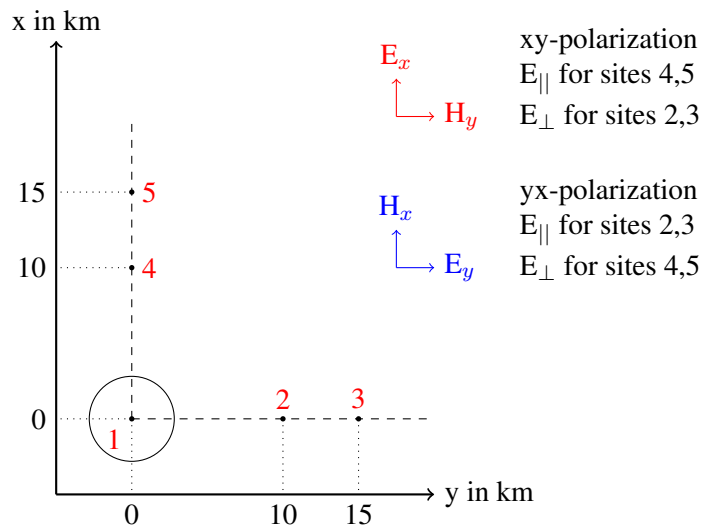


Fig. 5: Experimental design

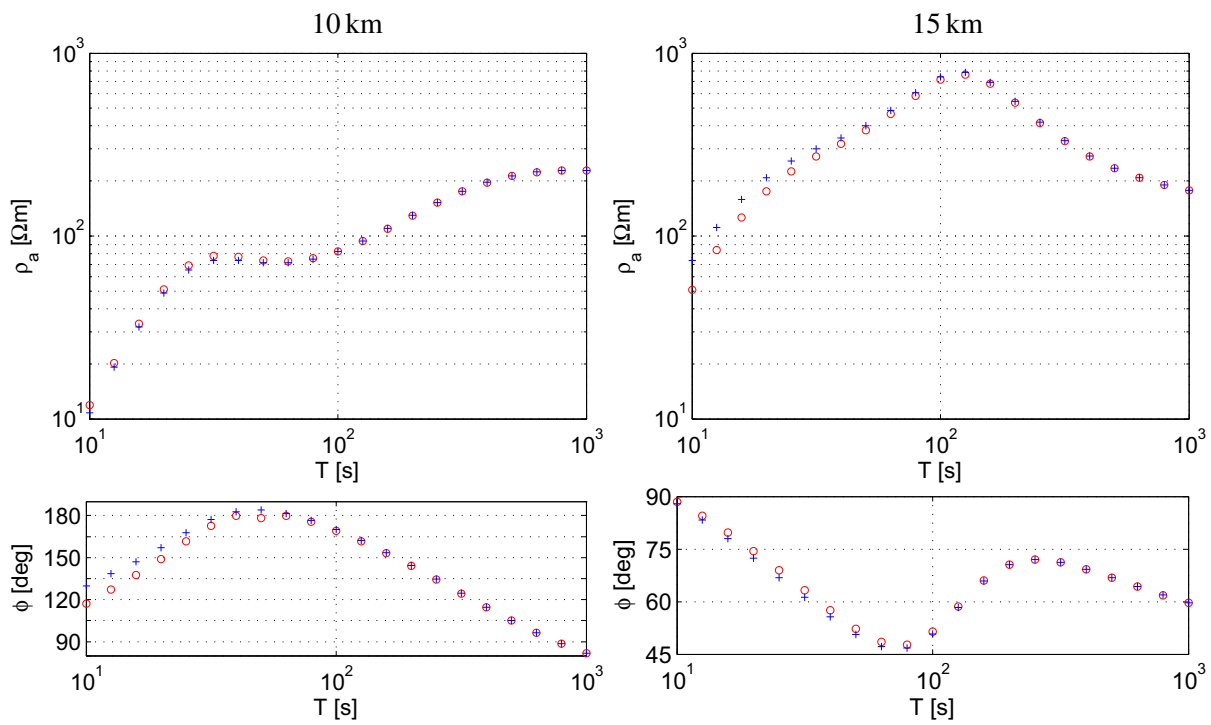


Fig. 6: Sounding curves of apparent resistivity ρ_a (top) and phase ϕ (bottom) at a distance of 10 km at data points 2 'o' and 4 '+' (left) and 15 km at data points 3 'o' and 5 '+' (right) for $E_{\perp}$, 'o' xy-polarization, '+' yx-polarization

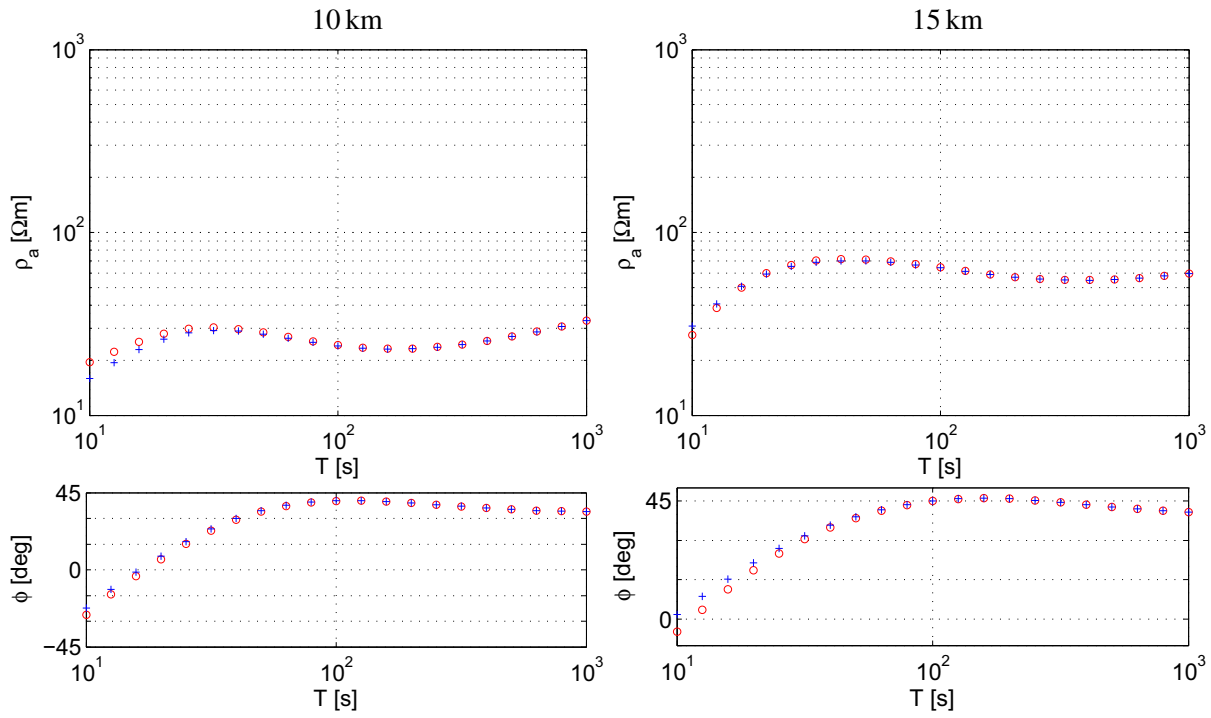


Fig. 7: Sounding curves of apparent resistivity ρ_a (top) and phase ϕ (bottom) at a distance of 10 km at data points 4 'o' and 2 '+' (left) and 15 km at data points 5 'o' and 3 '+' (right) for $E_{||}$, 'o' xy-polarization, '+' yx-polarization

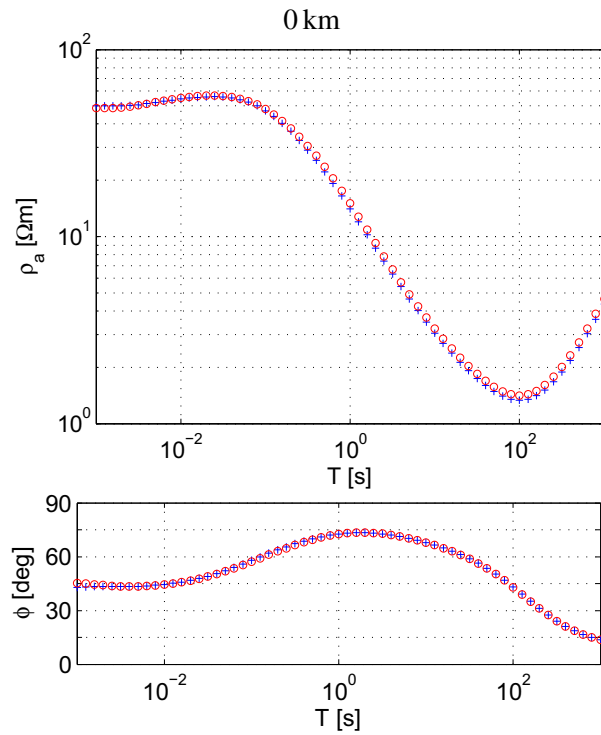


Fig. 8: Sounding curves of apparent resistivity ρ_a (top) and phase ϕ (bottom) on top of the volcano at data point 1 (x=0 km), o xy-polarization, + yx-polarization

6 Conclusions

We have presented our first promising results for simulating MT data at volcano Stromboli. In order to provide detailed information about the interior structure of the volcano that is of great interest with regard to eruption processes the electromagnetic fields need to be computed for a wide frequency range and interpreted on the volcano as well as on the seafloor. We have applied the finite element method using unstructured triangular and tetrahedral grids that are well suited to take into account the topographic and bathymetric effects of the volcano's slopes. By examining the distribution of the current density and the electromagnetic fields themselves a more fundamental understanding of the underlying physical phenomena might be achieved. In the future, more detailed model studies aim at resolving rising gas bubbles associated with Strombolian eruption processes. Furthermore, to be even closer to reality we intend to use real topography and bathymetry data of Stromboli in the form of digital elevation models.

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Integral equations for the interpretation of MT and GDS results

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1. The linearization of the EM inverse problem

We assume that electromagnetic (EM) response estimates are available for a set of discrete frequencies ω_n , in case of 2D or 3D interpretations also for various field sites at surface locations $\underline{r} = (x, y, 0)$. Cartesian coordinates are used, with z positive down. To be found are models of conductivity or resistivity within a modelling domain A in the lower half-space. Regardless of their specific nature, the complex-valued estimates will be denoted with $y_n(\underline{r})$ and called henceforth *the datum*, with $\Delta y_n(\underline{r})$ as the stochastic error of its absolute value. Let the model generate the theoretical datum $y_n^{(\text{mod})}(\underline{r})$ for the same frequency and location. Then the difference $\delta y_n(\underline{r}) = y_n(\underline{r}) - y_n^{(\text{mod})}(\underline{r})$ represents the misfit residual for the respective datum and model, with

$$\delta y^2 = \langle |\delta y_n(\underline{r})|^2 \rangle \quad (1)$$

as mean squared residual, when $\langle \rangle$ implies an average over all data. Let correspondingly

$$\Delta y^2 = \langle \Delta y_n(\underline{r})^2 \rangle \quad (2)$$

be the mean squared data error. Then the interpretation will be regarded to be within error limits, when δy and Δy are equal. Data of very uneven quality should be weighted with their reciprocal rms errors Δy_n prior to their interpretation.

A functional F_n shall connect datum and model in $y_n^{(\text{mod})}(\underline{r}) = F_n(x | \underline{r})$, depending on the entire model in A . It can be an algorithm or for example a numerical FD solution. A frequently used interpretation method, based on a Taylor expansion of a starting model, converts the non-linear inverse problem into an approximated linear problem with the aid of derivatives $\partial F_n / \partial x$. Its solution leads to successive model improvements $\delta x(\underline{r}')$ as they follow from the misfit residuals $\delta y_n(\underline{r})$ for the model in the foregoing iteration or the starting model, yielding a better fit to the data. Here we shall adopt a different approach by formulating the forward problem as an integral equation with an integrand which is decomposed into a *known* data kernel $K_n(x | \underline{r}, \underline{r}')$ and an *unknown* model parameter $x(\underline{r}')$ at the internal point $\underline{r}'$, in the 1D case for example the logarithm of resistivity:

$$y_n(\underline{r}) = \int_A K_n(x | \underline{r}, \underline{r}') \cdot x(\underline{r}') d\underline{r}' + \delta y_n. \quad (3)$$

The non-linearity of the inverse problem is preserved in the indicated model-dependence of the data kernel. The key point is now to define data and model for a data kernel, which depends only weakly on the model, and that a completely model-independent data kernel can be formulated to begin an iterative process, during which an entirely new model is derived in each iteration rather than stepwise model improvements.

2. The iterative process of modelling and its inherent problems

The process is initiated not with a *preconceived* starting model as it is usually done, but with an approximated starting kernel $K_n^{(0)}(\underline{r}, \underline{r}')$ which is model-independent. It replaces in eq. (3) the data kernel $K_n(x | \underline{r}, \underline{r}')$ for the not yet known model $x(\underline{r}')$. Solving then this equation toward the model parameters leads to a *data-derived* starting model $x^{(0)}(\underline{r}')$. It is used to calculate a now model-dependent data kernel $K_n^{(1)}(x^{(0)} | \underline{r}, \underline{r}')$, which may or may not be closer to the data kernel for the final model. Eq. (3) is solved again with this kernel towards a second model $x^{(1)}(\underline{r}')$, and so on. If iterations converge, data kernels and models become more and more consistent with each other, and the process can be terminated, when the change of models and kernels between consecutive iterations is below chosen thresholds..

The advantages are twofold. (i) Because models follow directly from the data and not from misfit residuals, data errors are readily converted into model errors as shown in the next section.. (ii) For the same reason the model resolution can be quantified in terms of a resolution kernel. Due to the limited number of error-bearing data, the obtained model parameters have to be understood in general as spatial mean values $\bar{x}(\underline{r}'_k)$ of the “true” model in some neighbourhood of an internal point $\underline{r}' = \underline{r}'_k$ according to

$$\bar{x}(\underline{r}'_k) = \int_A a(\underline{r}'_k, \underline{r}') x(\underline{r}') d\underline{r}' , \quad (4)$$

with $a(\underline{r}'_k, \underline{r}')$ as resolution kernel for the so-called *target point* $\underline{r}'_k$. The closeness of $a(\underline{r}'_k, \underline{r}')$ to a delta- function for a perfect resolution expresses the achieved degree of resolution at the target point. Measures of resolution are

$$\Delta(\underline{r}'_k) = \int_A [a(\underline{r}'_k, \underline{r}') - \delta(\underline{r}'_k - \underline{r}')]^2 d\underline{r}' \quad (5a)$$

or, with $J(\underline{r}'_k, \underline{r}')$ as “anti-delta-function, the *Backus-Gilbert spread*

$$\Delta(\underline{r}'_k) = \int_A a(\underline{r}'_k, \underline{r}')^2 J(\underline{r}'_k, \underline{r}') d\underline{r}' . \quad (5b)$$

The smaller these measures, the better the model resolution around the target point.

Problems which may arise in the course of the above described iterative process are likewise twofold. Firstly, the iterative process will not converge, when the approximated starting kernel $K_n^{(0)}(\underline{r}, \underline{r}')$ is too different from the kernels in the following iterations. In that case iterations should be started with the kernel for a preconceived model which can be thought to be closer to the final model. Secondly, in 2D and 3D interpretations conductivity or resistivity may turn negative. Such non-physical results can be encountered when interpreting error-bearing data which are inconsistent with *any* model, or when the data base is too small for the chosen complexity of the model. In principle negative values can be avoided by increased smoothing of the model by stronger regularisation, but on the expense that the misfit residuals may become too large in comparison to the data errors. Obviously, the second problem does

not exists when the logarithm of conductivity or resistivity serves as model parameter, as it is the case here for 1D interpretations, and which is common practice for interpretations based on incremental improvements of a starting model.

The iterative process has to be interrupted, when one or more model parameters attain a non-physical value, and an intermediate iterative process is conducted in the form of an algorithm described by Lawson & Hanson (1974, Section 23.3). Its purpose is to solve linear problems by least squares under side conditions in the form of inequalities, for example that no discrete model parameter x_m may be smaller than a specified threshold value x_L . After completion the algorithm assigns the model parameters to two classes. The first class contains all parameters with $x_m > x_L$ and $\partial Q / \partial x_m = 0$ as least squares condition, when Q denotes the squared residual norm. Parameters assigned to the second class have values at the lower limit and exclusively positive derivatives $\partial Q / \partial x_m$. Hence, their increase beyond x_L increases Q and thereby leads to a poorer than least squares fit to the data. Consequences for model interpretations will be one of the topics in Section 8..

Kalscheuer & Pedersen (2007) have considered in similar ways modelling errors and the resolution of 2D models, which have been obtained on the basis of derivatives of the functional and successive model improvements, as briefly outlined above. Their conclusions involve the following underlying assumptions. The model response y_n^* for the next to final iteration has to be regarded as error-free, which implies that the data errors are also the errors of the misfit residuals in the last iteration, which lead to the final model. Correspondingly, the next to last model has to be regarded as error-free in order that the errors of the last model improvement as derived now from the data errors represent also the errors for the final model.

3. The four modes of model derivation

For clarity we consider in this section a modelling space which is subdivided into M uniform layers or grid cells with a constant discrete model parameter x_m . Furthermore, the data are now numbered in sequence for frequency *and* field sites. If N_1 is the number of frequencies and N_2 the number of field sites, then the total number of data is $N = N_1 \cdot N_2$. In the notations of eqs (1) and (3) the resulting linear system of N equations for M unknowns is

$$y_n = \sum_{m=1}^M K_{nm} x_m + \delta y_n, \quad (6)$$

for $n=1,2,\dots,N$ and without explicitly expressing the model-dependence of the data kernel. Its general solution in terms of a spatially averaged model parameter for a chosen target layer or cell $m = k$ is

$$\bar{x}_k = \sum_{n=1}^N h_{kn} y_n, \quad (7)$$

while the model resolution for this layer or cell is expressed in correspondence to eq. /(4) by

$$\bar{x}_k = \sum_{m=1}^M a_{km} x_m \quad (8)$$

with the now discrete resolution coefficients a_{km} . The squared modelling error in terms of the solution coefficients h_{kn} is

$$\Delta \bar{x}_k^2 = \sum_{n=1}^N h_{kn}^2 \Delta y_n^2, \quad (9)$$

as it is readily inferred from eq. (7) for statistically independent random data errors Δy_n . We consider now the data kernel K_{nm} in eq. (7) as element of an $N * M$ data kernel matrix $\underline{\underline{K}}$ and the solution coefficient h_{kn} in eq. (8) as element of an $M * N$ solution matrix $\underline{\underline{H}}$, which represent the so-called pseudo-inverse of matrix $\underline{\underline{K}}$. Inserting y_n from eq. (6) into eq. (7) and combining then this equation with eq. (8) shows that the resolution coefficients are elements of an $M * M$ resolution matrix $\underline{\underline{A}} = \underline{\underline{H}} \underline{\underline{K}}$.

We distinguish now between two models on the basis of *global* solution criteria, which involve either *all* data or *all* model parameters, and two models on the basis of *local* solution criteria in reference to the mean model at a specified target layer or grid cell. Standard models of the first kind are least squares models with

$$\underline{\underline{H}} = (\underline{\underline{K}}^T \underline{\underline{K}})^{-1} \underline{\underline{K}}^T, \quad (10)$$

which minimise the sum of all squared misfit residuals δy_n^2 , and minimum norm models with

$$\underline{\underline{H}} = \underline{\underline{K}}^T (\underline{\underline{K}} \underline{\underline{K}}^T)^{-1} \quad (11)$$

which minimise the sum of all squared model parameters x_m^2 under the side condition that the model accounts for the data with zero misfit residuals. As a rule, either model may explain the data too well, in case of the minimum norm model that is obviously always so for error-bearing data. In addition the matrices $\underline{\underline{K}}^T \underline{\underline{K}}$ and $\underline{\underline{K}} \underline{\underline{K}}^T$ usually are ill-conditioned for inversion. Therefore a stabilised pseudo-inverse of matrix $\underline{\underline{K}}$ is in common use according to

$$\underline{\underline{H}} = (\underline{\underline{K}}^T \underline{\underline{K}} + \alpha^2)^{-1} \underline{\underline{K}}^T = \underline{\underline{K}}^T (\underline{\underline{K}} \underline{\underline{K}}^T + \alpha^2)^{-1}. \quad (12)$$

Interpretations on its basis lead to “regularized” models, which neither yield the best possible fit to the data nor do they provide the smoothed possible model with zero misfit residuals. This trade-off between model fit and model smoothness is controlled by Tikhonov’s regularisation parameter α^2 . See Protokoll EMTF Kolloquium Wohldenbergl, p. 88-89, (2005), how to conduct an efficient search for an appropriate value of α^2 , providing equality of δy and Δy from eqs (1) and (2) for an interpretation within error limits.

Models based on local criteria minimise either the resolution measures Δ_k of eqs (5) or the squared model error $\Delta \bar{x}_k^2$ according to eq. (9). Since in the first case model errors become unacceptably large and in the second case the same applies to the poorness of the resolution, the minimum of a linear combination of Δ_k and $\Delta \bar{x}_k^2$ leads to the here indispensable trade-off between model resolution and model accuracy. There are no established rules how to conduct

this Backus-Gilbert second trade-off. In the above cited reference it is suggested to adjust the trade-off that the regularized model has the same error for the respective target layer or cell.

4. 1D interpretations

The foregoing sections have presented the basic ideas behind our treatment of the non-linear inverse problem. We turn now to their implementation when interpreting data of increasing complexity. As in the first two sections we return to spatially continuous models, but it is clear that in practical applications, as they will follow in Section 7, the models will be subdivided again into uniform domains and represented by sets of discrete model parameters.

Starting with 1D model, they are derived from the surface response at a single site, i.e. from the impedance $Z(\omega_n)$ for a quasi-uniform inducing field. The functional F_n will be the Ψ -algorithm. See the Protokoll EMTF Kolloquium Wohldenber, p. 85-87, (2005) for a description of this algorithm. Its key property is that it allows a straightforward decomposition of the integrand of the basic integral equation into a to second order model-independent data kernel, connecting the logarithm of the impedance with the logarithm of resistivity. Hence, the 1D version eq. (3) is

$$y_n = \int_0^{\infty} K_n(x | 0, z') \cdot x(z') dz' + \delta y_n \quad (13).$$

with

$$y_n = \ln\{Z(\omega_n)/Z_0(\omega_n)\} = \ln\{\rho_a(\omega_n)/\rho_0\} + 2i[\pi/4 - \varphi(\omega_n)] , \quad (13a)$$

as datum,

$$x(z') = \ln\{\rho(z')/\rho_0\} \quad (13b)$$

as model parameter and

$$K_n^{(0)}(0, z') = 2\alpha_n \exp\{2\alpha_n z'\} \quad (13c)$$

as model-independent approximated data kernel, where $\alpha_n = \sqrt{\mu_0 \omega_n / \rho_0}$; ρ_0 is an arbitrary scaling resistivity and $Z_0(\omega_n) = \sqrt{i\omega_n \rho_0 / \mu_0}$ the surface impedance of a uniform half-space of resistivity ρ_0 , while $\rho_a = \mu_0 / \omega_n \cdot |Z(\omega_n)|^2$ and $\varphi = \arg\{Z(\omega_n)\}$ are apparent resistivity and phase in their usual definitions.

The depth z' is not the real depth z , however, but a conductivity-weighted depth

$$z'(z) = \int_0^z \sqrt{\rho_0 / \rho(\hat{z})} d\hat{z} . \quad (14)$$

This transformation of $\rho(z)$ into $\rho(z')$ expands low resistivity sections with $\rho(z) < \rho_0$ and thus strong attenuation of the downward diffusing EM field, while it compresses high resistivity sections with $\rho(z) > \rho_0$ and small attenuation, thus balancing the overall rate of downward attenuation. When in practice a layered model concept is used, with d_m denoting

the thickness of the m -th layer in real depth, then eq. (14) implies that $d_m / \sqrt{\rho_m} = d_0 / \sqrt{\rho_0}$ is the same for all layers, when $\rho(z')$ is equally subdivide into constant depth increments d_0 above a concluding uniform half-space. Once the layer resistivities have been determined, the model in true depth is readily reconstructed from $d_m = d_0 \sqrt{\rho_m / \rho_0}$, noting that in this way layer thicknesses are not part of the interpretation. For a quasi-continuous model with many layers, the specific choice of d_0 is practically without influence, since any resistivity distribution can be approximated with a sufficiently finely subdivided model. If the model consists, however, only of a few layers to keep the solution with $M \ll N$ closely to a least squares solution, then the choice of d_0 matters and it should be optimized to obtain the best possible fit. Under certain circumstances it may be useful to relax the stringent relation from above between thickness and resistivity by introducing layer weights w_m in $d_m = w_m d_0 \sqrt{\rho_m / \rho_0}$. They can be preset to place boundaries at the depth of seismic discontinuities, for example, or they are derived in a separate least squares analysis for a sequence of sedimentary strata of more or less known resistivity.

The 1D version of eq. (5b) is

$$\Delta(z'_k) = \int_0^{\infty} a(z'_k, z')^2 J_0 \cdot (z' - z'_k)^2 dz' \quad (15)$$

with $J_0 = 12$, and the resulting Backus-Gilbert spread can be regarded as the half-width of the resolution kernel on a z' depth scale. In general it is sufficient to assess the resolution of 1D models with the uses of the approximated data kernel of eq. (13c), that is without reference to a specific model.. Furthermore, the integrals involved in the minimisation of the spread can be solved then in closed form. It has been found that their numerical integration with the use of mode-dependent data kernels leads more or less to the same results. See Protokoll EMFT Kolloquium Wohldenber (2006) for demonstrations.

5. 2D interpretations in E-polarisation

The modelling space A will be the cross-section of a 2D structure in the (y, z) plane, striking in x -direction.. We assume induction by a quasi-uniform field with a linearly polarized electric vector in strike direction, and the data to be interpreted are the magneto-telluric impedances and the transfer functions of geomagnetic depth sounding (GDS), which have been derived from observations in a chain of field sites across the 2D structure. It is imbedded into a normal structure of conductivity $\sigma_n(z')$ which we presume to be known from 1D interpretations of the impedance $Z_n = E_{nx} / B_{ny}$ at a safe distance. To be found is $\sigma_a(y', z') = \sigma(y', z') - \sigma_n(z')$ within A from the anomalous parts of the observed surface field in terms of their transfer functions. Omitting their non-existing dependencies on x and observing that for quasi-uniform fields B_{nz} is zero, these field components are

$$E_{ax}(y, 0) = E_x / y, 0) - E_{nz}(0) = [Z_{xy}(y) - Z_n] \cdot B_{ny}(0) \quad (16)$$

with Z_{xy} as E-polarisation impedance tensor element, and

$$B_{ay}(y,0) = B_y(y,0) - B_{ny}(0) = d_D(y) \cdot B_{ny}(0), \quad B_{az}(y,0) = z_D(y) \cdot B_{ny}(0) \quad (17)$$

with d_D and z_D as GDS transfer functions. Note that it is necessary in this context to relate the electric field as well as the two magnetic field components to the *normal* part of the horizontal magnetic surface field B_{ny} as it would have been observed in the hypothetical absence of the 2D structure and which can be inferred from observations at a great distance..

For the derivation of the integral equation to interpret these data, we start from the 2D forward problem in E-polarisation. With $\underline{r} = (y, z)$ and $\underline{r}' = (y', z')$ dependencies on x are omitted again, also dependencies on frequency are not expressed explicitly. Then according to the integral method the internal electric field follows for a given model and frequency from

$$E_{ax}(\underline{r}) = -i\omega_n\mu_0 \int_A G^{(TE)}(\sigma_n | \underline{r}, \underline{r}') E_x(\underline{r}') \sigma_a(\underline{r}') d\underline{r}', \quad (18)$$

where $E_{nx}(z')$ within the normal structure is assumed to be known and with $G^{(TE)}$ as Green's function for E-polarisation. It connects an oscillating electric line current in x-direction, passing through $\underline{r}'$, with the electric field which it generates in the same direction at $\underline{r}$. The sole dependence of $G^{(TE)}$ on the normal structure is indicated with σ_n among its arguments. Cf. Aarhus Lecture Notes, p.42 (1975). Since this structure is known, we presume the same for Green's function. Details about their derivation can also be found in the cited reference and in Vorlesungs-Skript 1992/93, Blatt 17-20.

We assume that $E_x(\underline{r}')$ within A has been determined, either by solving eq. (18) or by another method of forward modelling. Then the 2D version of eq. (3) to interpret E_{ay} as datum follows from eq. (18) readily as

$$E_{ax}(y,0) = -i\omega_n\mu_0 \int_A G^{(TE)}(\sigma_n | y,0;\underline{r}') E_x(\underline{r}') \sigma_a(\underline{r}') d\underline{r}' + \delta E_{ax}(y,0) . \quad (19a)$$

For a corresponding derivation of integral equations for the anomalous magnetic field as datum, we note that according to Faraday's law $B_{ay} = -i\omega_n \partial E_{ax} / \partial z$ and $B_{az} = +i\omega_n \partial E_{ax} / \partial y$ when $E_{ay} = E_{az} = 0$, and that in the integrand of eq. (18) only Green's function depends on the field point coordinates. Denoting with subscripts their respective derivative for $z = 0$, these equations follow in the same way from eq.(18) as

$$B_{ay}(y,0) = +\mu_0 \int_A G_z^{(TE)}(\sigma_n | y,0;\underline{r}') E_x(\underline{r}') \sigma_a(\underline{r}') d\underline{r}' + \delta B_{ay}(y,0) , \quad (19b)$$

$$B_{az}(y,0) = -\mu_0 \int_A G_y^{(TE)}(\sigma_n | y,0;\underline{r}') E_x(\underline{r}') \sigma_a(\underline{r}') d\underline{r}' + \delta B_{az}(y,0) . \quad (19c)$$

Evidently the anomalous part of the conductivity, preferably in the dimensionless form

$$S(\underline{r}') = \sigma_a(\underline{r}') / \sigma_n(z') , \quad (20)$$

is a natural choice for a model parameter, which identifies the product of Green's function times $\sigma_n(z')$ with the internal electric field E_x as data kernel. The latter consists of a known normal part $E_{nx}(z')$ and an initially unknown anomalous part $E_{ax}(y', z')$, which when ignored defines the model-independent data kernel for the derivation of the starting model. The iterative evaluation of eqs (19) follows then the same general path as outlined in Section 2. There are three data sets to choose from, which can be used either alone or in combination.

The 2D version of eq. (5b) is

$$\Delta(\underline{r}'_k) = \int_A a(\underline{r}'_k, \underline{r}')^2 J_0 \cdot (y' - y'_k)^2 \cdot (z' - z'_k)^2 d\underline{r}', \quad (21)$$

yielding an areal measure of resolution. A circle with radius $\sqrt{\Delta(\underline{r}'_k)/\pi}$, drawn around the target point, visualizes the cross-section, over which the respective model parameter represents an average.

6. 2D interpretations in B-polarisation

The set-up of the model and field sites is the same as in the previous section, except that now the *magnetic* vector of the quasi-uniform inducing field is in strike direction. Since the magnetic surface field is without anomalous part and thereby without information about internal conditions, the sole field component available for interpretation is the horizontal component E_y of the electric surface field. In principle also its vertical component just below the surface could be used. It is not easily accessible to observations, however. We presume again that the normal conductivity structure with the 1D impedance $Z_n = -E_{ny} / B_{nx}$ is known and thus base the interpretation on the anomalous part of E_y given by

$$E_{ay}(y, 0) = E_y(y, 0) - E_{ny}(0) = [Z_{yx}(y) + Z_n] \cdot B_{nx}(0) \quad (22)$$

with Z_{yx} as impedance tensor element for B-polarisation.

The simple boundary condition for the magnetic field conceals the forthcoming difficulties, which we shall face when deriving the integral equation for E_{ay} . They arise from the presence of up and down going currents within and around the anomalous cross-section, also from the presence of electric charges where the conductivity changes gradually or abruptly. Therefore the integral method is rarely used for solving the forward problem in B-polarisation, with such notable exemptions as Fluche's thesis (1992). In the integral B_x takes the place of E_x and resistivity ρ the place of conductivity σ .

Let in analogy to eq. (20)

$$R(\underline{r}') = \rho_a(\underline{r}') / \rho_n(z') \quad (23)$$

denote a dimensionless measure for the anomalous part of resistivity at the internal point $\underline{r}'$ and $R(\underline{r})$ the same for the field point $\underline{r}$. Then the integral equation for deriving $B_x(\underline{r}')$ for a known normal part $B_{nx}(z')$ and model is

$$[1 + R(\underline{r})] \cdot B_{ax}(\underline{r}) + R(\underline{r}) \cdot B_{nx}(z) = \int_A \{i\omega_n \mu_0 G^{(TM)}(\rho_n | \underline{r}, \underline{r}') \cdot R(\underline{r}') + \rho_n(z') [\text{grad} R \cdot \text{grad} G^{(TM)}] \} \cdot B_x(\underline{r}') d\underline{r}' . \quad (24)$$

Cf. Aarhus Lecture Notes, p 46-47 (1975). The complications against the corresponding equation in the case of E-polarisation are obvious. On the left appears in addition to the anomalous part B_{ax} also its normal part. The first part of the integrand on the right resembles the integrand in eq. (18) for E-polarisation except that Green's function for B-polarisation has to be used. It connects a chain of oscillating magnetic dipoles in x-direction, passing through the internal point $\underline{r}'$, with the magnetic TM field it generates in the same direction at the field point $\underline{r}$. Green's function depends again solely on the known normal resistivity structure, as indicated, and we presume again that it has been derived beforehand for the involved set of points. Details about their actual determination can be found in the cited references in connection with eq. (18). Further complications arise when integrating over the second part. For a modelling cross-section subdivided into uniform grid cells this integration takes the form of integrals along cell boundaries and thereby accounts for accumulated charges on them. See the Appendix for details.

Also the connection between the magnetic field and the electric field is not as straightforward as for E-polarisation. The now relevant Ampere's law connects the spatial derivatives of the magnetic field with the current density $j_y = j_{ny} + j_{ay} = E_y / \rho$ rather than with the electric field itself. For $B_z = 0$ this law implies that $\partial B_{nx} / \partial z = \mu_0 j_{ny}$ and $\partial B_{ax} / \partial z = \mu_0 j_{ay}$, yielding

$$\mu_0 E_{ay} = \rho \partial B_{ax} / \partial z + \rho_a \partial B_{nx} / \partial z . \quad (25)$$

We multiply now eq. (24) on both sides with $\rho_n(z = +0)$ and differentiate both side with respect to z at $z = +0$ just below the surface, assuming that $\partial R / \partial z \rightarrow 0$ for $z \rightarrow +0$, i.e. that the topmost model is uniform in vertical direction. The result is

$$\rho_n(+0) \cdot \frac{\partial}{\partial z} \int_A \dots d\underline{r}' = \rho(y, +0) \frac{\partial B_{ax}}{\partial z} \Big|_{z=+0} + \rho_a(y, +0) \frac{\partial B_{nx}}{\partial z} \Big|_{z=+0} .$$

Equating the expression on the right, divided by μ_0 , with $j_{ay}(y, 0)$ from eq. (25) leads in combination with eq. (24) for $z = +0$ to the integral equation for E_{ay} as datum. Thus the 2D version of eq. (3) for B-polarisation is

$$E_{ay}(y, 0) = \rho_n(+0) / \mu_0 \cdot \int_A \{i\omega_n \mu_0 G_z^{(TM)}(\rho_n | y, +0; \underline{r}') \cdot R(\underline{r}') + \rho_n(z') [\text{grad} R \cdot \text{grad} G_z^{(TM)}] \} \cdot B_x(\underline{r}') d\underline{r}' + \delta E_{ay}(y, 0) . \quad (26)$$

The subscript to Green's function implies again differentiation with respect to z at $z = +0$. In the numerical solution of eq. (26) further problems arise from the involvement of second derivatives of Green's function in $\text{grad}(\partial G_z^{(TM)} / \partial z)$. They are overcome by integrations with Green's function split into "transient" and "quasi-static" parts, as outlined in the Appendix.

The integral corresponds in its first part to eq. (19b) for B_{ay} as datum, thereby suggesting in analogy R as the appropriate model parameter for B-polarisation. This choice defines for the first part the product of Green's function and B_x as data kernel. It will be a special topic of the Appendix to show that also the second part of the integrand can be decomposed into a data kernel, involving the integrals along boundaries, and the same model parameter $R(\underline{r}')$. In either case, the iterative process to solve eq. (26) towards this parameter will be initiated by replacing $B_x(\underline{r}')$ with its normal value $B_{nx}(z')$ for the derivation of a starting model. The definition of an areal measure of resolution will be the same as for E-polarisation in eq. (21).

Concluding herewith the sections on 2D interpretation, we note that the suggested different choices of model parameters for E- and B-polarisation exclude a joint inversion of both polarisations, at least within the framework of the here adopted method of interpretation. Only in the case of very moderate anomalies this may be possible. We infer from eqs (20) and (23) that $(1 + S) \cdot (1 + R) = 1$ or $R = -S / (1 + S)$. Thus, when S is small against unity for $\sigma_a \ll \sigma_n$, we can replace R in eq. (26) by $-S$ and facilitate thereby a joint inversion. But in general it seems that the specific information contents of 2D observations will be more fully exploited by deriving *conductivity models* from E-polarisation data and *resistivity models* from B-polarisation data. This distinction reflects the increased sensitivity of E-polarisation to interspersed good conductors due to the horizontal flow of induced currents, while vertical currents associated with B-polarisation account for their superior response to interspersed resistors.

7. 3D interpretations – an outlook

We assume that observations have been carried out in a network of field sites, and that for each site and a given polarisation MT impedances have been obtained for the two components of the electric field and GDS transfer functions for the three components of the anomalous magnetic field. The electric vector of a quasi-uniform inducing field shall be either in x-direction or in y-direction, and in this way we shall distinguish between transfer functions for *x-polarisation* and *y-polarisation*. The datum for interpretation can be anyone of these ten transfer functions, either alone within the network of sites or in combination with others.

As before far-away observation are assumed to have established the conductivity for the normal structure, into which the 3D modelling space is embedded, yielding $Z_n = E_{nx} / B_{ny}$ as known 1D impedance for x-polarisation and $Z_n = -E_{ny} / B_{nx}$ as the same impedance for y-polarisation. Hence, with $\sigma_n(z')$ being known, the interpretation will be based on the anomalous parts of the electric and magnetic surface fields to determine $\sigma_a(\underline{r}')$. In terms of their transfer functions the anomalous field components to be interpreted are

$$\begin{aligned} E_{ax} &= E_x - E_{nx} = (Z_{xy} - Z_n) \cdot B_{ny} & E_{ay} &= E_y = Z_{yy} \cdot B_{ny} & (27a) \\ B_{ax} &= B_x = h_D \cdot B_{ny}, & B_{ay} &= B_y - B_{ny} = d_D \cdot B_{ny}, & B_{az} &= B_z = z_D \cdot B_{ny} \end{aligned}$$

for x-polarisation and

$$\begin{aligned} E_{ax} &= E_x = Z_{xx} \cdot B_{nx}, & E_{ay} &= E_y - E_{ny} = (Z_{yx} + Z_n) \cdot B_{nx}, \\ B_{ax} &= B_x - B_{nx} = h_H \cdot B_{nx}, & B_{ay} &= B_y = d_H \cdot B_{nx}, & B_{az} &= B_{zz} = z_H \cdot B_{nx} \end{aligned} \quad (27b)$$

for y-polarisation, noting that for quasi-uniform inducing fields $B_{nz} = 0$. It should be observed that once more all transfer functions are in reference to the *normal* horizontal magnetic field as it may have been observed at a great distance from the anomaly. This is an absolute requirement for the here proposed approach of interpretation. If for one reason or the other some nearby field site is chosen as a substitute, it is necessary to re-calculate the transfer functions repeatedly within the iterative process, using for their transformation the modelled GDS transfer functions of the reference site for B_{ax} and B_{ay} .

For objective reasons the 3D forward problem is commonly formulated in terms of the electric field. Within the framework of the integral method the extension from 2D and E-polarisation to 3D implies that Green vectors

$$\underline{G}_i = G_{i1} \underline{\hat{x}} + G_{i2} \underline{\hat{y}} + G_{i3} \underline{\hat{z}} \quad (i, j = 1, 2, 3 \text{ for } x, y, z)$$

take the place of Green's function, field vectors $\underline{E}(z')$ the place of E_x and the scalar product of these two vectors the place of the product of Green's function with E_x . These modifications convert eq. (18) into

$$E_{ai}(\underline{r}) = -i\omega\mu_0 \int_A [\underline{G}_i(\sigma_n | \underline{r}, \underline{r}') \cdot \underline{E}(\underline{r}')] \sigma_a(\underline{r}') d\underline{r}' \quad (28)$$

as basic equation for 3D modelling, from which follow then the integral equations to interpret the various MT and GDS data. Cf. Aarhus Lecture Notes, p. 50 (1975). The components G_{ij} of the Green vector relate the electric field in i-direction at the field point $\underline{r}$ to the causing electric dipole in j-direction at the internal point $\underline{r}'$. Since these components are determined again solely by the surrounding normal structure, we assume them to be known.

Starting then with E_{ax} at the Earth's surface, we obtain from eq. (28) as the 3D version of eq. (3) for this datum

$$E_{ax}(x, y, 0) = -i\omega\mu_0 \int_A [\underline{G}_x(\sigma_n | x, y, 0; \underline{r}') \cdot \underline{E}(\underline{r}')] \sigma_a(\underline{r}') d\underline{r}' + \delta E_{ax}(x, y, 0) \quad (29)$$

with a corresponding integral equation for E_{ay} . Thus the data kernel for σ_a as model parameter is given by the scalar product of Green vectors $\underline{G}_x$ or $\underline{G}_y$ with the internal electric field vector.

Turning now to the integral equations for the components of the anomalous magnetic field, Faraday's law implies that with $\partial E_{az} / \partial x = \partial E_{ax} / \partial y = 0$ just below the Earth's surface

$$B_{ax} = \frac{1}{i\omega} \frac{\partial E_{ay}}{\partial z} , \quad B_{ay} = -\frac{1}{i\omega} \frac{\partial E_{ax}}{\partial z} , \quad B_{az} = \frac{1}{i\omega} \left(\frac{\partial E_{ax}}{\partial y} - \frac{\partial E_{ay}}{\partial x} \right) .$$

Differentiating eq. (28) with respect to the field point coordinates affects again only the components of the Green vectors. Denoting with the added subscript their respective differentiation at $z=0$, the integral equations for the three anomalous magnetic field components are

$$B_{ax}(x, y, 0) = -\mu_0 \int_A [\underline{G}_{yz}(\sigma_n | x, y, 0; \underline{r}') \cdot \underline{E}(\underline{r}')] \sigma_a(\underline{r}') d\underline{r}' + \delta B_{ax}(x, y, 0) , \quad (30a)$$

$$B_{ay}(x, y, 0) = +\mu_0 \int_A [\underline{G}_{xz}(\sigma_n | x, y, 0; \underline{r}') \cdot \underline{E}(\underline{r}')] \sigma_a(\underline{r}') d\underline{r}' + \delta B_{ay}(x, y, 0) , \quad (30b)$$

$$B_{az}(x, y, 0) = -\mu_0 \int_A [\{\underline{G}_{xy}(\sigma_n | x, y, 0; \underline{r}') - \underline{G}_{yx}(\sigma_n | x, y, 0; \underline{r}')\} \cdot \underline{E}(\underline{r}')] \sigma_a(\underline{r}') d\underline{r}' + \delta B_{az}(x, y, 0) . \quad (30c)$$

Depending on the polarisation, the components of the internal electric field vectors, as they appear in eqs (29 and (30), are

$$E_x(\underline{r}') = E_{nx}(z') + E_{ax}(\underline{r}') , \quad E_y(\underline{r}') = E_{ay}(\underline{r}') , \quad E_z(\underline{r}') = E_{az}(\underline{r}') \quad (31a)$$

for x-polarisation and

$$E_x(\underline{r}') = E_{ax}(\underline{r}') , \quad E_y(\underline{r}') = E_{ny}(z') + E_{ay}(\underline{r}') , \quad E_z(\underline{r}') = E_{az}(\underline{r}') \quad (31b)$$

for y-polarisation. Their anomalous parts, which are initially unknown, are omitted when defining the model-independent data kernel to start iterations according to Section 2. A 3D volume measure of resolution follows from eq. (21) after adding $(x-x')^2$ as factor to the integrand.

So far 3D interpretations appear as a straightforward extension of 2D interpretations for E-polarisation. But there exists a certain bias with regard to the resulting models. In 2D and E-polarisation MT as well as GDS data belong to anomalous fields in the same TE mode and, as pointed out in the previous section, they will be particularly responsive to conducting regions in the Earth. MT data in B-polarisation belong to an anomalous electric field which is exclusively in the TM mode. Their anomalous part is therefore more sensitive to resistive regions. In 3D, however, only GDS data represent surface fields in the TE mode and thus maintain their sensitivity to conductors, while MT data are related now to anomalous electric surface fields in both TE and TM modes. Consequently in their anomalous parts they should be less responsive to conducting regions than GDS data, but more responsive to resistive regions. Thus, the question arises, whether MT data in their anomalous parts can be split according to modes and then interpreted separately with either conductivity or resistivity models as in 2D.

In concluding this section on 3D interpretations we explore ways and means to achieve the just stated separation. There are two options to do so. Firstly, the vertical electric field just above the Earth's surface belongs exclusively to a TM-mode field in the air space. It is difficult to observe, however, but this may not be necessary. In a closely spaced network of

field sites it should be possible to determine with sufficient accuracy the spatial derivatives of the horizontal electric components $\partial E_x / \partial x$ and $\partial E_y / \partial y$. Taking their sum eliminates their TE-mode portions and, because the remaining TM-mode portion belongs to a potential field,

$$\partial E_x / \partial x + \partial E_y / \partial y = -\partial E_z / \partial z \big|_{z=0} . \quad (32)$$

This expression will be particularly responsive to internal charge accumulations. Cf. Extended Abstract EM Induction Workshop Hyderabad (2004).

In an alternative approach, based on Faraday's law, the vertical magnetic component within a network of sites is converted into the TE-mode portion of the anomalous electric surface field, which when subtracted from the total anomalous field yields its TM-mode portion. Cf. Becken & Pedersen (2003) for details. In order to interpret the thus isolated TM-modes with resistivity models, one has to proceed from a formulation of the forward problem in terms of the magnetic field vector, which is never done. Recalling the encountered difficulties, when implementing this approach in 2D for B-polarisation, it is not clear, whether the mounting problems in an extension to 3D would be justified by a sufficiently improved response towards resistive regions in the Earth.

8. Exemplary 2D interpretations with synthetic data in both polarisations

Since the Protokoll EMFT Kolloquium Wohlden (2006) contain numerous examples for 1D interpretations, this final section focuses on those in 2D. See also Schmucker (1993). First a short discourse is inserted to demonstrate that the occurrence of negative model conductivities or resistivities is not necessarily restricted to extraordinary situations. The model parameter $S = \sigma_a / \sigma_n = \varpi / \sigma_n - 1$ for E-polarisation or $R = \rho_a / \rho_n = \rho / \rho_n - 1$ for B-polarisation obviously has a common lower limit of -1 for non-negative conductivities or resistivities.

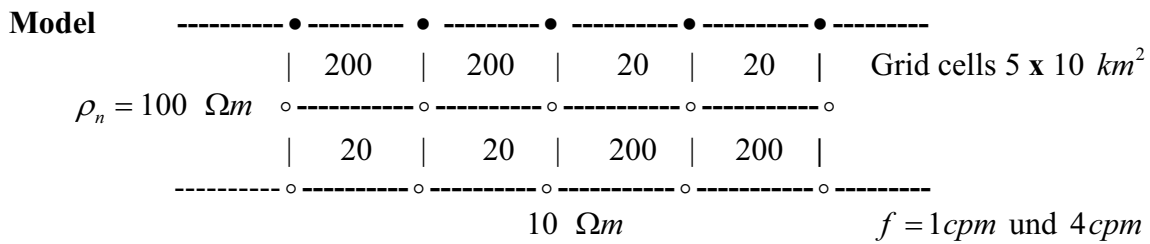
Consider then a 2D model with two resistivities, $\rho_1 = 200 \Omega m$ and $\rho_2 = 20 \Omega m$, embedded into a 1D structure with $\rho_n = 100 \Omega m$. With $S = \rho_n / \rho - 1$ in terms of resistivity we obtain

$$S_1 = -0.5 , \quad S_2 = +4.0 \quad \text{and} \quad R_1 = +1.0 , \quad R_2 = -0.8 .$$

If then a data-derived model parameter for E-polarisation is uncertain by more than 0.5, either because of poor data quality or poor convergence of iterations, this could push S_1 below its allowed lower limit. The Lawson-Hanson algorithm, as described in Section 1, would shift it back to $S_1 = -1$ which renders the respective parts of the model as undistinguishable from a perfect resistor. In B-polarisation even uncertainties beyond 0.2 could invoke the Lawson-Hanson algorithm to make other parts of the model to appear as perfectly conducting..

This numerical example shows that even models with moderate resistivity contrasts may be partially un-resolvable in terms of finite resistivities or conductivities. If model parameters, which are below the allowed limit, do not disappear in the course of the iterative process, the final model has "holes", where in E-polarisation the resistivity is too high to be distinguishable from a *perfect resistor* and in B-polarisation too low to be distinguishable from a *perfect conductor*.

Now the interpretation of synthetic data, comprising in the case of E-polarisation MT impedances Z_{xy} and GDS transfer functions d_D and z_D , while for B-polarisation the data set is restricted to MT impedances Z_{yx} . The modelling space is subdivided into eight grid cells of equal size with either $20\Omega m$ or $200\Omega m$. To the left and right are laterally uniform sections of $100\Omega m$ and below a uniform half-space of $10\Omega m$. This normal structure is assumed to be known and thus not part of the forthcoming interpretation. Large dots mark the position of five hypothetical field sites, for which data sets have been calculated for two frequencies as indicated, adding 5% normally distributed random errors to each datum. Skin depths vary between 17.7 km for $20\Omega m$ and 54.6 for $200\Omega m$ at 1 cpm, and they are half as big for 4 cpm, indicating that the modelling space is reasonably well exposed to an attenuated downward diffusing field.



Since three field sites are positioned atop vertical boundaries, the synthetic B-polarisation data are the anomalous current densities j_{ay} just below the surface, which are continuous across these boundaries. The following modelling results are regularized least squares models, for which a regularisation parameter has been determined, which lifts the mean misfit residuals to the level of the mean data errors, so that the data are interpreted within their error limits. With complex data for five field sites and two frequencies we have $N = 20$ real data to determine $M = 8$ model parameters.

The selected model is rather complicated. In E-polarisation currents are concentrated on the left in the two bottom cells and on the right in the two top cells. B-polarisation currents which enter into the model from the left are first guided downwards to the conducting bottom section and then sharply bent upwards into the conducting top section on the right. Hence, we may expect charges of both signs to be accumulated on most boundaries, thus causing strong quasi-static contributions to the anomalous electric surface field E_{ay} as outlined in the Appendix. We may guess that the conductors in the model are well perceived by E-polarisation data, in particular on the right without overburden. But there will be problems with the resistive parts, particularly beneath a conducting overburden. B-polarisation data in contrast will be most responsive to the high resistivity sections, and it is not clear, to which extent they can recognize the conducting grid cells beneath a resistive cover on the left.

These expectations are verified by the outcome of the data interpretations as shown below. Numbers are the thereby obtained resistivities in Ωm , with modelling errors below them in parenthesis. Starting with the interpretation of E-polarisation data, we note correctly determined low resistivities in the top row to the right, while they appear as slightly overestimated on the bottom on the left, possibly due to a certain bias towards the high resistivities above. As to be expected, resistivities are largely underestimated in all resistive grid cells and here also with substantial uncertainties. This applies in particular to the cells in the bottom row on the right below a shielding conducting overburden. Comparing the quality

of these modelling results from MT and GDS data, there are no large differences except that results from GDS data seem to be of slightly superior quality.

E-Polarisation

Eax	•-----•-----•-----•-----•				
	154 54 22 22				
	(160) (18) (4) (4)				
	○-----○-----○-----○-----○				
	34 30 35 39				
	(8) (5) (7) (10)				
	○-----○-----○-----○-----○				

Bay	•-----•-----•-----•-----•				
	141 130 21 21				
	(38) (17) (1) (1)				
	○-----○-----○-----○-----○				
	26 27 55 97				
	(1) (2) (14) (38)				
	○-----○-----○-----○-----○				

Baz	•-----•-----•-----•-----•				
	156 158 21 21				
	(19) (34) (1) (1)				
	○-----○-----○-----○-----○				
	27 25 146 343				
	(2) (3) (109) (608)				
	○-----○-----○-----○-----○				

B-Polarisation

Jay	•-----•-----•-----•-----•				
	228 200 43 11				
	(27) (27) (16) (15)				
	○-----○-----○-----○-----○				
	0 3 197 201				
	(-) (16) (12) (8)				
	○-----○-----○-----○-----○				

Turning to the modelling results from B-polarisation, here with j_{ay} substituting E_{ay} as datum, it is impressive to observe that the high resistivity values are all correctly determined within error limits. Evidently a conducting overburden as on the right has no shielding effect upon a resistive section beneath it, quite in contrast to E-polarisation and possibly reflecting the diagnostic effect of charged boundaries. But as anticipated their resolving power for conductors is limited, in particular when they are in greater depth. In the first cell in the bottom row on the left the resistivity is not resolvable at all, i. e. this cell appears as a perfectly conducting hole in the final model.

Even though these models have been obtained without taking model resolution and model accuracy into account, it has been found that the Backus-Gilbert trade-off between them has been optimized in the sense that the resulting models have the same model fit as the

regularized least squares models and that they interpret also the data within error limits. Furthermore, the areal resolution measure of eq. (21) corresponds closely to the chosen size of grid cells. Thus, any finer subdivision of the modelling space into, say, 32 grid cells would not have improved the resolution and any then appearing details in the models would be insignificant, when errors are taken into account.

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Appendix Numerical and analytical integrations of integrals along grid cell boundaries in B-polarisation

The integral in eq. (24) contains in its second part the scalar product of $\text{grad } R$ and $\text{grad } G^{(TM)}$; $R = \rho_a / \rho_n$ and differentiations are with respect to the internal point coordinates (y', z') . In order to simplify notations, we omit for Green's function references to their mode and to their dependence on σ_n , using subscripts to indicate derivatives, i. e. $G_z(\underline{r}, \underline{r}')$ stands for $\partial G(\sigma_n | \underline{r}, \underline{r}') / \partial z$. We assume that the modelling space is equally subdivided into uniform rectangular grid cells with horizontal and vertical boundaries. Then $[\text{grad } R \cdot \text{grad } G]$ reduces to $\partial R / \partial z' \cdot \partial G / \partial z'$ on horizontal and to $\partial R / \partial y' \cdot \partial G / \partial y'$ on vertical boundaries.

Let z_i denote the depth of the upper boundary of the i -th grid cell, which has R_i as model parameter, and let this boundary extend from $y' = y_i$ to $y' = y_i + h$, with h as width of the grid cells. We place now this boundary into a narrow strip of width 2ε and length h and

assume that R changes within the strip with depth gradually from R_{i-1} for the grid cell above to R_i . Then the contribution of this strip to the integral is given by

$$\int_{y_i}^{y_i+h} \int_{z_i-\varepsilon}^{z_i+\varepsilon} \rho_n(z') \frac{\partial R}{\partial z'} G_{z'}(r; y', z') B_x(y', z') dz' dy' .$$

We observe that B_x and $\rho_n G_{z'}$ are continuous across the boundary, the latter as a measure for the vertical current. Then for $\varepsilon \rightarrow 0$

$$\begin{aligned} \int_{y_i}^{y_i+h} \int_{z_i-\varepsilon}^{z_i+\varepsilon} \dots dz' dy' &= (R_i - R_{i-1}) \rho_n(z_i) \int_{y_i}^{y_i+h} G_{z'}(r, y', z_i) B_x(y', z_i) dy' \\ &\approx (R_i - R_{i-1}) \rho_n(z_i) G_{z'}(r, y_i + h/2, z_i) B_x(y_i + h/2, z_i) h \end{aligned} \quad (A1)$$

with a corresponding relations for the vertical boundary involving $G_{y'}$. Their summation over all boundaries finishes the numerical integration towards the internal magnetic field.

In principle it would be possible to evaluate in the same way the second part of the integral in eq. (26) towards the anomalous electric surface field. But as to be seen a decisive portion of the integral should be solved analytically. We return to the upper boundary of the i -th grid cell and observe that also $\rho_n G_{zz'}$ will be continuous at horizontal boundaries. Then the contribution of this boundary to the integral is $(R_i - R_{i-1}) H_i$ in analogy to eq. (A1) with

$$H_i = \rho_n(z_i) \int_{y_i}^{y_i+h} G_{zz'}(y, 0; y', z_i) B_x(y', z_i) dy' . \quad (A2)$$

Numbering the grid cells from top to bottom with $i = 1, 2, \dots$, their total contribution will be $R_1 \cdot H_1 + (R_2 - R_1) \cdot H_2 + (R_3 - R_2) \cdot H_3 + \dots$. Re-ordering terms leads also for the second part of the integral to the decomposition into a data kernel $[(H_i - H_{i+1}) + (V_i - V_{i+1})]$ for the model parameter R_i , with V_i and V_{i+1} as integrals on the left and right vertical boundary.

Complications arise from the involvement of second derivatives of Green's function. They are overcome as follows: The products $\rho_n(z') G_{zz'}(y, z; y', z')$ and $\rho_n(z') G_{zy'}(y, z; y', z')$ are cosine and sine transforms of $\hat{G}_{zz'}(z, z')$ with $\cos(ku)$ and $\sin(ku)$ as distance factors, k as wave number and $u = y - y'$ as horizontal distance between field and internal point. The asymptotic value of $\hat{G}_{zz'}$ for $k \rightarrow \infty$ is the purely geometric attenuation factor $k \cdot \exp(-k|z - z'|)$. Hence, for $z = 0$ the cosine and sine transforms of $[\hat{G}_{zz'} - k \cdot \exp(kz')]$ define the frequency-dependent *transient* parts of $\rho_n(z') G_{zz'}$ and $\rho_n(z') G_{zy'}$. Their remaining *static* parts as transforms of $k \cdot \exp(kz')$ are

$$\frac{1}{\pi} \frac{z_i^2 - u^2}{(z_i^2 + u^2)^2} \quad \text{for } \rho_n(z_i) G_{zz'}(y, 0; y', z_i) \quad (A3a)$$

on horizontal boundaries at $z' = z_{ii}$, and

$$\frac{1}{\pi} \frac{2z'v}{(z'^2 + v^2)^2} \quad \text{for } \rho_n(z') G_{zy'}(y, 0; y_i, z') \quad (\text{A3b})$$

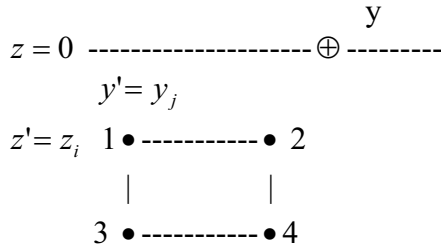
on vertical boundaries at $y' = y_j$ and with $v = (y - y_j)$.

We solve integrations along boundaries numerically as in eq. (A1), when we use the transient parts of $\rho_n G_{zz'}$ and $\rho_n G_{zy'}$, but analytically with their static parts in the following approximated manner. The magnetic field within the i -th grid cell, including its boundaries, is expanded into a two-dimensional Taylor series. Then to first order

$$B_x(y', z') = B_x(y_0, z_0) + (y' - y_0) \cdot \frac{\partial B_x}{\partial y'} + (z' - z_0) \cdot \frac{\partial B_x}{\partial z'}$$

with (y_0, z_0) denoting the coordinates of the centre of the grid cell. For horizontal boundaries we insert the thus approximated magnetic field for $z' = z_i$ into eq. (A2), replace $\rho_n G_{zz'}$ by its static part from eq. (3A) and integrate the obtained purely geometric expression in closed form. The same is performed on the vertical boundary at $y' = y_i$, replacing now $\rho_n G_{zy'}$ by its static part from eq. (A3b). Adding the resulting contributions of all four boundaries of the i -th grid cell yields

$$S_i = 0 \cdot B_x(y_0, z_0) + \frac{1}{\pi} \left[\log \frac{r_3}{r_1} - \log \frac{r_4}{r_2} \right] \cdot \frac{\partial B_x}{\partial y'} + \frac{1}{\pi} [\varphi_{34} - \varphi_{12}] \cdot \frac{\partial B_x}{\partial z'}. \quad (\text{A4})$$



The numbers refer to the four corners of the grid cell, the angle $\varphi_{k\ell}$ implies the angle under which the boundary between the corners k and ℓ is seen from field point $\oplus$, and r_k is the distance between corner k and $\oplus$.

The first conclusion to be drawn from eq. (A4) is that the internal magnetic field contributes to this sum only through its spatial derivatives. Secondly, the coefficients of the magnetic field derivatives represent the potential field, which positive and negative monopoles, here electric charges, sitting on opposing boundaries generate at the field point. But since the derivatives of B_x are frequency-dependent, their contribution to E_{ay} should be named *quasi-static*, even though they arise from the static parts of $\rho_n G_{zz'}$ and $\rho_n G_{zy'}$. This applies also to characteristic so-called *static effects* for B-polarisation, among them the large “adjustment distance” from anomalies, within which E_y returns to its normal level.

Unstructured grid based 2D inversion of plane wave EM data for models including topography

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SUMMARY

We present a 2D damped least-squares inversion code for plane wave electromagnetic (EM) methods using an adaptive unstructured grid finite element forward operator. Unstructured triangular grids permit efficient discretization of arbitrary 2D model geometries and, hence, allow for modeling arbitrary topography. The inversion model is parameterized on a coarse parameter grid which constitutes a subset of the forward modeling grid. We investigate two types of parameter grids: a regular type, however, containing trapezoidal cells and hanging nodes, and an unstructured triangular type. The transformation from parameter to forward modeling grid is obtained by adaptive mesh refinement. Sensitivities are determined by solving a modified sensitivity equation system obtained from the derivative of the finite element equations with respect to the model parameters.

Firstly, the inversion of a COPROD2 data set in E-polarization is presented as an example to show that our inversion code produces reasonable results for real data and flat earth models. Secondly, we demonstrate that surface topography may induce significant effects on the EM response and the inversion result, and that it cannot be ignored when the scale length of topographic variations is in the order of magnitude of the skin depth. Finally, we demonstrate the inversion of a synthetic data set from a model with topography.

Keywords: Unstructured grids, finite elements, topography, VLF, MT, inversion

FORWARD MODELING

The forward computations are carried out using an adaptive unstructured triangular grid finite element algorithm (Franke, Börner and Spitzer, 2004). In the case of plane, diffusive, time-harmonic electromagnetic fields in 2D conductivity structures Maxwell's equations can be combined to yield two decoupled equations of induction reading

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = i\omega\mu\sigma E_y, \quad (1)$$

$$\frac{\partial}{\partial x} \left(\frac{1}{\sigma} \frac{\partial H_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{1}{\sigma} \frac{\partial H_y}{\partial z} \right) = i\omega\mu H_y \quad (2)$$

for E- and H-polarizations, respectively, in a right-handed Cartesian coordinate system with the positive z -axis pointing upwards. E_y is the y -component of the electric field and H_y is the y -component of the magnetic field. y denotes the strike direction. ω , μ , i , and σ are angular frequency, magnetic permeability, imaginary unit, and electrical conductivity, respectively. To solve for the unknown fields, inhomogeneous Dirichlet boundary conditions are applied that assign the field values of a horizontally layered half-space to the boundaries.

The finite element discretization leads to a system of equations that can be expressed in matrix-vector form as

$$(\mathbf{K} + \mathbf{M}) \vec{u} = f, \quad (3)$$

where $\vec{u}$ is either a column vector of the electric field E_y or the magnetic field H_y at each node in E- and H-polarization, respectively, and f is the right-hand side. $\mathbf{K}$ and $\mathbf{M}$ are referred to as stiffness and mass matrices.

The remaining field components H_x , H_z for E-polarization and E_x , E_z for H-polarization can be determined at each grid node by

$$\begin{aligned} H_x &= \frac{1}{i\omega\mu} \frac{\partial E_y}{\partial z}, \quad \text{and} \quad H_z = -\frac{1}{i\omega\mu} \frac{\partial E_y}{\partial x}, \\ E_x &= -\frac{1}{\sigma} \frac{\partial H_y}{\partial z}, \quad \text{and} \quad E_z = \frac{1}{\sigma} \frac{\partial H_y}{\partial x}. \end{aligned} \quad (4)$$

The apparent resistivity ρ_a and the phase ϕ for E- and H-polarizations (in the case of VLF-R and MT methods) can be computed as

$$\rho_a = \frac{1}{\omega\mu} \left| \frac{E_y}{H_x} \right|^2, \phi = \tan^{-1} \left(\frac{\text{imag}(E_y/H_x)}{\text{real}(E_y/H_x)} \right), \quad (5)$$

for E-polarization,

$$\rho_a = \frac{1}{\omega\mu} \left| \frac{E_x}{H_y} \right|^2, \phi = \tan^{-1} \left(\frac{\text{imag}(E_x/H_y)}{\text{real}(E_x/H_y)} \right), \quad (6)$$

for H-polarization.

The real part Re and the imaginary part Im of the magnetic transfer function in the case of VLF can be computed as

$$Re = \text{real} \left(\frac{H_z}{H_x} \right) \cdot 100\%,$$

$$Im = \text{imag} \left(\frac{H_z}{H_x} \right) \cdot 100\% \quad (7)$$

INVERSION PROCEDURE

We apply a damped least-squares method for the minimization of the objective function ψ given by

$$\psi = \left(\Delta \vec{d} - \mathbf{S} \Delta \vec{p} \right)^T \left(\Delta \vec{d} - \mathbf{S} \Delta \vec{p} \right) + \lambda \left(\Delta \vec{p}^T \Delta \vec{p} - p_0^2 \right), \quad (8)$$

where $\Delta \vec{d} = \vec{d}^{obs} - \vec{d}^{comp}$ describes the discrepancy between the observed data $\vec{d}^{obs}$ and the computed data $\vec{d}^{comp}$. $\mathbf{S}$ and $\Delta \vec{p}$ denote the sensitivity matrix and the model parameter update, respectively. The logarithm of the conductivities are considered as model parameters. The Lagrange parameter λ is introduced to constrain the energy of the model parameter update to a finite quantity p_0^2 . To get the minimum of the objective function ψ , its partial derivatives $\partial\psi/\partial\Delta p_j$ are required to be zero for all model cells j . The resulting normal equation reads

$$\left(\mathbf{S}^T \mathbf{S} + \lambda \mathbf{I} \right) \Delta \vec{p} = \mathbf{S}^T \Delta \vec{d}, \quad (9)$$

where $\mathbf{I}$ is the identity matrix. Equation (9) is solved applying a direct solver at each stage of the iterative inversion process. Model parameters are updated in each iteration. In the first step of our approach, we find that the maximum singular value of $\mathbf{S}^T \mathbf{S}$ proves to be a good guess as the starting value for the Lagrange parameter λ . To get fast convergence, λ is decreased by a factor of less than one (e.g. 0.6) in each iteration.

The root mean square (RMS) error and χ^2 -value can be calculated by

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^n \Delta d_i^2},$$

$$\chi^2 = \frac{1}{n} \sum_{i=1}^n \frac{\Delta d_i^2}{\epsilon_i^2}, \quad (10)$$

where ϵ_i and n denote the standard deviation of the data and number of the data, respectively. We stop the iteration if one of the following criteria is met: (1) the maximum number of iterations is reached, (2) the convergence of RMS error stagnates, (3) $\chi^2 \approx 1$.

SENSITIVITY CALCULATION

The element S_{ij} of the sensitivity matrix $\mathbf{S}$ for the i^{th} observation site and j^{th} model parameter is calculated using the modified sensitivity equation method presented by Rodi (1976) which requires $(n + 1)$ forward computations for each

frequency

$$S_{ij} = \left(\frac{\vec{a}_i}{\vec{a}_i^T \vec{u}} - \frac{\vec{b}_i}{\vec{b}_i^T \vec{u}} \right) (\mathbf{K} + \mathbf{M})^{-1} \left(-\frac{\partial(\mathbf{K} + \mathbf{M})}{\partial(\ln \sigma_j)} \right) \vec{u}, \quad (11)$$

where $\vec{a}_i$ and $\vec{b}_i$ are column vectors to calculate the electric and the magnetic fields in the case of E-polarization and vice-versa in the case of H-polarization for the i^{th} datum from $\vec{u}$. $\vec{a}_i$ is formed by simply keeping 1 at the position of the i^{th} datum and zeros at the other nodes. If the observation site is not located exactly at grid node then field values are interpolated by two nearby grid nodes. $\vec{b}_i$ is designed in such a way that it performs a numerical differentiation over $\vec{u}$ according to eq. (4).

The sensitivities for the logarithm of the apparent resistivity $S_{ij}^{\ln \rho_a}$ and for the phase S_{ij}^ϕ can be computed as follows:

$$S_{ij}^{\ln \rho_a} = 2 \operatorname{real}(S_{ij}) \quad , \quad S_{ij}^\phi = \operatorname{imag}(S_{ij}). \quad (12)$$

An analogous strategy is used to calculate sensitivities for the real and imaginary parts of the magnetic transfer function in the case of VLF that corresponds to the E-polarization case. The only difference is that $\vec{a}_i$ and $\vec{b}_i$ both are now designed to perform numerical differentiation over $\vec{u}$ to get H_z and H_x according to eq. (4).

For details of these derivations, the reader is referred to Rodi (1976) and Farquharson and Oldenburg (1996).

INVERSION OF A FLAT EARTH COPROD2 DATA SET

In this section, we show that our code is basically working for real field data, however, for reasons of comparability and due to the lack of available examples we restrict ourselves to a flat target area. We therefore invert, as an example, a COPROD2 data set (Jones, 1993) consisting of 20 sites and 4 periods in E-polarization to show that our code produces results comparable to other flat earth inversion codes. Here we have chosen the Occam inversion code by deGroot-Hedlin and Constable (1993). Fig. 1a shows our inverted model obtained in 15 iterations starting from a 100 $\Omega \cdot \text{m}$ half-space. The χ^2 -value is 1.1 when the error floor is set to 10% in ρ_a and 2.9° in ϕ . The presence of a conductive overburden down to 5 km depth and three distinct anomalous regions below 10 to 50 km depth are clearly visible. Fig. 1b shows the inverted model using the Occam code starting from the same half-space model and assuming the same error floor, however, considering the data from both E- and H- polarizations. Both results agree well.

PARAMETERIZATION OF A MODEL INCLUDING SURFACE TOPOGRAPHY

In the following, we discuss two possibilities of parameterizing a model whose surface is associated with a varying topography. We perform the parameterization by segmentation either in rectangles and trapezoids (Fig. 2) that form a rather regular type of grid or in unstructured triangular cells (Fig. 3) that closely correspond to the forward modeling grid. The rectangular/trapezoidal grid comprises hanging nodes which enhance the flexibility with respect to resolution. Both types are adaptively refined into unstructured triangular grids for forward modeling. In Figs 2 and 3, the parameter grid is indicated in red and the first refinement stage of the unstructured triangular forward modeling grid in blue. Note that the latter is further refined using an adaptive refinement strategy to actually perform the simulation.

THE TOPOGRAPHY EFFECT

We now investigate the influence of the topography on the VLF-R and VLF response and the inverse process. For this purpose, we disassemble a model in a first step to separately examine its response originating from the subsurface and from topographic undulations. Since these features are inductively coupled, both superposed responses are certainly not giving the total response. However, it instructively displays the order of magnitude of the associated effects. In a second

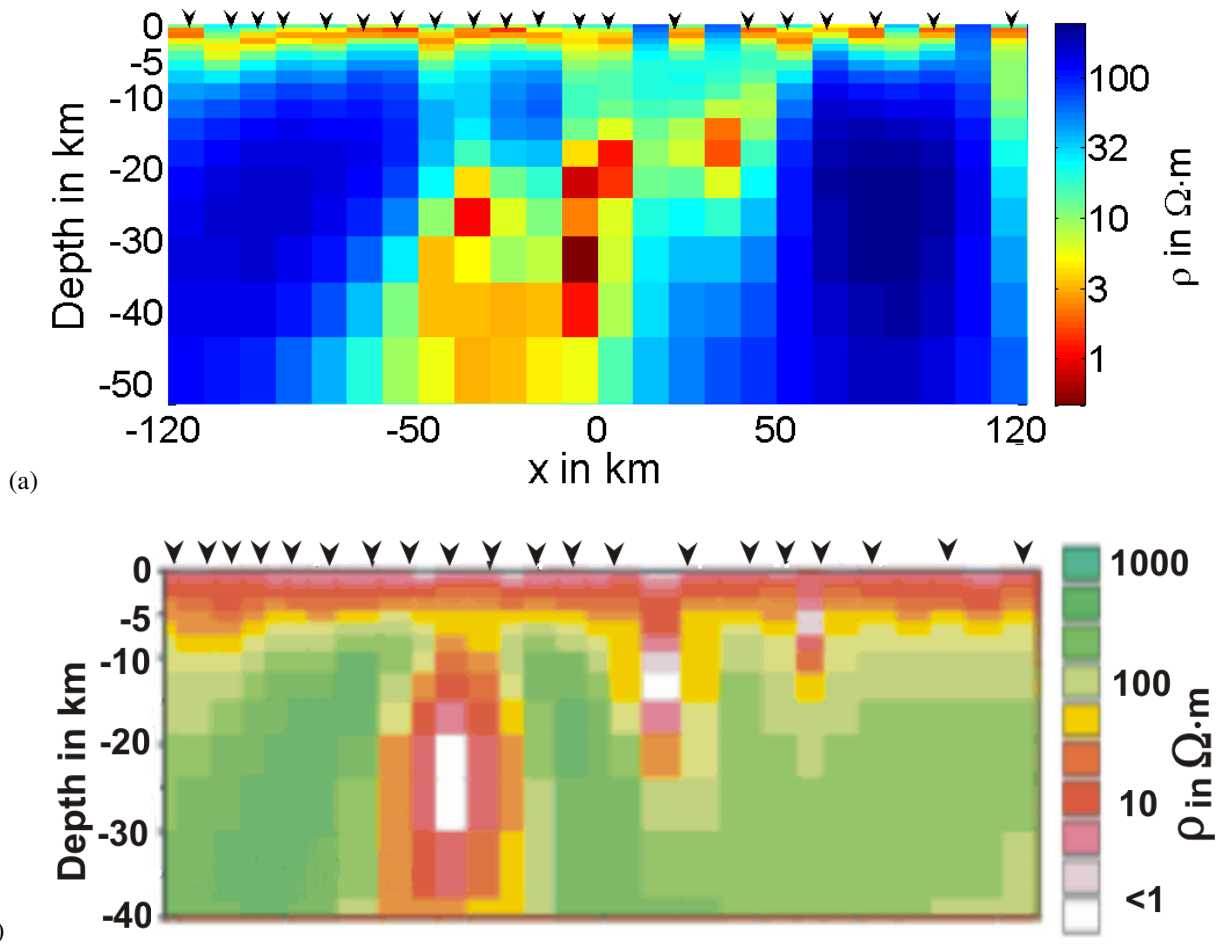


Figure 1: (a) Model obtained from the inversion of the COPROD2 data in E-polarization using the inversion code presented here. (b) Model obtained from a smoothness-constrained joint inversion of the COPROD2 data in E- and H-polarizations according to deGroot-Hedlin and Constable (1993) (modified after Jones, 1993).

step, we take data of a homogeneous earth model with surface topography and perform a flat earth inversion to point out topography induced artifacts.

DECOMPOSING THE RESPONSE FROM SURFACE TOPOGRAPHY AND SUBSURFACE CONDUCTIVITY STRUCTURES

The synthetic model displayed in Fig. 4 consists of two anomalous regions having resistivities of $100 \Omega \cdot m$ and $20 \Omega \cdot m$, respectively, within a $1000 \Omega \cdot m$ half-space with a smooth, but pronounced topography. The observation sites are located at 50 m intervals from -575 m to 575 m and marked by arrows. Synthetic data are generated for three frequencies in the VLF range: 5, 16 and 25 kHz. In Fig. 4a, the total synthetic response of the complete model is displayed in terms of apparent resistivity and phase according to eq. 6 and real and imaginary part of the magnetic transfer function according to eq. 7. In Fig. 4b, the perturbing bodies are removed so that a homogeneous model remains. The response clearly shows the influence of the topography. In Fig. 4c, the topography undulations are replaced by an average flat earth level so that the remaining lateral variation in the response is only due to the perturbing bodies. Note that the order of magnitude of both effects is comparable.

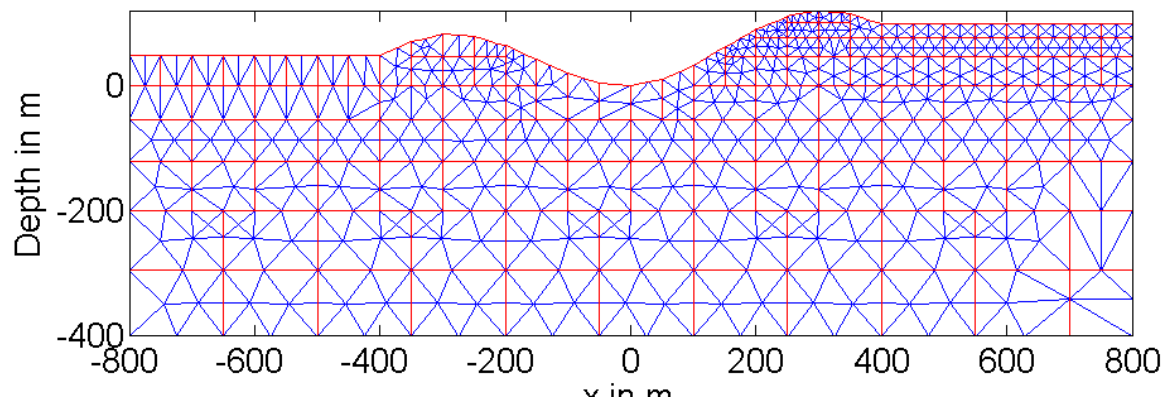


Figure 2: Parameterizing the model in rectangles and trapezoids (red lines). The unstructured forward modeling grid is indicated in blue.

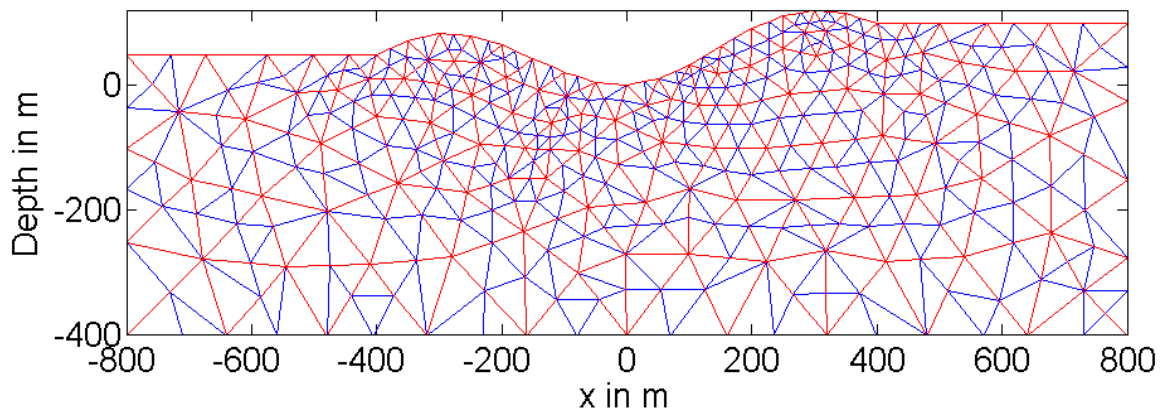


Figure 3: Parameterizing the model in unstructured triangular grids (red lines). The forward modeling grid is again outlined in blue.

FLAT EARTH INVERSION OF DATA FROM MODELS WITH TOPOGRAPHY

In this section we investigate how data from a model with topography influence the results of our inversion algorithm if the topography is not taken into account. For this purpose, we consider the synthetic VLF-R data set shown in Fig. 4b which is generated for a homogeneous $1000 \Omega \cdot \text{m}$ model with topography and without anomalous regions. We invert this data set using a flat earth assumption. The starting model is a $2000 \Omega \cdot \text{m}$ half-space. The inversion result obtained in 11 iterations is shown in Fig. 5. There are clear artifacts associated with the topography undulations. Conductive structures having resistivities of $\approx 500 \Omega \cdot \text{m}$ appear below the central valley and the transitions from the hills to the planes on the left- and right-hand side whereas resistive anomalies around $2500 \Omega \cdot \text{m}$ show up beneath the hills. Knowing the true resistivity, the maximum deviation of the inverted resistivities is more than a factor of 2 in both directions of the resistivity scale. This example demonstrates that the topography effect may become significant. It is therefore necessary to take into account any arbitrary topography for simulation and inversion. Approximate data correction schemes then become needless.

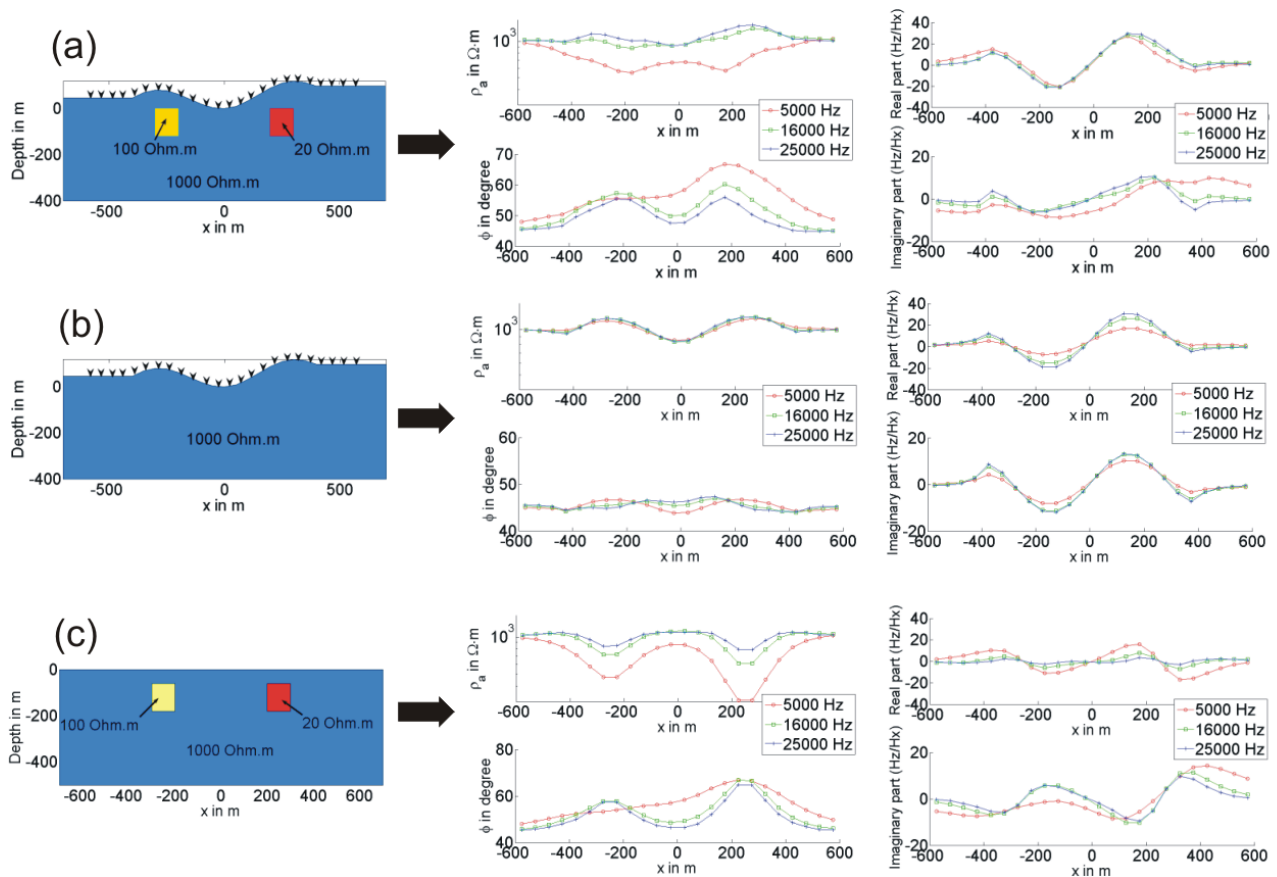


Figure 4: Synthetic responses for different models: (a) with topography and conductive regions (b) only with topography and (c) only with conductive regions within a flat earth.

INVERSION OF SYNTHETIC DATA FROM MODELS INCLUDING TOPOGRAPHY

To show that our code is able to cope with the problem of topography induced artifacts we take the synthetic data set from Fig. 4a for both the VLF-R and VLF case and add 5 % random noise for each frequency. We invert these data using both parameterization schemes presented in Figs 2 and 3. Starting model is always a homogeneous 2000 $\Omega \cdot m$ model.

For brevity, we are only going to show here the resulting models obtained by inversion of VLF-R data (Figs 6 and 7). The original rectangular anomalous regions are indicated by dashed lines. Both parameterization schemes recover the synthetic models satisfactorily after reaching the χ^2 -criterion (in 8 to 9 iteration steps).

At first glance, the parameterization using rectangles and trapezoids seems to give better results in comparison with the parameterization using unstructured grids. This, however, is due to the perfect match of structure and grid. The future strategy is to adapt unstructured grids in each iteration step to some arbitrary structure obtained during the inversion process.

CONCLUSIONS

We have developed a 2D inversion code for inverting plane wave EM data from models including topography. At first, we have shown that our inversion code is able to cope with real data in the form of a COPROD2 data set acquired in a flat earth environment. Using forward modeling, we have then demonstrated that the topography effect may become significant. A flat earth inversion of data generated from a homogeneous model including topography exhibits characteristic artifacts

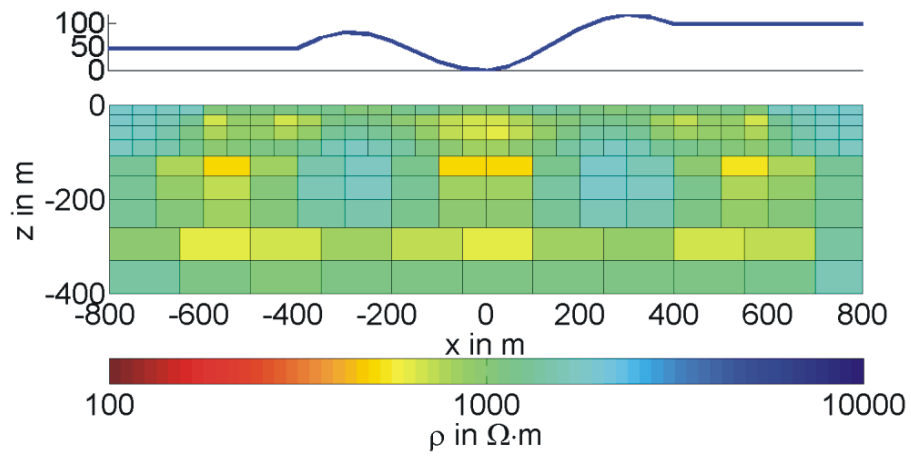


Figure 5: Flat earth inversion results of VLF-R data generated from a model including topography. For reasons of comparison, the original topography is plotted at the top.

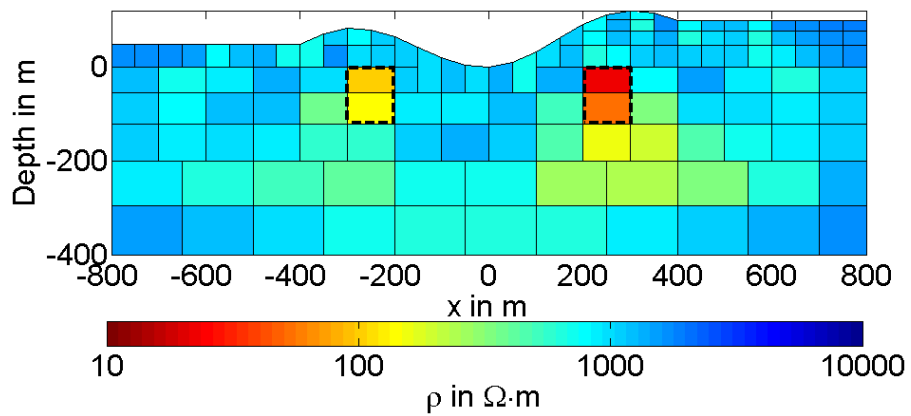


Figure 6: Inverted model obtained by inversion of VLF-R data using the rectangular/trapezoidal parameterization scheme.

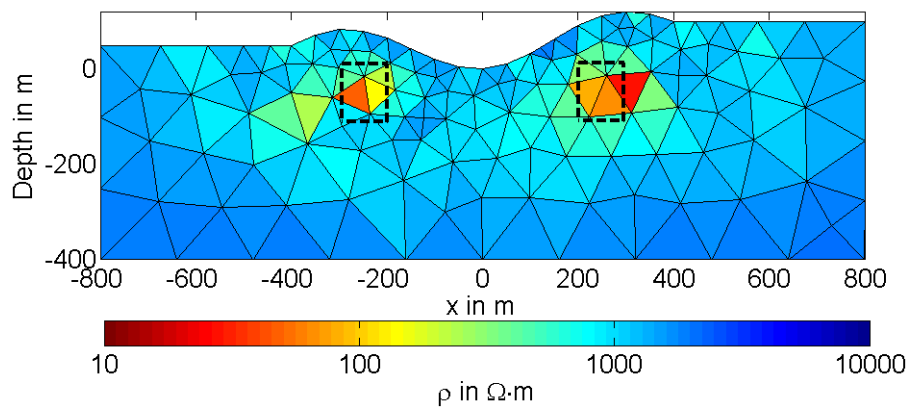


Figure 7: Inverted model obtained by inversion of VLF-R data using the unstructured parameterization scheme.

and, thus, corroborates the necessity to incorporate the topography into the inversion process.

After demonstrating the effect of topography, we have shown by inversion of VLF-R data that our code is able to resolve anomalous regions in the presence of topography. Two parameterization schemes are tested for models including topography. The best inversion results are obtained when the grid is adapted to the structures in the inverse model. Future inversion strategies will therefore incorporate adaptive parametrization schemes during the inversion process.

Concluding, the inversion of models including surface or subsurface topography, i.e., seabed topography, voids, mining galleries, tunnels, caves etc. opens up new ways for field surveys and specific applications and enhances the interpretation techniques available at present.

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The empirical mode decomposition (EMD) method in MT data processing

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Abstract

The natural magnetotelluric (MT) geophysical prospecting method utilizes the spectra of associated time-varying horizontal electric and magnetic fields at the Earth's surface to determine a frequency-dependent impedance tensor. Most current methods of analysis determine the spectra based on Fourier transform and therefore must assume either that the signals under analysis are stationary over the record length or that any distortion in the spectral estimations due to non-stationarity will occur in an equivalent manner in the spectra of both the electric and magnetic fields and thus have no effect on the impedance estimates. A new method for dealing with non-stationarity of the MT time series based upon empirical mode decomposition (EMD) method and Hilbert spectrum is proposed in this paper. In this paper, we use the EMD method, Hilbert transform and Marginal Hilbert spectrum to determine the impedance tensor and compare the results with the traditional data processing method.

1 Introduction

The classic Fourier transform is based on the decomposition of the signal according to fixed basis functions and a fixed frequency set determined by the sampling frequency and the data or window length. It assumes that the signal is periodic or stationary. The Fourier spectrum defines uniform harmonic components globally, therefore, it needs many additional harmonic components to simulate non-stationary data that are non-uniform globally. As a result, it spreads the energy over a wide frequency range. Constrained by the energy conservation, these spurious harmonics and the wide frequency spectrum cannot faithfully represent the true energy density in the frequency space. The geomagnetic time series, however, are characterized by a change of frequency content with time and non-stationarity. This change of frequency content may not be imaged by the Fourier analysis as the Fourier spectra averages over time. A way out is Hilbert spectrum, which does not assume a set of fixed frequencies and allows the imaging of frequency content as a function of time. However calculation of Hilbert spectra is unstable applied on geomagnetic time series directly. But with the development of the EMD method, a new way of decomposing data exists, which lead to stable calculation of the Hilbert spectra.

Hilbert-Huang transform (HHT), introduced by Huang on the basis of the classic Hilbert transform (Huang et al., 1998) [1] [2] [3], is a new non-stationary signal processing technique. The key part of the method is the *Empirical Mode Decomposition* (EMD), with which any complicated data set can be decomposed into a finite and often small number of *intrinsic mode functions* (IMFs) that admit a well-behaved *Hilbert Transform* to obtain the physical meaningful *instantaneous frequencies* (IF). The final presentation of the results is an energy-frequency-time distribution, designated as the *Hilbert spectrum*. The EMD is an adaptive decomposition of the data based on local characteristic time scales of a signal, it is applicable to non-stationary processes, and therefore, it is highly efficient. With the Hilbert transform, the frequency content in each IMF is not fixed but determined by the signal itself, and may change with time. Furthermore, since the method eliminates the need for spurious harmonics to represent non-stationary signals, the corresponding Hilbert spectrum will not lead to energy diffusion and leakage.

In this paper, the Empirical Mode Decomposition (EMD) method will be introduced in section 2. Based on a simple example, we show how to apply the EMD to a time series to obtain the Intrinsic Mode Function (IMF) and we illustrate the concepts of the Hilbert transform, instantaneous amplitude, instantaneous frequency, Hilbert spectrum and marginal spectrum. In section 3, we apply the EMD method to the real

MT data set and calculate the impedance tensor based on the Hilbert spectra. The results are compared to the impedance calculated by the classical Fourier approach. Conclusion and outlook is given in section 4.

2 EMD method

The empirical mode decomposition (EMD) is based on the direct extraction of the energy associated with various intrinsic time scales to generate a collection of intrinsic mode functions (IMF). Each IMF allows a well-behaved Hilbert transform, from which the instantaneous frequency can be calculated. Thus, we can localize any event on the time as well as the frequency axis. The decomposition can be viewed as an expansion of the data in terms of the IMFs. Each IMF can be, just like the underlying time series, non-stationary. Most important of all, the IMFs are adaptive. The requirements for locality and adaptivity are very crucial for non-stationary data, since we need the instantaneous frequency and energy rather than the global frequency and energy defined by the Fourier spectral analysis.

In order to define a meaningful instantaneous frequency, the intrinsic mode functions (IMF) have to satisfy two conditions [1] [2] [3]:

- (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and
- (2) at any point, the mean value of the envelope defined by the local maxima and the envelope by the local minima is zero.

These requirements to intrinsic mode function are adopted because they represent the oscillation mode imbedded in the data. Each IMF is capable of containing a modulated frequency and amplitude and therefore might be of non-stationary character.

An IMF represents a simple oscillatory mode as opposed to a simple harmonic function. Based on the above definitions, any signal $x(t)$ can be decomposed as follows [1] [2] [3]:

- (1) Identify all the local extrema, and then connect all the local maxima by a cubic spline line as the upper envelope.
- (2) Repeat the procedure for the local minima to produce the lower envelope. The upper and lower envelopes should cover all the data between them.
- (3) The mean of upper and low envelope value is designated as m_1 ; and the difference between the signal $x(t)$ and m_1 is the first component, h_1 ; i.e.

$$x(t) - m_1 = h_1$$

Ideally, if h_1 is an IMF, then h_1 is the first component of $x(t)$.

- (4) If h_1 is not an IMF, h_1 is treated as the original signal and repeat (1), (2), (3); then

$$h_1 - m_{11} = h_{11}$$

After repeated sifting, i.e. up to k times, h_{1k} becomes an IMF, that is

$$h_{1(k-1)} - m_{1k} = h_{1k}$$

Then it is designated as

$$c_1 = h_{1k}$$

The first IMF component is obtained from the original data. c_1 should contain the finest scale or the shortest period component of the signal.

- (5) Separate c_1 from $x(t)$ by

$$r_1 = x(t) - c_1$$

where r_1 is treated as the original data and repeat the above processes until the second IMF component c_2 of $x(t)$ has been derived. The above process is repeated n times until n -IMFs of the signal $x(t)$ have been determined.

$$\begin{aligned} r_1 - c_2 &= r_2 \\ &\vdots \\ r_{n-1} - c_n &= r_n \end{aligned}$$

The decomposition process can be stopped when r_n becomes a monotonic function from which no more IMFs can be extracted. We finally obtain

$$x(t) = \sum_{j=1}^n c_j + r_n$$

Thus, one can achieve a decomposition of the signal into n -empirical modes and a residue r_n , which is the mean trend of $x(t)$. The IMFs $c_1, c_2, \dots, c_n$ include different frequency bands, where highest frequencies are usually found in the first IMF and lower frequencies in subsequent IMFs. The frequency content in each IMF changes with time and the frequency band found in one IMF might overlap with the frequency band in another IMF. However, at each point in time, no two IMFs contain the same frequency.

For example, figure 1 shows a time series that is the sum of two sine waves with an modulated amplitude wave given by:

$$x(t) = 2\sin(2\pi \times 15t) + \sin(2\pi \times 5t)\sin(2\pi \times 0.1t) + 4\sin(2\pi \times t)$$

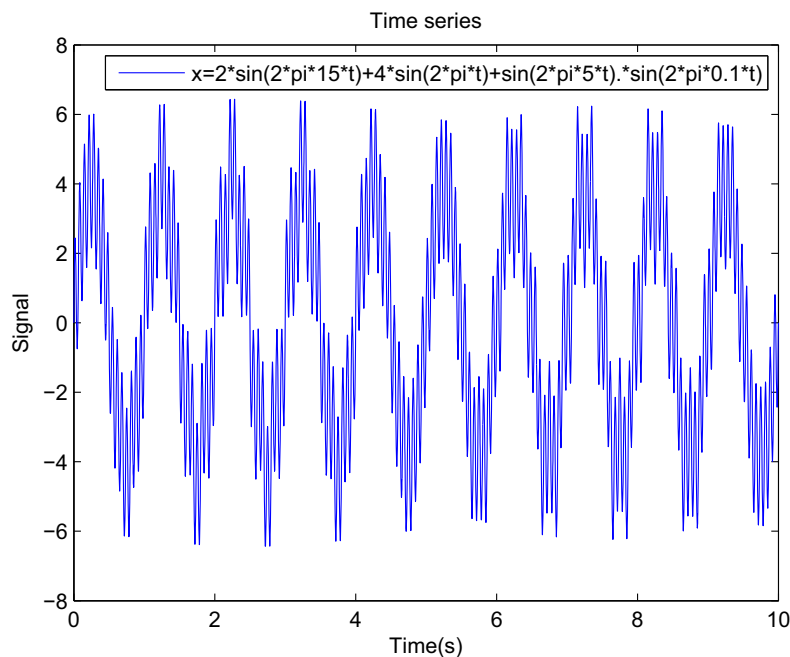


Figure 1: The time series example.

Figure 2 shows the IMFs of the time series. It is clear that the first IMF shows the 15Hz component of the time series, the second IMF the 5Hz modulated signal and the third IMF the 1Hz signal, respectively. The last IMF shows the trend of the signal.

In a second step, the IMFs are submitted to the Hilbert transformation process, which is defined as:

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt'$$

where P indicates the Cauchy principle value. With this definition, $X(t)$ and $Y(t)$ form the complex conjugate pair, which can be composed to an analytic signal $Z(t)$, as

$$Z(t) = X(t) + iY(t) = A(t)e^{i\theta(t)}$$

where

$$A(t) = \sqrt{X^2(t) + Y^2(t)}$$

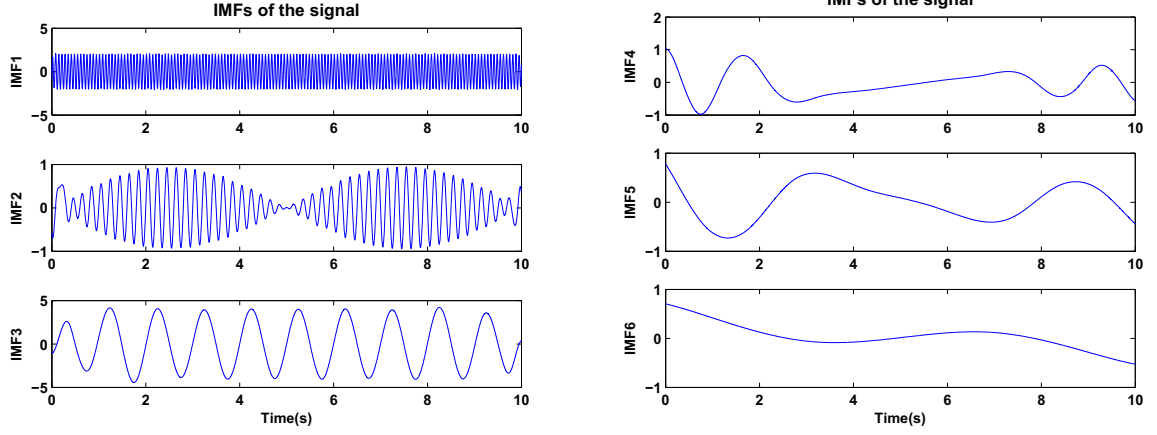


Figure 2: The 6 IMF components of time series.

and

$$\theta(t) = \text{atan}\left(\frac{Y(t)}{X(t)}\right)$$

We call $A(t)$ *instantaneous amplitude* and $\theta(t)$ *instantaneous phase*.

If we express the phase with a Taylor series then

$$\theta(t) = \theta(t_0) + (t - t_0)\theta'(t_0) + R$$

where R is small when t is close to t_0 . The analytic signal becomes

$$Z(t) = X(t) + iY(t) = A(t)e^{i\theta(t)} = A(t)e^{i(\theta(t_0) - t_0\theta'(t_0))}e^{it\theta'(t_0)}e^{iR}$$

and we see that $\theta'(t_0)$ has the role of frequency if R is neglected. This makes it natural to introduce the notion of *instantaneous (angular) frequency*, that is

$$\omega(t) = \frac{d\theta(t)}{dt}.$$

After performing the Hilbert transform to each IMF component, the original signal can be expressed as the real part (RP) of the analytic signal in the following form:

$$x(t) = \text{RP} \sum_{j=1}^n A_j(t)e^{i\theta_j(t)} = \text{RP} \sum_{j=1}^n A_j(t)e^{i \int \omega_j(t) dt}$$

Here we left out the residue r_n on purpose, for it is either a monotonic function or a constant.

The above equation enables us to represent the amplitude and the instantaneous frequency as functions of time in a three-dimensional plot, in which the amplitude can be contoured on the frequency-time plane. This frequency-time distribution of the amplitude is designated as the *Hilbert spectrum* $H(\omega, t)$.

With the Hilbert spectrum defined, we can also define the marginal spectrum, $h(\omega)$ as

$$h(\omega) = \int_0^T H(\omega, t) dt$$

where T is the total data length. The Hilbert spectrum offers a measure of amplitude contribution from each frequency and time, while the marginal spectrum offers a measure of the total amplitude contribution from each frequency.

Instantaneous Frequency An ambiguity inherent in the instantaneous phase renders equation above impractical for calculating the instantaneous frequency: only the principal values of the phase are computed, which causes 2π phase discontinuities. Instantaneous frequency is instead calculated by another equation, directly derived from equation above:

$$\omega(t) = \frac{X(t)Y'(t) - X'(t)Y(t)}{X^2(t) + Y^2(t)}$$

where the primes denote differentiation with respect to time [4].

Above equation requires two differentiations to calculate the instantaneous frequency. To avoid these differentiations, three formulas that approximate instantaneous frequency and are faster to compute are used[4].

$$\begin{aligned}\omega_a(t) &= \frac{1}{T} \text{atan} \frac{X(t)Y(t+T) - X(t+T)Y(t)}{X(t)X(t+T) + Y(t)Y(t+T)} \\ \omega_b(t) &= \frac{1}{2T} \text{atan} \frac{X(t-T)Y(t+T) - X(t+T)Y(t-T)}{X(t-T)X(t+T) + Y(t-T)Y(t+T)} \\ \omega_c(t) &= \frac{4}{T} \text{atan} \frac{X(t)Y(t+T) - X(t+T)Y(t)}{(X(t) + X(t+T))^2 + (Y(t) + Y(t+T))^2}\end{aligned}$$

where T is sample period.

Now we can calculate the instantaneous frequencies and instantaneous amplitudes of each IMFs of the above example. Figure 3 shows the instantaneous frequencies (left) and amplitude (right). Frequencies components can be clearly seen in the left figure. It is obvious that the time series just contains discrete frequencies as opposed to the continues frequency band in Fourier analysis. The small oscillations in the frequency band and the inaccurate frequencies at the boundary are due to the numerical calculation.

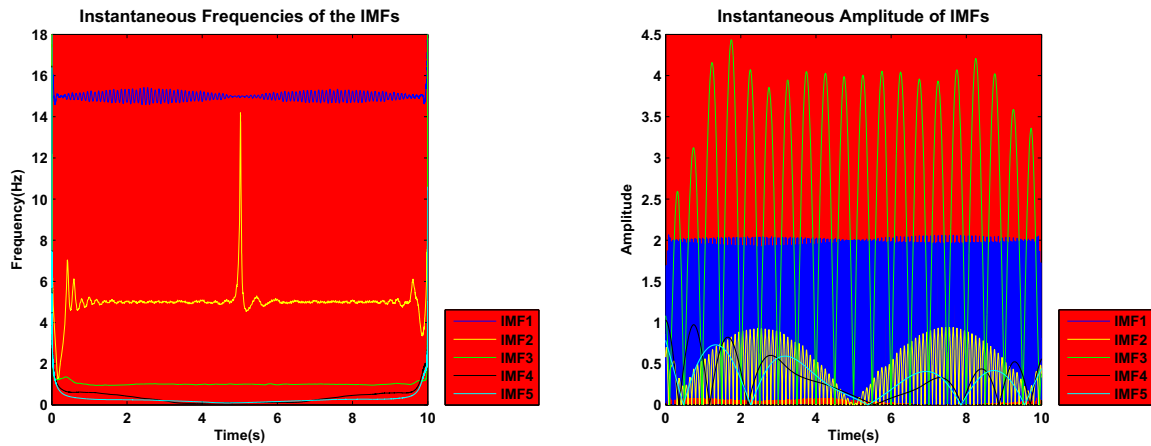


Figure 3: The instantaneous frequency and amplitude of IMF components of the time series.

We can calculate the Hilbert marginal spectrum of the time series. The comparison with the Fourier spectrum is shown in Figure 4. One can see that the peaks of the three components are clearly separated and the resolution ratio is higher in the Hilbert marginal spectrum since there is severe energy leakage in Fourier spectrum. This gives us an idea to apply this efficient method to estimate the apparent resistivity from measured electro- and magneto- fields time series.

3 EMD and HT to the MT data processing

Now, we apply the EMD and HT to the magnetotelluric "FLARE10" raw data measured near Faroe island. The MT stations and the E- and B-field time series of MT station 11 are shown in Figure 5.

We applied EMD method to the E- and B-field time series to obtain the IMFs of both time series, which for the E-field is shown in Figure 6. Through a Hilbert transform of each IMF, one can obtain the

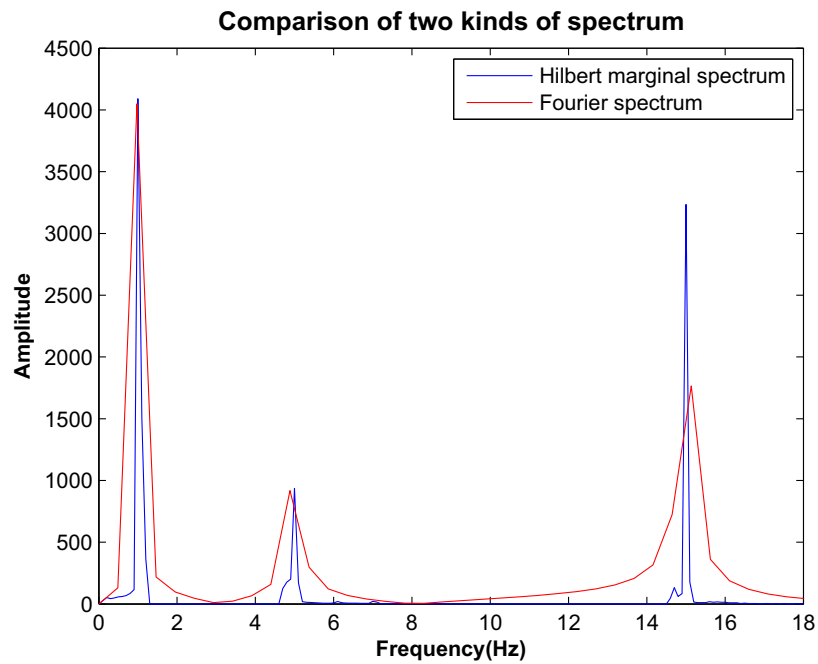


Figure 4: The comparison of Hilbert marginal spectrum and Fourier spectrum of the time series.

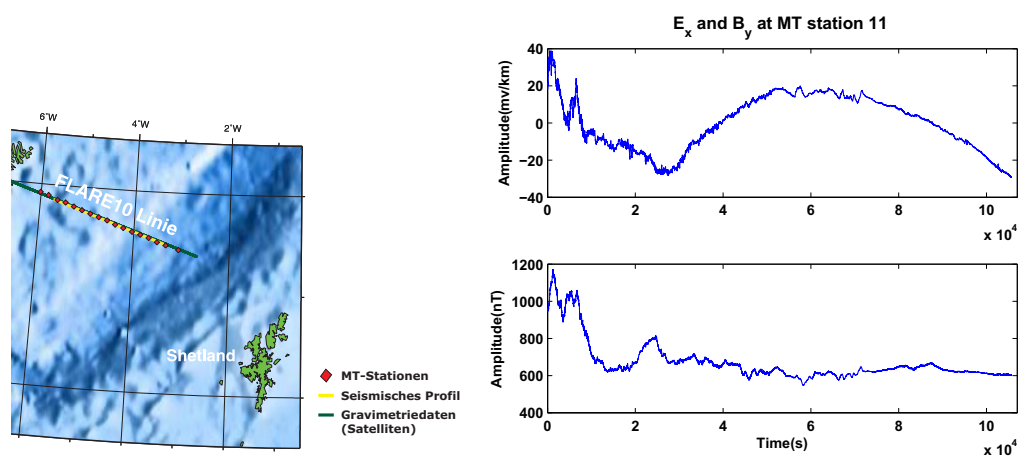


Figure 5: MT stations near Faroe island. The time series of E- and B-field at MT station 11.

instantaneous frequencies and amplitudes of the signal. The instantaneous frequencies for all IMFs are shown in Figure 7. Through the Hilbert spectra it becomes obvious that the frequencies contained in the signal are discrete a fact that is not visible in the Fourier analysis.

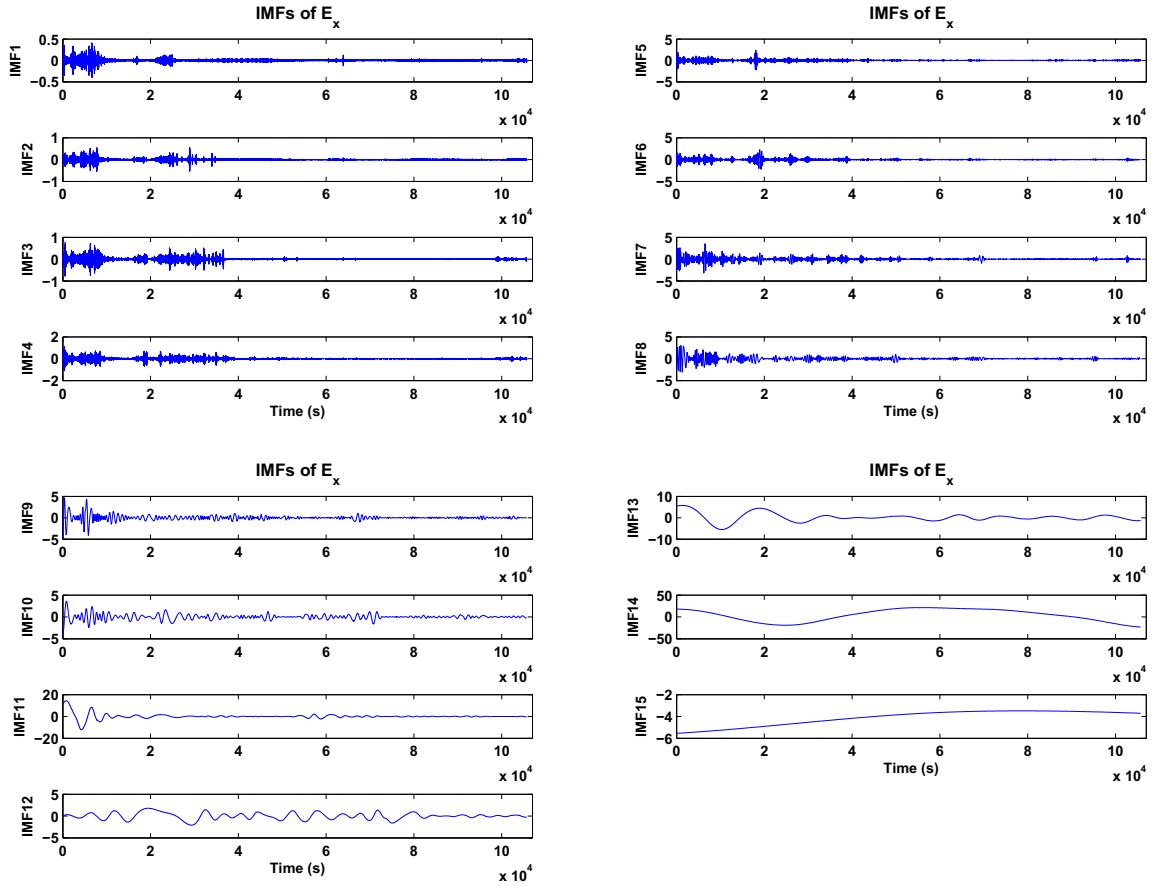


Figure 6: The 15 IMF components of $e(t)$.

Now, we can choose some frequency bands to calculate the Hilbert marginal spectrum of each frequency band to calculate the impedance given by [5] [8] [9] [7] [10] [6]

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} Z_{xx}(\omega) & Z_{xy}(\omega) \\ Z_{yx}(\omega) & Z_{yy}(\omega) \end{pmatrix} \begin{pmatrix} B_x(\omega) \\ B_y(\omega) \end{pmatrix}$$

to calculate the transfer function.

For simplification, we just consider a 2D case in which case the above formula reduces to:

$$\begin{cases} Z_{xy} = \frac{\mu_0 E_x(\omega)}{B_y(\omega)} \\ Z_{yx} = \frac{\mu_0 E_y(\omega)}{B_x(\omega)} \end{cases}$$

In order to obtain the impedances, we introduce three strategies:

- Chose a time window, compute the Hilbert marginal spectrum pair of E- and B-field w.r.t. certain frequency band.
- Shift the window with some percentage overlapping and compute another spectrum pair, and so on.
- For all Hilbert marginal spectrum pairs, make a robust least square fit to obtain the ratio $E(\omega)/B(\omega)$ and the error.

One of the robust fit for frequency 0.0045Hz is shown in Figure 8.

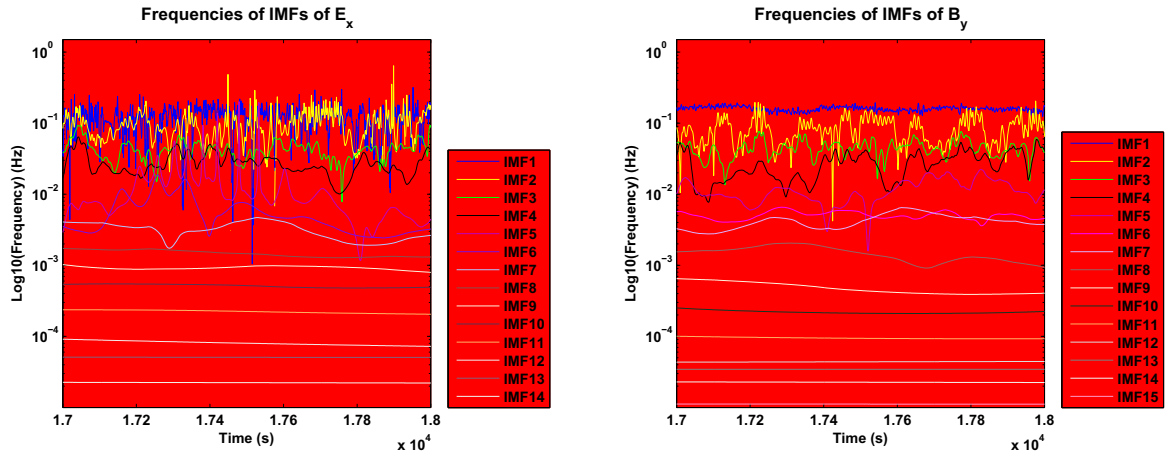


Figure 7: The instantaneous frequencies of 15 IMF components of $e(t)$ and $b(t)$. section: 1000s

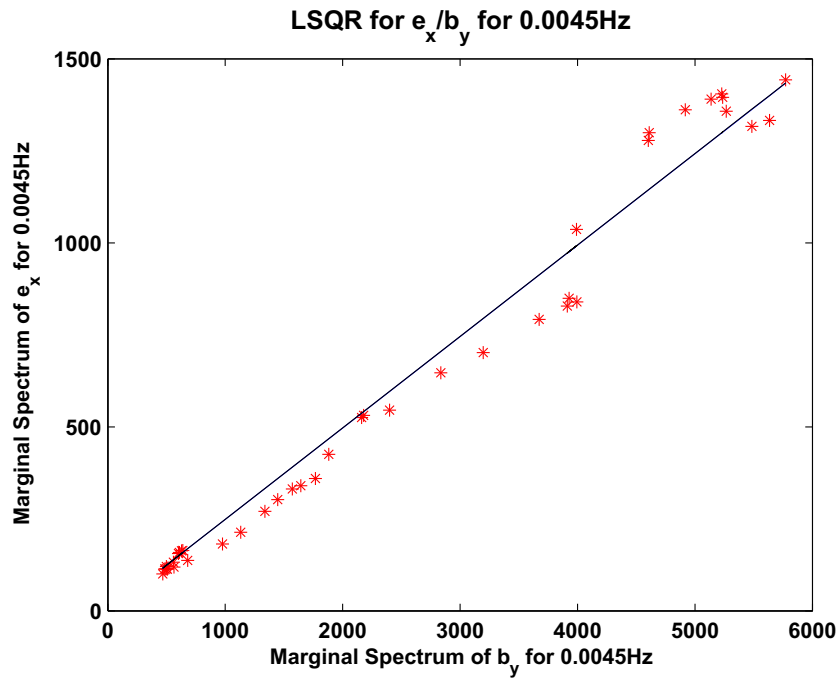


Figure 8: The robust least square fit of the Hilbert marginal spectrum pairs.

After estimation of the ratios $E(\omega)/B(\omega)$ for each chosen frequency band, we can calculate the impedance ρ_{xy} and ρ_{yx} and obtain the apparent resistivities. One estimation of the apparent resistivity of TM mode is shown in Figure 9.

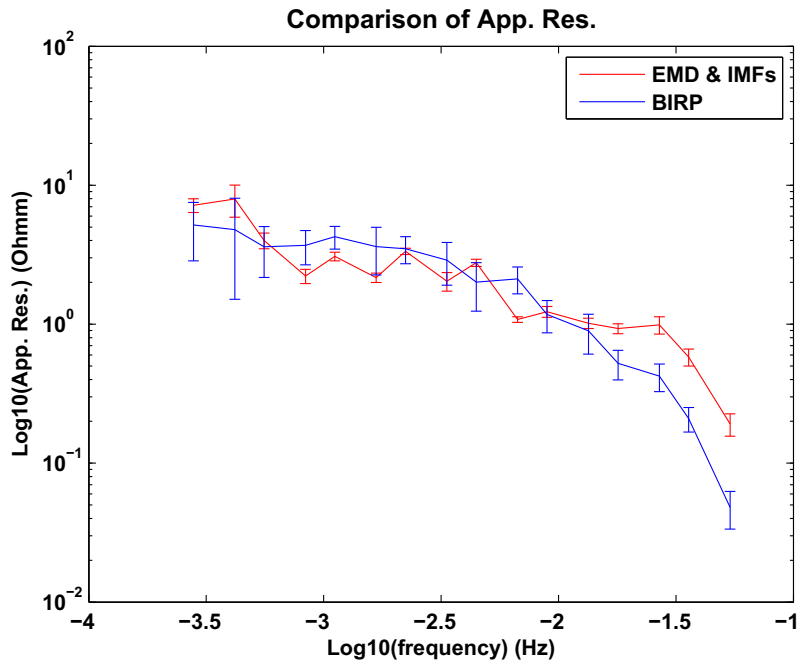


Figure 9: The robust least square fit of the Hilbert marginal spectrum pairs.

4 Conclusion and outlook

In this paper, a new method to deal with the non-stationary MT time series is introduced. The method is easily handled, delivers satisfactory results and may be applied to raw data. Since the EMD method is not tied to specific basis functions but on the data itself, it is adaptive and highly efficient. EMD and HT provide means to analysis the change in frequency content of geomagnetic time series at a high resolution. We are investigating ways of using EMD and HT for transfer function calculations. We believe that it is an interesting new approach to MT data processing which is worth developing forward.

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Krylov Subspace Approximation for TEM Simulation in the Time Domain

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Summary

Forward transient electromagnetic modeling requires the numerical solution of a linear constant-coefficient initial-value problem for the quasi-static Maxwell equations. After discretization in space this problem reduces to a large system of ordinary differential equations, which is typically solved using finite-difference time-stepping. We compare standard time-stepping schemes such as the explicit and unconditionally stable Du Fort-Frankel scheme with the more recent Runge-Kutta-Chebyshev methods, which are designed specifically for parabolic initial value problems, with Krylov subspace techniques for the explicit solution of the initial value problem using the matrix exponential. Besides the classic Arnoldi/Lanczos approximation we also consider restarted Arnoldi approximations as were recently proposed in (Eiermann & Ernst, 2006). These restarted schemes have the advantage of requiring only an a priori fixed amount of memory storage, a significant aspect in the context of 3D simulations.

We also present a recent efficient implementation (Afanasjew, Ernst, Güttel, & Eiermann, to appear) of the restarted Arnoldi method for evaluating the matrix exponential.

1 TEM – Governing Equations

Geophysical exploration using transient electromagnetic fields (TEM) is a technique for inferring properties of the subsurface by observing the response over time to controlled electromagnetic sources. Here we consider the forward problem of computing the electromagnetic field due to a vertical magnetic dipole, a configuration often used in practice.

The governing equations are the quasi-static Maxwell's equations

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{e} \right) + \partial_t \sigma \mathbf{e} = -\partial_t \mathbf{j}^e, \quad (1)$$

where

$\mathbf{e} = \mathbf{e}(\mathbf{x}, t)$	is the electric field,
$\mu = \mu(\mathbf{x})$	is the magnetic permeability,
$\sigma = \sigma(\mathbf{x})$	is the electric conductivity and
$\mathbf{j}^e = \mathbf{j}^e(\mathbf{x}, t)$	is the impressed source current density.

The spatial domain is typically a parallelepiped $\Omega \subset \mathbb{R}^3$ whose upper boundary is either at ground surface level or above it. In the simplest model, the perfect conductor boundary condition $\mathbf{n} \times \mathbf{e} = \mathbf{0}$ is imposed on all six faces of $\partial\Omega$.

The impressed source current is typically of *shut-off* type, i.e., of the form

$$\mathbf{j}^e(\mathbf{x}, t) = \mathbf{q}(\mathbf{x})H(-t), \quad (2)$$

where H denotes the Heaviside unit step function and the vector field $\mathbf{q}$ describes the spatial current pattern.

2 Semidiscretization in Space

Omitting the impressed source current $\mathbf{j}^e(\mathbf{x}, t)$ in (1)—since we are looking at times $t > 0$ —the PDE becomes

$$\partial_t \mathbf{e} = -\frac{1}{\sigma} \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{e} \right).$$

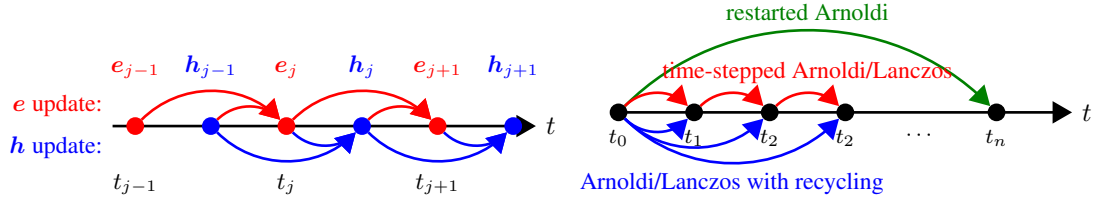


Figure 1: Leap-frog iteration of the Du Fort-Frankel method with time-interleaved electric and magnetic fields (left). Considered computational strategies for Krylov subspace methods (right).

To discretize this equation in space, we introduce a graded tensor-product mesh on the spatial domain Ω , which is refined near the source. The curl-curl equation is then discretized using the well-known Yee finite-difference scheme (Yee, 1966), which transforms the PDE to the linear first-order ordinary differential equation

$$\partial_t \mathbf{e} = A\mathbf{e}, \quad \mathbf{e}(t_0) = \mathbf{e}_0, \quad (\text{ODE})$$

where the matrix A represents the discrete action of $-1/\sigma \nabla \times (1/\mu \nabla \times \cdot)$ on the spatial discretization of the electric field $\mathbf{e}$. The solution of (ODE) is explicitly given by

$$\mathbf{e}(t) = e^{(t-t_0)A} \mathbf{e}_0. \quad (3)$$

Our objective, given an initial field $\mathbf{e}_0$ at $t = t_0$, is to evaluate the solution $\mathbf{e}(t)$ at given discrete time values $t_1 < t_2 < \dots < t_n$ in an integration interval $[t_0, t_n]$.

3 Time-Stepping

Semi-discretized initial-value problems like (ODE) are traditionally solved using a time-stepping scheme. The reference scheme for our comparisons is the Du Fort-Frankel scheme, an explicit time-integrator with a weak stability constraint on the time step, which was proposed for TEM forward modelling in (Wang & Hohmann, 1993).

Given an initial electric field $\mathbf{e}_0$ at time t_0 , and an initial magnetic field $\mathbf{h}_0$ at time $t_0 + \Delta t_0/2$ we perform a leap-frog iteration (Figure 1, left). In each step we first compute the electric field $\mathbf{e}_j$ from $\mathbf{e}_{j-1}$ and $\mathbf{h}_{j-1}$ and then the magnetic field $\mathbf{h}_j$ from $\mathbf{h}_{j-1}$ and $\mathbf{e}_j$. With $\delta_{\min}$ denoting the smallest mesh size this method is stable if

$$\Delta t_j = t_{j+1} - t_j < \delta_{\min} \sqrt{\frac{\mu_{\min} \sigma_{\min} t_j}{6}}.$$

Apart from Du Fort-Frankel we tested ROCK4, an implementation of fourth order Chebyshev methods. These explicit Runge-Kutta methods are particularly well suited for parabolic initial values problems like those discussed here.

4 Krylov Subspace Methods

As an alternative to time-stepping, one could evaluate (3) directly, which entails multiplying a matrix exponential with the initial vector $\mathbf{e}_0$. As the matrix A is large and sparse, this can be achieved efficiently with a Krylov subspace approximation. This gives rise to various implementations depending on time and memory constraints. Some computational strategies based on Krylov subspace methods are illustrated in Figure 1 (right).

4.1 Krylov Subspace Methods for Matrix Functions

Given a square matrix $A \in \mathbb{R}^{N \times N}$ (large and sparse), a vector $\mathbf{b} \in \mathbb{R}^N$ and a scalar function $f(x)$ which is defined in a neighborhood of the eigenvalues of A , then

$$f(A) = p(A),$$

where $p(\cdot)$ is a polynomial of degree $< N$ that Hermite-interpolates f in the eigenvalues of A . By $\mathcal{K}_m(A, \mathbf{b})$ we denote the m -th Krylov space of $\mathbf{b}$ and A , that is

$$\mathcal{K}_m(A, \mathbf{b}) = \text{span}\{\mathbf{b}, A\mathbf{b}, A^2\mathbf{b}, \dots, A^{m-1}\mathbf{b}\}.$$

Since, by definition, $f(A)\mathbf{b}$ is a polynomial in A of degree $< N$ there holds

$$f(A)\mathbf{b} \in \mathcal{K}_N(A, \mathbf{b}).$$

Krylov subspace methods approximate $f(A)\mathbf{b}$ by suitable vectors $\mathbf{f}^m = p_m(A)\mathbf{b} \in \mathcal{K}_m(A, \mathbf{b})$ from lower order Krylov spaces, i.e., with $m \ll N$. The basic steps are as follows:

- Generate an orthonormal basis $V_m = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_m]$ of $\mathcal{K}_m(A, \mathbf{b})$ using the *Arnoldi process*, yielding the projected matrix $H_m := V_m^T A V_m \in \mathbb{R}^{m \times m}$, which is an upper Hessenberg matrix.
- The *Krylov subspace approximation of order m* is defined as

$$\mathbf{f}^m := V_m f(H_m) V_m^T \mathbf{b} = \|\mathbf{b}\| V_m f(H_m) [1, 0, \dots, 0]^T.$$

If A is Hermitian then the *Lanczos process*, in place of the Arnoldi process, may be used to generate the basis. In this case H_m is tridiagonal and, instead of orthonormalizing each vector $\mathbf{v}_m$ against all preceding vectors $\mathbf{v}_1, \dots, \mathbf{v}_{m-1}$, there exists a three-term recurrence involving only $\mathbf{v}_{m-2}$, $\mathbf{v}_{m-1}$ and $\mathbf{v}_m$.

4.2 Time-Stepped Arnoldi Method

For each time-step j we compute the Arnoldi approximation of order $m = m(j)$

$$\mathbf{f}_{j+1}^m \in \mathcal{K}_m(A, \mathbf{f}_j^m) \text{ for } f(x) = e^{(t_{j+1}-t_j)x},$$

where $\mathbf{f}_0^m = \mathbf{e}_0$. From error analysis of Krylov subspace methods it is known that to guarantee a certain relative error of the Krylov approximation $\mathbf{f}_{j+1}^m$ we should choose

$$m = m(j) \sim \|(t_{j+1} - t_j)A\|^{1/2}.$$

The drawback of this method is that we build a new Krylov space for each time-step which may be computationally unfeasible.

4.3 Arnoldi Method with Recycling

For each time step j we compute the Arnoldi approximation

$$\mathbf{f}_j^m \in \mathcal{K}_m(A, \mathbf{e}_0) \text{ for } f(x) = e^{(t_j-t_0)x},$$

where we choose $m = m(j) \sim \|(t_j - t_0)A\|^{1/2}$.

Our proposed method reuses the computed basis vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{m(j)}$ for the time-step $j + 1$, just adding the vectors $\mathbf{v}_{m(j)+1}, \mathbf{v}_{m(j)+2}, \dots, \mathbf{v}_{m(j+1)}$.

This approach was found to be most efficient, although the number $m(j)$ of required Krylov vectors is slightly larger than that for the *time-stepped Arnoldi method*, since the time interval is longer.

4.4 Restarted Arnoldi Method

The restarted Arnoldi method introduced in (Eiermann & Ernst, 2006) generates a succession of Krylov spaces of a fixed maximal size m . The approximations are then chosen from the union of all Krylov spaces generated up to that point. The advantage is that one never needs to store more than m basis vectors. Moreover, for the exponential function required in this context, the superlinear convergence of the unrestarted method is retained.

In (Afanasjew et al., to appear), this method was further enhanced by evaluating $f(H_m)$ using a suitable rational approximation of fixed order, which amounts computationally to the solution of a small number of linear systems of equations with coefficient matrix H_m , a small, constant amount of work in each restart cycle. Moreover, error estimators giving upper and lower bounds of the approximation error can be generated inexpensively in each step.

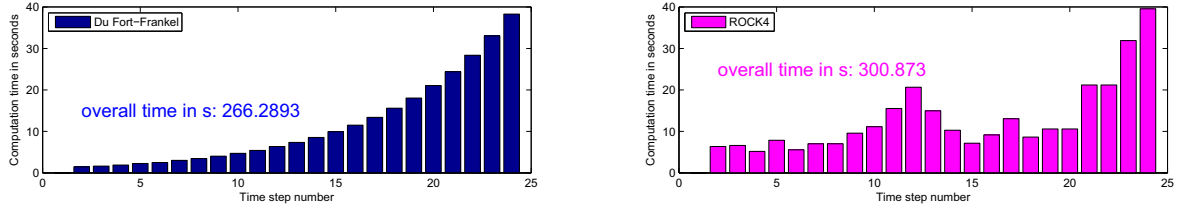


Figure 2: Du Fort-Frankel/ROCK4. Computational effort.

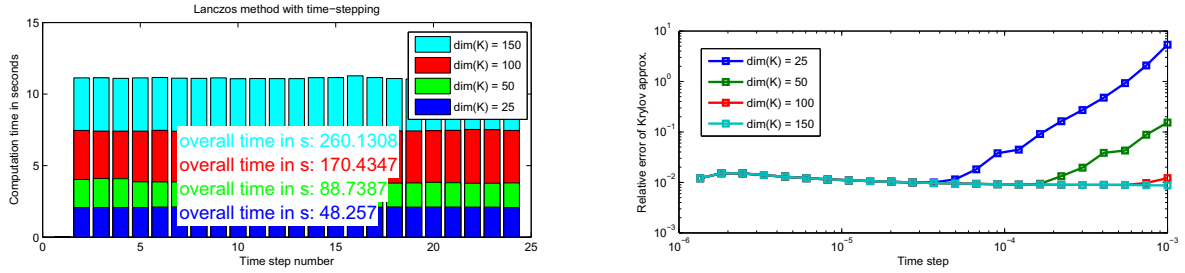


Figure 3: Lanczos method with time-stepping. Here m is constant for all time steps j .

5 Numerical Experiments

We solve (1) on a cube with constant conductivity and permittivity. The finite-difference mesh consists of $58 \times 58 \times 58$ cells, resulting in 565,326 degrees of freedom for the electric field. We evaluate the solution at 24 logarithmically equispaced times between 10^{-6} s and 10^{-3} s. All computations were performed until the relative error was below 10^{-2} .

In Figure 2 we compare the performance of the Du Fort-Frankel method with the more sophisticated ROCK4 method. As can be seen in the bar graphs, ROCK4 gets outperformed despite performing fewer time steps. We believe that this is due to its relatively high per-step overhead compared to the simplistic iteration scheme in Du Fort-Frankel.

Turning to the Krylov subspace methods, Figure 3 shows the computational effort for the time-stepping strategy. The times are merely illustrative since using a constant Krylov subspace size for every (exponentially growing) time step is wasteful. The figure nicely relates the size of the Krylov subspace to the achievable relative error.

The tradeoff between computation time and memory consumption can be seen in Figure 4. We compare the running time—total and per time step—and the size of the required Krylov subspace, that directly corresponds to the required memory. In this example, having enough memory available can save up to a third of the overall computation time.

Table 1 summarizes the performance of the restarted Arnoldi method for a single large time step from 10^{-6} s to 10^{-3} , requiring an error below 10^{-12} . While the non-restarted Krylov algorithms perform faster they require a—in most cases—prohibitively big amount of memory compared to the constant storage requirements for the restarted variant.

Finally, Figure 5 contains a plot of the transient of the electric field at a distance of 26.2 m from the source. We see a good agreement between the transients produced by both methods. The faster Krylov method is even somewhat closer to the analytic solution since it—in contrast to the Du Fort-Frankel method—does not require discretization in time.

m	time[s]	mvp	error
70	112	1400	9.13e-13
90	118	1350	2.01e-13
full 2-pass	144	2144	9.93e-13
full 1-pass	86	1072	9.93e-13

Table 1: Computing times for the restarted Arnoldi method for various restart lengths.

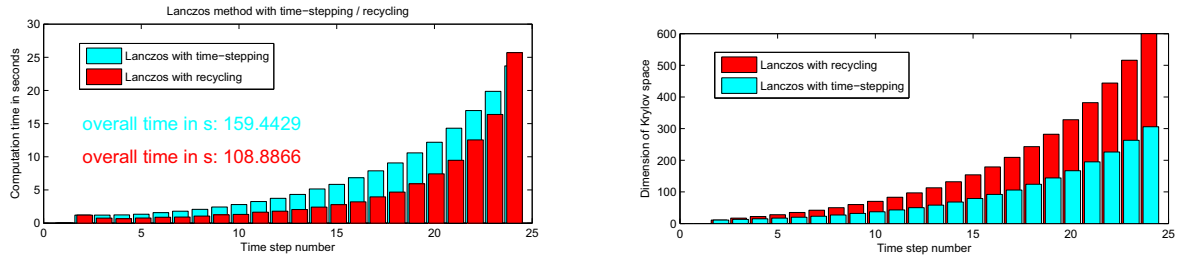


Figure 4: Comparing Lanczos time-stepping and recycling.

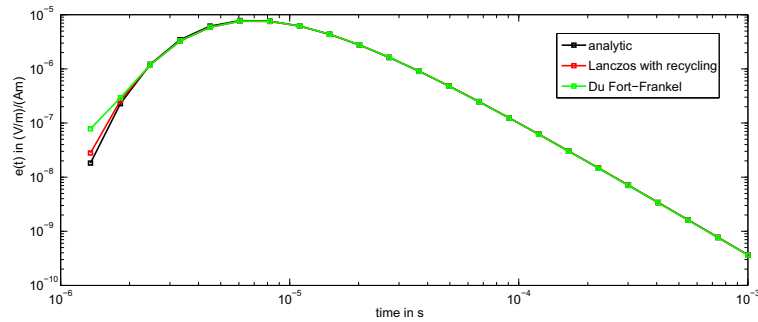


Figure 5: Transient electric field at a distance of 26.2 m from the source.

Conclusions

Krylov subspace approximation is an efficient computational tool for integrating the initial value problem (ODE), arising in TEM forward modelling. The restarted Arnoldi method for the matrix exponential offers the possibility for the user to tradeoff storage requirements against speed, a possibility not offered by competing Krylov subspace methods such as SLDM.

Acknowledgments

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Electromagnetic induction in the spherical rotating earth due to asymmetric current loops or belts

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Abstract

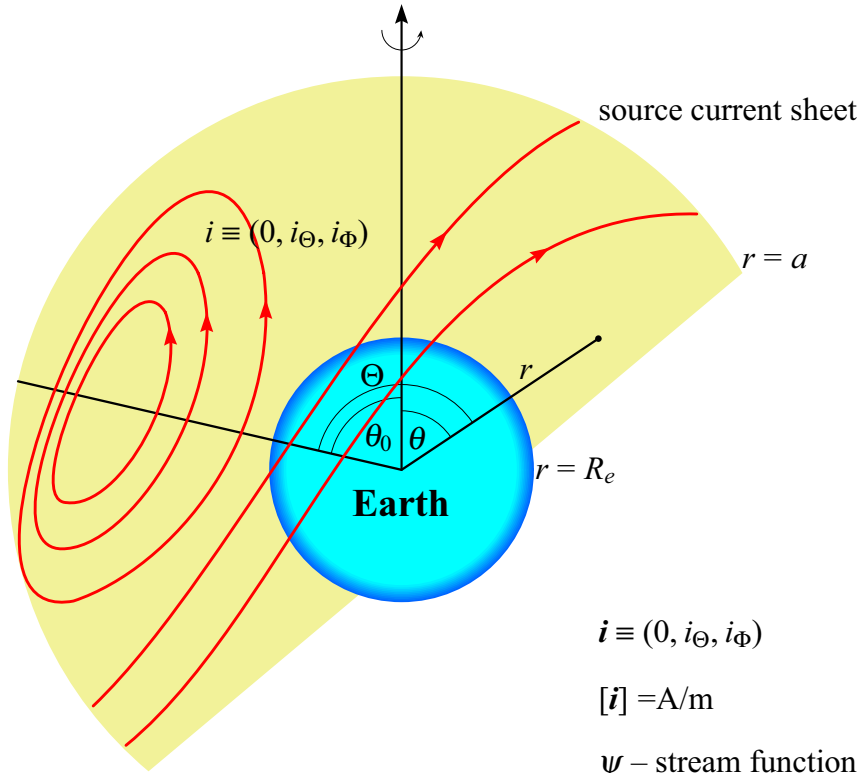
The paper presents theoretical formulae for calculation of the spherical harmonic expansion of magnetic potential for current loop or belt model for stationary current systems in the high ionosphere and magnetosphere. The axis of symmetry for the current system does not coincide with the axis of Earth's rotation. Due to inclination of these axes there occurs azimuthal asymmetry of the exciting magnetic field which causes time-harmonic field for the observator on the rotating Earth. Theoretical EM field can be calculated by means of theory EM induction for the multilayered sphere for the superposition of spherical waves for discrete angular frequencies $m \cdot \Omega$, where m is order of the spherical harmonics and Ω is angular frequency of the Earth's rotation. The paper presents theoretical graphs of time variations of components (B_x, B_y, B_z) at selected observatory on the surface of the rotating Earth for near auroral (polar) or distant quasi equatorial current belts. There is shown that in the time course of magnetic field is dominant diurnal time period, corresponding to $m = 1$, while $m = 0$ corresponds to the steady external field.

Introduction

The advanced geomagnetic research of the Earth's space has discovered that in the Earth's ionosphere and magnetosphere there exist numerous huge electric current systems of complex geometry and time changes. The most known and closest to the Earth's surface is the ionospheric system generating the S_q geomagnetic variations (Campbell, 1989), another current systems occur in the auroral oval (Akasofu, 1972) which cause the substorms, etc. Very important is the ring current system at the distances $2-5 R_e$, which persists also in quiet magnetospheric state and during the perturbed solar wind conditions becomes the source of geomagnetic storms. This ring current is volume distributed, its intensity is dependent on both the distance from the Earth's centre and polar angle Θ . The analyses presented e.g. in Nishida (1978) show interesting property that the current intensity in the interior ring at $r \approx 3 R_e$ is directed eastward (intensity about $I_1 \doteq + 80000$ A) and in the outer oval at $r \approx 4.5 R_e$ is current directed westward (intensity about $I_2 \doteq - 1100000$ A). Using the Amper's law we can easily find that this westward current is clearly dominant during the main phase of the magnetic storm, since during this phase the horizontal (northward) component of the geomagnetic field on the middle latitudes strongly decreases.

The circular current loop/belt model

Let us consider the source of the external magnetic field the steady current spherical sheet of radius $a > R_e$. The surface current density $\mathbf{i} \equiv (0, i_\Theta, i_\Phi)$ as shows scheme in Fig. 1. The basic formulae for the exciting magnetic field we derived by using stream function Ψ concept according to Smythe, 1950, where the magnetic field components are derived by the vector potential $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}}$. The "hat" symbol denotes fields in the stationary (non-rotating) co-ordinates (r, Θ, Φ) , firmly linked to the current source spherical sheet. We will use equivalent formulae for the magnetic field potential



$$\mathbf{i} \equiv (0, i_\Theta, i_\Phi)$$

$$[\mathbf{i}] = \text{A/m}$$

ψ – stream function

$$i_\Theta = \frac{-1}{a \sin \Theta} \frac{\partial \psi}{\partial \Phi}$$

$$i_\Phi = \frac{1}{a} \frac{\partial \psi}{\partial \Theta}$$

Fig. 1. Simplified spherical source current sheet

$\hat{V}(r, \Theta, \Phi)$ by putting $\hat{V} = -\partial(rW)/\partial r$ and $\hat{\mathbf{B}} = -\text{grad } \hat{V}$. If the stream function $\Psi(\Theta, \Phi)$ is expressed by the sum of spherical harmonics $S_n^m(\Theta, \Phi)$:

$$\Psi(\Theta, \Phi) = \sum_{n=1}^{\infty} \sum_{m=0}^n S_n^m(\Theta, \Phi), \quad (1)$$

then the magnetic field potential outside of source sheet ($r > a$) is:

$$\hat{V}_o = -\mu_0 \sum_{n=1}^{\infty} \frac{n}{2n+1} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^n S_n^m(\Theta, \Phi), \quad (2)$$

and in the interior ($r < a$):

$$\hat{V}_i = \mu_0 \sum_{n=1}^{\infty} \frac{n+1}{2n+1} \left(\frac{r}{a}\right)^n \sum_{m=0}^n S_n^m(\Theta, \Phi). \quad (3)$$

These potential are discontinuous on the source spherical sheet, there must be:

$$\mu_0 \Psi(\Theta, \Phi) = [\hat{V}_i - \hat{V}_o]_{r=a}. \quad (4)$$

For the zonal currents in the spherical sheet (current flow along parallels of co-latitude Θ) there is axial symmetry and we have:

$$\Psi(\Theta) = \sum_{n=1}^{\infty} c_n P_n(\cos \Theta). \quad (5)$$

Current density has only Φ component:

$$i_\Phi = \frac{1}{a} \frac{\partial \Psi}{\partial \Theta} = - \sum_{n=1}^{\infty} c_n P_n^1(\cos \Theta). \quad (6)$$

The potential of magnetic field which excites Earth $r < a$:

$$\hat{V}_i = \mu_0 \sum_{n=1}^{\infty} \frac{n+1}{2n+1} \left(\frac{r}{a}\right)^n c_n P_n(\cos \Theta). \quad (7)$$

In next explanations this potential we will denote as $\hat{V}^e$, since is external with respect to the Earth's body. As a simple model of the ring current is a circular loop of radius $a (> 3 R_e)$ bearing electric current I , situated near the geomagnetic equatorial plane. The angle between the axis of current circle and axis of symmetry of the belt we denote as α . The axis of current loop/belt we consider running through the Earth's centre and inclined by the angle θ_0 to the earth's north semi axis. The situation is shown in the Fig. 2 and $c_n = -I(2n+1) \sin \alpha P_n^1(\cos \alpha) / [2n(n+1)]$.

The magnetic field components for the region $r < a$ due to this single loop in the r, Θ variables are known e.g. from *Smythe (1950)* in the form:

$$\begin{aligned} \hat{B}_r^e &= \frac{\mu_0 I \sin \alpha}{2a} \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^{n-1} P_n^1(\cos \alpha) P_n(\cos \Theta), \\ \hat{B}_\Theta^e &= \frac{-\mu_0 I \sin \alpha}{2a} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{a}\right)^{n-1} P_n^1(\cos \alpha) P_n^1(\cos \Theta). \end{aligned} \quad (8)$$

Here $P_n^m(\cos \Theta)$ are the Legendre functions degree n orders $m = 0, 1$. These formulae were derived from the magnetic vector potential with non zero azimuthal component A_ϕ . For the

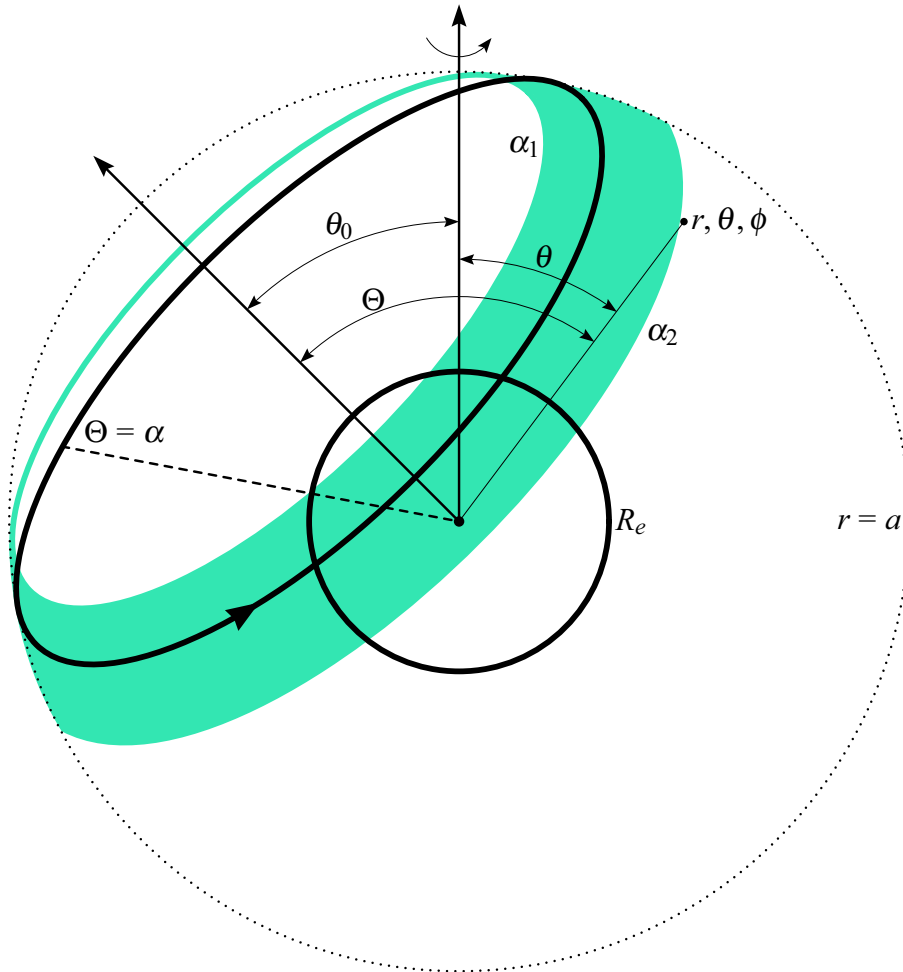


Fig. 2. The geometrical scheme of the current belt (green) at the sphere $r = a$ distributed at polar angle distances $\Theta \in \langle \alpha_1, \alpha_2 \rangle$ above the spherical Earth. The black circle refers to the current loop.

geomagnetic purposes there is more suitable find the scalar potential which negative gradient gives the components (1). After some modifications we can find this potential in the form:

$${}^e\hat{V}(r, \Theta) = -R \sum_{n=1}^{\infty} (r/R)^n A_n P_n(\cos \Theta), \quad (9)$$

where $R = R_e$ is the radius of the Earth and coefficients A_n are:

$$A_n = \frac{\mu_0 I \sin \alpha}{2a} \cdot \frac{1}{n} \left(\frac{R}{a} \right)^{n-1} \cdot P_n^1(\cos \alpha), \quad (10)$$

we can easily find that B_r and B_{Θ} are given by the derivatives:

$$\hat{B}_r = -\frac{\partial {}^e\hat{V}}{\partial r}, \quad \hat{B}_{\Theta} = -\frac{1}{r} \frac{\partial {}^e\hat{V}}{\partial \Theta}, \quad (11)$$

since $P_n^1(\cos \Theta) = -\partial P_n(\cos \Theta)/\partial \Theta$.

From the principle of superposition for stationary electromagnetic field it is clear that potential (2) can be generalized to the system of current loops carrying the intensity I_k , their position is given by the support sphere radius a_k and polar angle α_k . Then we obtain from (3) formula for coefficients of the exciting potential

$$\tilde{A}_n = \frac{\mu_0}{2n} \sum_{k=1}^N \frac{I_k \sin \alpha_k}{a_k} \cdot \left(\frac{R}{a_k} \right)^{n-1} \cdot P_n^1(\cos \alpha_k). \quad (12)$$

We can also transform this summation formula to the case of continuous distribution electric current density in the azimuthal direction $J_{\phi}(\Theta')$, which is distributed in the polar angle distances $\langle \alpha_1, \alpha_2 \rangle$ above the sphere $r > R$. Then we will have:

$$\tilde{A}_n = \frac{\mu_0}{2na} \left(\frac{R}{a} \right)^{n-1} \int_{\alpha_1}^{\alpha_2} J_{\phi}(\Theta') \sin \Theta' P_n^1(\cos \Theta') d\Theta'. \quad (13)$$

The magnetic field potential in co-ordinates (r, Θ) can be easily transformed into stationary spherical system (r, θ, ϕ) with polar axis identical with Earth rotation axis. Let the θ, ϕ co-ordinate angles of north pole crosssection of the current system axis symmetry are (θ_0, ϕ_0) . Then the expression for the $\cos \Theta$ will be:

$$\cos \Theta = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos(\phi - \phi_0). \quad (14)$$

Using the additional theorem for $P_n(\cos \Theta)$ (e.g. *Stratton, 1941*) we will have:

$$\begin{aligned} P_n(\cos \Theta) &= P_n(\cos \theta) P_n(\cos \theta_0) + \\ &+ 2 \sum_{m=1}^{\infty} \frac{(n-m)!}{(n+m)!} P_n^m(\cos \theta_0) P_n^m(\cos \theta) \cos m(\phi - \phi_0). \end{aligned} \quad (15)$$

The magnetic potential ${}^e\hat{V}$ will be real part of the complex expression:

$${}^e\hat{V}(r, \theta, \phi) = -R \sum_{n=1}^{\infty} (r/R)^n \sum_{m=0}^n C_{n,m} P_n^m(\cos \theta) \exp[-i m(\phi - \phi_0)], \quad (16)$$

where the spherical harmonics coefficients C_n are calculated from A_n using relation:

$$C_{nm} = A_n (2 - \delta_{m,0}) \frac{(n-m)!}{(n+m)!} P_n^m(\cos \theta_0), \quad (17)$$

$\delta_{m,0} = 1$ for $m = 0$, $\delta_{m,0} = 0$ for $m \geq 1$ is Kronecker's symbol. Here we use the associated Legendre functions in the form:

$$P_n^m(x) = (1-x^2)^{m/2} \frac{d^m P_n(x)}{dx^m}. \quad (18)$$

We will have e.g. $P_1^1(x) = \sqrt{1-x^2}$, $P_2^1(x) = 3x\sqrt{1-x^2}$, $P_2^2(x) = 3(1-x^2)$ etc. We can see, that each coefficient A_n generates a family of $n+1$ coefficients C_{nm} . The components of the exciting magnetic field can be easily calculated by means of $-\text{grad } e\hat{V}(r, \theta, \phi)$:

$$e\hat{B}_r = -\frac{\partial}{\partial r}(e\hat{V}), \quad e\hat{B}_\theta = -\frac{1}{r} \frac{\partial}{\partial \theta}(e\hat{V}), \quad e\hat{B}_\phi = -\frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(e\hat{V}). \quad (19)$$

Let us note that these formulae are more suitable for geomagnetic studies in comparison to the formulae using the elliptic integrals in the paper *Fuji and Schulz (2002)*.

The current density in the stationary system (r, θ, ϕ) we suppose as stationary, but for the observer (geomagnetic observatory) on the rotating Earth with geographical co-ordinates (r, θ, λ) the external field potential becomes time-harmonic, since the azimuthal function $\exp[-i m(\phi - \phi_0)]$ must be transformed into $\exp[-i m(\lambda - \phi_0 + \Omega t)]$, where Ω is angular frequency of Earth's rotation. This means that on the rotating Earth we observe asymmetric stationary external magnetic field as time-harmonic, with discrete angular frequencies $\omega = m \cdot \Omega$.

The problem of EM induction in the rotating sphere (Earth) can be solved by means of low-velocity relativistic electrodynamics (e.g. *Bullard, 1949; Sochelnikov, 1979*) the appropriate Maxwell equations in the non rotating reference frame are:

$$\nabla \times \hat{\mathbf{B}} = \hat{\mathbf{j}}\mu_0, \quad \nabla \times \hat{\mathbf{E}} = -\partial \hat{\mathbf{B}} / \partial t, \quad \nabla \cdot \hat{\mathbf{B}} = 0. \quad (20)$$

The density of the electric current $\hat{\mathbf{j}}$ is:

$$\hat{\mathbf{j}} = \begin{cases} \sigma(\hat{\mathbf{E}} + \hat{\mathbf{v}} \times \hat{\mathbf{B}}), & \text{for } r \leq R_e, \\ 0 & \text{for } r > R_e. \end{cases} \quad (21)$$

We can see that for the region of the rotating sphere (Earth), $r \leq R_e$ the magnetic field obeys equation:

$$\nabla \times \nabla \times \hat{\mathbf{B}} = -\sigma\mu_0 [\partial \hat{\mathbf{B}} / \partial t - \nabla \times (\hat{\mathbf{v}} \times \hat{\mathbf{B}})]. \quad (22)$$

In the region outside the sphere the magnetic field satisfies equation

$$\nabla \times \hat{\mathbf{B}} = 0, \quad r > R_e, \quad (23)$$

so it can be calculated by gradient of the scalar potential in spherical functions of variables r, θ, ϕ . In our case we have only rotational motion of the conducting sphere around the polar axis $\theta = 0$ of the co-ordinate system $\hat{\mathbf{S}}$, so the velocity vector will have only ϕ -component:

$$\hat{\mathbf{v}} \equiv (0, 0, v_\phi), \quad \hat{v}_\phi = \Omega r \sin \theta. \quad (24)$$

Careful calculation of the expression $\nabla \times (\hat{\mathbf{v}} \times \hat{\mathbf{B}})$ will give:

$$\nabla \times (\hat{\mathbf{v}} \times \hat{\mathbf{B}}) = -\Omega \partial \hat{\mathbf{B}} / \partial \phi. \quad (25)$$

Considering that in our non rotating reference frame we have $\partial \hat{\mathbf{B}} / \partial t = 0$, we obtain from (22) equation:

$$\nabla \times \nabla \times \hat{\mathbf{B}} + \sigma\mu_0 \Omega \partial \hat{\mathbf{B}} / \partial \phi = 0. \quad (26)$$

The magnetic field potential (16) due to stationary electric currents depends on ϕ co-ordinate via $\exp[-i m(\phi - \phi_0)]$, where $m = 0, 1, 2, \dots$ is (azimuthal) order number of spherical harmonics. The same dependence will be transmitted to the magnetic field, then the derivative with respect to ϕ for m -harmonics in equation (26) can be easily performed and we obtain for m -th harmonics ${}_m\hat{\mathbf{B}}$ from (26):

$$\nabla \times \nabla \times ({}_m\hat{\mathbf{B}}) = i \mu_0 m \sigma \Omega ({}_m\hat{\mathbf{B}}). \quad (27)$$

For the observer (observatory) on the rotating Earth with co-ordinates (r, θ, λ) ; θ is the geographical co-latitude angle and λ is the geographical longitude angle, we must put transformation:

$$\phi = \lambda + \Omega t, \quad (28)$$

where t denotes UT. The magnetic field on the surface or inside of rotating Earth we denote $\mathbf{B} \equiv (B_r, B_\theta, B_\lambda)$. It will be time harmonic dependence, since from (26) we obtain for its m -harmonics ${}_m\mathbf{B}$ equation:

$$\nabla \times \nabla \times ({}_m\mathbf{B}) = i \omega \mu_0 \sigma ({}_m\mathbf{B}), \quad (29)$$

where $\omega = m\Omega$. This is well known EM induction equation for angular frequency ω , which can be solved by standard treatment using spherical wave functions for poloidal type of magnetic field.

Time harmonic EM induction in the multilayered rotating Earth

The problem of the time-harmonic excitation field and its induction effect in the sphere has been gradually developed during last century e.g.: Lamb, Schuster, Chapman, Rikitake, Lahiri, Price. Very detailed monographies in this topic is are (*Rotanova and Pushkov 1982; Berdichevsky and Zhdanov, 1984; Campbell, 1987*). Valuable knowledge to the basic EM induction problem for the spherical Earth can be also found in papers (*Pěč, Martinec and Pěčová, 1985*) and more recently in (*Maus and Lühr, 2005*) as well as (*Velímský and Martinec, 2005; Velímský, Martinec and Everett, 2006*). In our study we present formulae which use more suitable form of spherical Bessel functions, which simplifies expressions for reflection and transmission coefficients on spherical boundaries. The harmonic time variability of the exciting potential ${}^eV(r, \theta, \lambda)$ we suppose in the form $\exp(-i \omega t)$, where $\omega = m\Omega$ and introduce the complex potential:

$${}^eU(r, \theta, \lambda) = -R \sum_{n,m} C_{nm} (r/R)^n P_n^m(\cos \theta) G_m(\lambda, t), \quad (30)$$

where $G_m(\lambda, t) = \exp[-i m(\lambda - \phi_0 + \Omega t)]$.

Physical reality we assign to the real part of the ${}^eU(r, \theta, \lambda)$ and its gradient. The individual components of the exciting magnetic field are of individual spherical harmonics (n, m) are:

$$\begin{aligned} {}^eB_{nm} &= {}^0C_{nm} (r/R)^{n-1} \cdot n P_n^m(\cos \theta) G_m(\lambda, t) \\ {}^\theta B_{nm} &= {}^0C_{nm} (r/R)^{n-1} \cdot \frac{d P_n^m(\cos \theta)}{d \theta} G_m(\lambda, t) \\ {}^\lambda B_{nm} &= {}^0C_{nm} (r/R)^{n-1} \cdot \frac{P_n^m(\cos \theta)}{\sin \theta} \frac{\partial G_m(\lambda, t)}{\partial \lambda}. \end{aligned} \quad (31)$$

The Earth we consider as a spherical multilayered of radius $R (= R_e)$, consisting of L concentric spherical layers till the core mantle boundary at the depth 2900 km, the core ($r \leq r_L$) is considered as a uniform sphere of conductivity σ_L . We introduce $r_1 = R_e$ and the conductivity σ_j is assumed constant in the layer $r_{j+1} \leq r \leq r_j$. The magnetic permeability is assumed constant in all layers, as

well in the non-conducting region “0” outside the sphere and equal $\mu_0 = 4\pi \times 10^{-7}$ Henry/m. The slowly time varying EM field in the individual layers obeys the Maxwell’s equations

$$\nabla \times \mathbf{B} = \sigma\mu_0\mathbf{E}, \quad \nabla \times \mathbf{E} = +i\omega\mathbf{B}. \quad (32)$$

The magnetic induction $\mathbf{B}$ now represents the time-harmonic field of selected angular frequency ω . In a view of (28) and (31) this field obeys the vector wave equation

$$\nabla \times \nabla \times \mathbf{B} = i\omega\mu_0\sigma\mathbf{B} \quad (33)$$

Its solution in spherical co-ordinate system does not reduce to the scalar Helmholtz equation as in the Cartesian co-ordinates. The $\mathbf{B}$ vector must be separated into toroidal and poloidal parts as is proved in details by *Stratton, 1941; Born and Wolf, 1964*. If the exciting field is poloidal like (31), the induced magnetic field in a spherically concentric layers will be poloidal too and its (r, θ, λ) components will have the same θ, λ dependence as in (31). Their radial dependence will be expressed by the spherical functions $f_n = \psi_n(kr), \zeta_n(kr)$ which obey the ordinary differential equation

$$f_n''(z) + \left[1 - n(n+1)/z^2\right]f_n = 0, \quad z = kr. \quad (34)$$

It was proved in *Born, Wolf, 1964* that the solutions are functions:

$$\psi_n(kr) = (\pi kr/2)^{1/2} J_{n+1/2}(kr), \quad \zeta_n(kr) = (\pi kr/2)^{1/2} H_{n+1/2}^{(1)}(kr), \quad (35)$$

where $J_{n+1/2}(kr), H_{n+1/2}^{(1)}(kr)$ are Bessel function and Hankel function of the first kind half integer index $n + 1/2$ and complex argument kr :

$$kr = r(i\omega\sigma\mu_0)^{1/2} = r(1+i)(\omega\sigma\mu_0/2)^{1/2}. \quad (36)$$

In the j -th layer this argument will carry the dependence on the electrical conductivity σ_j , this means:

$$k_j r = r(1+i)(\omega\sigma_j\mu_0/2)^{1/2}. \quad (37)$$

The spherical (n, m) mode of the magnetic field will have in the j -th layer components:

$$\begin{aligned} {}^j B_{nm} &= [{}^j C_{nm} \psi_n(k_j r) + {}^j D_{nm} \zeta_n(k_j r)] n(n+1)(k_j r)^{-2} P_n^m(\cos \theta) G_m(\lambda, t), \\ {}^\theta B_{nm} &= [{}^j C_{nm} \psi_n'(k_j r) + {}^j D_{nm} \zeta_n'(k_j r)] (k_j r)^{-1} d P_n^m(\cos \theta) / d \theta G_m(\lambda, t), \\ {}^\lambda B_{nm} &= [{}^j C_{nm} \psi_n'(k_j r) + {}^j D_{nm} \zeta_n'(k_j r)] (k_j r)^{-1} P_n^m(\cos \theta) / \sin \theta \partial G_m(\lambda, t) / \partial \lambda. \end{aligned} \quad (38)$$

In the bottom sphere $r \leq d_L$ (the Earth’s core) we must put ${}^L D_{nm} \equiv 0$, since in this region $kr \rightarrow 0$, where $\zeta_n(kr)$ and its derivative is singular. In the non-conducting region $r \in \langle R, a \rangle$ we will have secondary magnetic field with potential

$${}^S U(r, \theta, \lambda) = -R \sum_{n,m} (R/r)^{n+1} \cdot {}^0 D_{nm} P_n^m(\cos \theta) G_m(\lambda, t). \quad (39)$$

Pertinent components for this potential can be expressed by negative grad ${}^S U(r, \theta, \lambda)$. From the EM theory we know that on each conductivity spherical boundary must be continuous transition of radial (rB_{nm}) and tangential (θB or λB_{nm}) components. In this manner we obtain the system of linear equations for unknown coefficients ${}^j C_{nm}, {}^j D_{nm}$ while only the exciting field coefficients ${}^0 C_{nm}$

we consider to be known (calculated for the external exciting electric currents). We shall briefly give algorithm from our previous papers *Hvoždara, 1976, 1980*. Then we have: for $r = r_1 = R$:

$$\begin{aligned} n^0 C_{nm} - (n+1)^0 D_{nm} &= \left[{}^1 C_{nm} \psi_n(k_1 r_1) + {}^1 D_{nm} \zeta_n(k_1 r_1) \right] n(n+1)(k_1 r_1)^{-2}, \\ {}^0 C_{nm} + {}^0 D_{nm} &= \left[{}^1 C_{nm} \psi'_n(k_1 r_1) + {}^1 D_{nm} \zeta'_n(k_1 r_1) \right] (k_1 r_1)^{-1}. \end{aligned} \quad (40)$$

On the boundary $r = r_j$, ($j \neq 1, j \neq L$):

$$\begin{aligned} {}^{j-1} C_{nm} \psi_n(k_{j-1} r_j) + {}^{j-1} D_{nm} \zeta_n(k_{j-1} r_j) &= \gamma_{j-1}^2 \left[{}^j C_{nm} \psi_n(k_j r_j) + {}^j D_{nm} \zeta_n(k_j r_j) \right] \\ {}^{j-1} C_{nm} \psi'_n(k_{j-1} r_j) + {}^{j-1} D_{nm} \zeta'_n(k_{j-1} r_j) &= \gamma_{j-1} \left[{}^j C_{nm} \psi'_n(k_j r_j) + {}^j D_{nm} \zeta'_n(k_j r_j) \right] \end{aligned} \quad (41)$$

where $\gamma_{j-1} = (\sigma_{j-1}/\sigma_j)^{1/2}$. For the deepest boundary $r = d_L$ (surface of the core) we have:

$$\begin{aligned} {}^{L-1} C_{nm} \psi_n(k_{L-1} d_L) + {}^{L-1} D_{nm} \zeta_n(k_{L-1} d_L) &= \gamma_{L-1}^2 {}^L C_{nm} \psi_n(k_L d_L) \\ {}^{L-1} C_{nm} \psi'_n(k_{L-1} d_L) + {}^{L-1} D_{nm} \zeta'_n(k_{L-1} d_L) &= \gamma_{L-1} {}^L C_{nm} \psi'_n(k_L d_L). \end{aligned} \quad (42)$$

This system of equations is solvable by the elimination methods, because equations for $r = r_2, r_3, \dots, r_L$ are homogeneous. Then we introduce the proportionality ${}^j W_n$ and reflection coefficients ${}^j F_n$ as follows:

$${}^L W_n = \psi'_n(k_L r_L) / [\gamma_{L-1} \psi_n(k_L r_L)], \quad {}^{L-1} D_{nm} = (-1) \cdot {}^{L-1} F_n \cdot {}^{L-1} C_{nm}, \quad (43)$$

where:

$${}^{L-1} F_n = \frac{\psi'_n(k_{L-1} r_L) - {}^L W_n \psi_n(k_{L-1} r_L)}{\zeta'_n(k_{L-1} r_L) - {}^L W_n \zeta_n(k_{L-1} r_L)}.$$

On the boundaries $r = d_{L-1}, \dots, d_2$ we have similarly:

$${}^j W_n = \frac{\psi'_n(k_j r_j) - {}^j F_n \zeta'_n(k_j r_j)}{\gamma_{j-1} [\psi_n(k_j r_j) - {}^j F_n \zeta_n(k_j r_j)]}. \quad (44)$$

$${}^{j-1} D_{nm} = (-1) \cdot {}^{j-1} F_n \cdot {}^{j-1} C_{nm}, \quad (45)$$

$${}^{j-1} F_n = \frac{\psi'_n(k_{j-1} r_j) - {}^j W_n \psi_n(k_{j-1} r_j)}{\zeta'_n(k_{j-1} r_j) - {}^j W_n \zeta_n(k_{j-1} r_j)}, \quad (46)$$

In fact the general expressions (44)–(46) also include the interface $r = d_L$, it should be put ${}^L F_n \equiv 0$. Now we have to consider equations (40) relevant to the boundary $r = r_1 (\equiv R)$ which can be written in the form

$$\begin{aligned} (n+1)^0 D_{nm} + {}^1 C_{nm} [\psi_n(k_1 r_1) - {}^1 F_n \zeta_n(k_1 r_1)] n(n+1)(k_1 r_1)^{-2} &= n^0 C_{nm}, \\ -{}^0 D_{nm} + {}^1 C_{nm} [\psi'_n(k_1 r_1) - {}^1 F_n \zeta'_n(k_1 r_1)] / (k_1 r_1) &= {}^0 C_{nm}. \end{aligned} \quad (47)$$

From this system of two linear equations we can easily find that the

$${}^1 C_{nm} = \frac{(k_1 r_1) (2n+1)^0 C_{nm}}{(n+1) [\psi_{n-1}(k_1 r_1) - {}^1 F_n \zeta_{n-1}(k_1 r_1)]}, \quad (48)$$

$${}^0 D_{nm} = \frac{-n^0 C_{nm} [\psi_{n+1}(k_1 r_1) - {}^1 F_n \zeta_{n+1}(k_1 r_1)]}{(n+1) [\psi_{n-1}(k_1 r_1) - {}^1 F_n \zeta_{n-1}(k_1 r_1)]}. \quad (49)$$

In derivation of these equations we have used recurrence properties of the radial functions:

$$f'_n(z) + (n/z)f_n(z) = f_{n-1}(z), \quad (2n+1)f_n(z) - zf_{n-1}(z) = zf_{n+1}(z), \quad (50)$$

where $f_n(z) = \psi_n(z)$ or $\zeta_n(z)$. In this manner we can calculate the electromagnetic response function for our multilayered spherical Earth:

$$\frac{{}^0D_{nm}}{{}^0C_{nm}} = \frac{-n}{n+1} \frac{\psi_{n+1}(k_1 r_1) - {}^1F_n \zeta_{n+1}(k_1 r_1)}{\psi_{n-1}(k_1 r_1) - {}^1F_n \zeta_{n-1}(k_1 r_1)} = {}^0F_n. \quad (51)$$

By using the recurrence relations (50) we can obtain alternative expression for 0F_n in the form:

$${}^0F_n = \frac{n}{n+1} \left\{ 1 - \frac{(2n+1)[\psi_n(k_1 r_1) - {}^1F_n \zeta_n(k_1 r_1)]}{k_1 r_1 [\psi_{n-1}(k_1 r_1) - {}^1F_n \zeta_{n-1}(k_1 r_1)]} \right\}. \quad (52)$$

This is more suitable for numerical calculations since it contains the radial functions of neighbouring indices n and $n-1$. In the classical geomagnetic EM induction theory the coefficient 0F_n corresponds to the ratio I_n/E_n , where I_n is the amplitude factor of the scalar potential of induced (interior) field and E_n is amplitude of the inducing (exciting) field for the spherical harmonics degree n . The coefficients E_n , I_n correspond to our ${}^0C_{nm}$, ${}^0D_{nm}$ respectively. We can see that by using the coefficient 0F_n the amplitudes of radial and tangential components of the n -th ($m=0$) harmonics have on the surface of the Earth are:

$${}_r b_n = [n - (n+1){}^0F_n]E_n, \quad {}_\theta b_n = [1 + {}^0F_n]E_n. \quad (53)$$

Their ratio can be used also for calculation of the surface impedance using *Berdichevsky and Zhdanov, (1984)* formulae:

$$Z_n = \frac{-i \omega \mu_0 R_e}{n(n+1)} \frac{{}_r b_n}{{}_\theta b_n} = \frac{-i \omega \mu_0 R_e}{n(n+1)} \frac{n - (n+1){}^0F_n}{1 + {}^0F_n}. \quad (54)$$

In this manner our formulae link to the global magnetovariational theory. Let us stress that EM response coefficients 0F_n are independent of azimuthal number m . The same holds true for ratio ${}_r b_n / {}_\theta b_n$ for the concentric layered Earth conductivity model.

Numerical calculation of EM response functions and apparent resistivity

Recently we have prepared fast and reliable computer FORTRAN code for calculation of EM response coefficients 0F_n for wide interval of time periods T , ranging from 3 hours till 6 years, using T_s as period T in seconds. These calculations were performed for various known depth-conductivity Earth's models, i.e. for $L \geq 5$ till $L = 12$. According to present knowledge, for the spherical Earth there must be considered two kinds of the crust and upper mantle electrical conductivity distribution models: continental (A), oceanic (B) ones.

The **continental model** is characterized by the superficial layer "1" of conductivity $\sigma_1 \approx 0.002$ – 0.02 S/m, thickness 8–15 km, then the conductivity σ_2 is about $\sigma_1/10$, because of dehydration of rocks, its thickness is about 30 km. From the bottom of continental crust in depths ~ 30 – 50 km the conductivity grows due to increasing temperature and attains about 0.05 S/m in the continental asthenosphere. In the mantle there is gradual growth of σ , attaining about 2 – 10 S/m on the mantle-core boundary in the depth 2900 km. Due to phase transitions of the mantle minerals and also growing temperature there are known boundaries with increase of electrical conductivity at depths about 410–4500 km, 800–1000 km, 1500–1800 km. In the Earth's core the electrical conductivity of hot Fe-Ni-S melt we consider to be uniform: $\sigma_L = 5000$ S/m. It means, that on this boundary

we have the ratio $\sigma_L/\sigma_{L-1} \approx 10^3$, which enables us to confine only on the calculation of transition coefficients jF_n in the layers in the mantle and crust, i.e. $j = L - 1, \dots, 1$ where the $r \in \langle r_c, R_E \rangle$, while $r_c = 0.5453 \times R_E$.

The **oceanic model** (B) is characterized by the sea water layer of the high conductivity $\sigma_1 \approx 0.4$ – 0.6 S/m, its mean thickness is about 4 km, which causes high attenuation of EM field of periods shorter than ~ 60 min. The rocks layers below the oceanic bottom have as a rule higher electrical conductivity in comparison with the continental ones, since the temperature and temperature gradient is higher about 20% in comparison to the continental lithosphere.

The numerical calculations were tested for numerous adequate models, but here we present results for the “continental” model shown in Fig. 3a. The moduli and phases of 0F_n for this model are in Fig. 3b together with apparent resistivity curves. The periods for one day and its fractions T/m

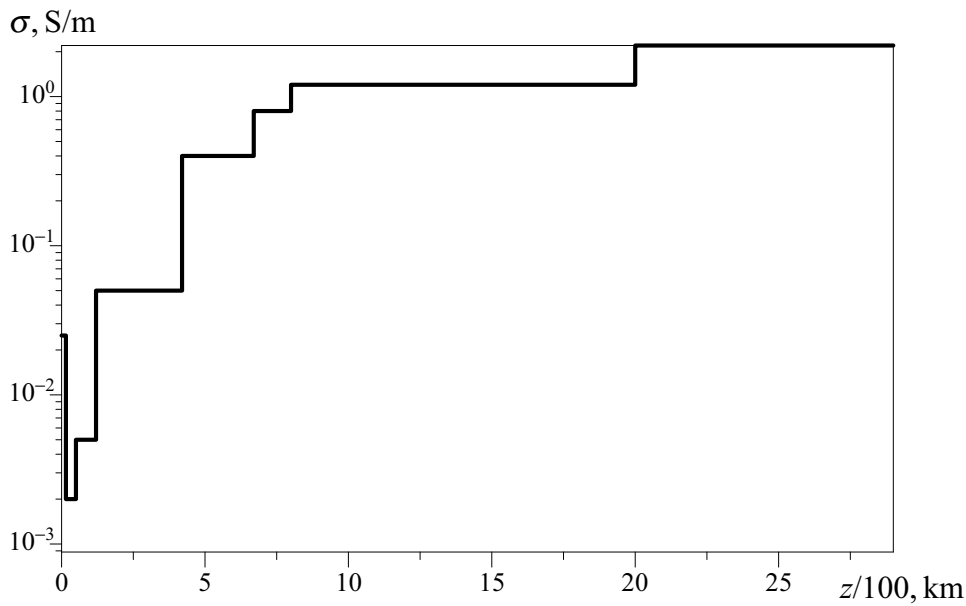


Fig. 3a. The graph of conductivity depth distribution in the Earth's crust and mantle (the “continental model,”) as function of depth (z), used in present study.

can be found at the range $\log \sqrt{T_s} \approx 2.1$ as shows also the Tab. 1.

m	1	2	3	4	5
$T/m, \text{ day}$	1	0.5	0.333	0.25	0.2
$\log \sqrt{T_s}$	2.468	2.317	2.229	2.167	2.118

For better resolution we plotted moduli of 0F_n multiplied by the factor 10 and the apparent resistivity values were normed to $\rho_1 = (\sigma_1)^{-1}$ (the resistivity of the first layer). We can see, that the values of $|{}^0F_n|$ as a rule drop with $\sqrt{T_s}$ and the decrease of these values is steeper for increased degree n of spherical harmonics. The phases of 0F_n attain values from 0° till $\sim 75^\circ$ and the phase shift grows with degree number n and period T_s . The values of $\log(\rho_a/\rho_1)$ decrease almost linearly with $\log \sqrt{T_s}$ and the curve for $n = 1$ is above those for $n \geq 2$, which is in agreement with results of *Berdichevsky and Zhdanov (1984)*. Let us note, that for calculations of 0F_n for shorter period (less than 0.5 day) and high conductive layer $j = 1$ there must be used large value asymptotics of $\psi_n(z)$, $\zeta_n(z)$.

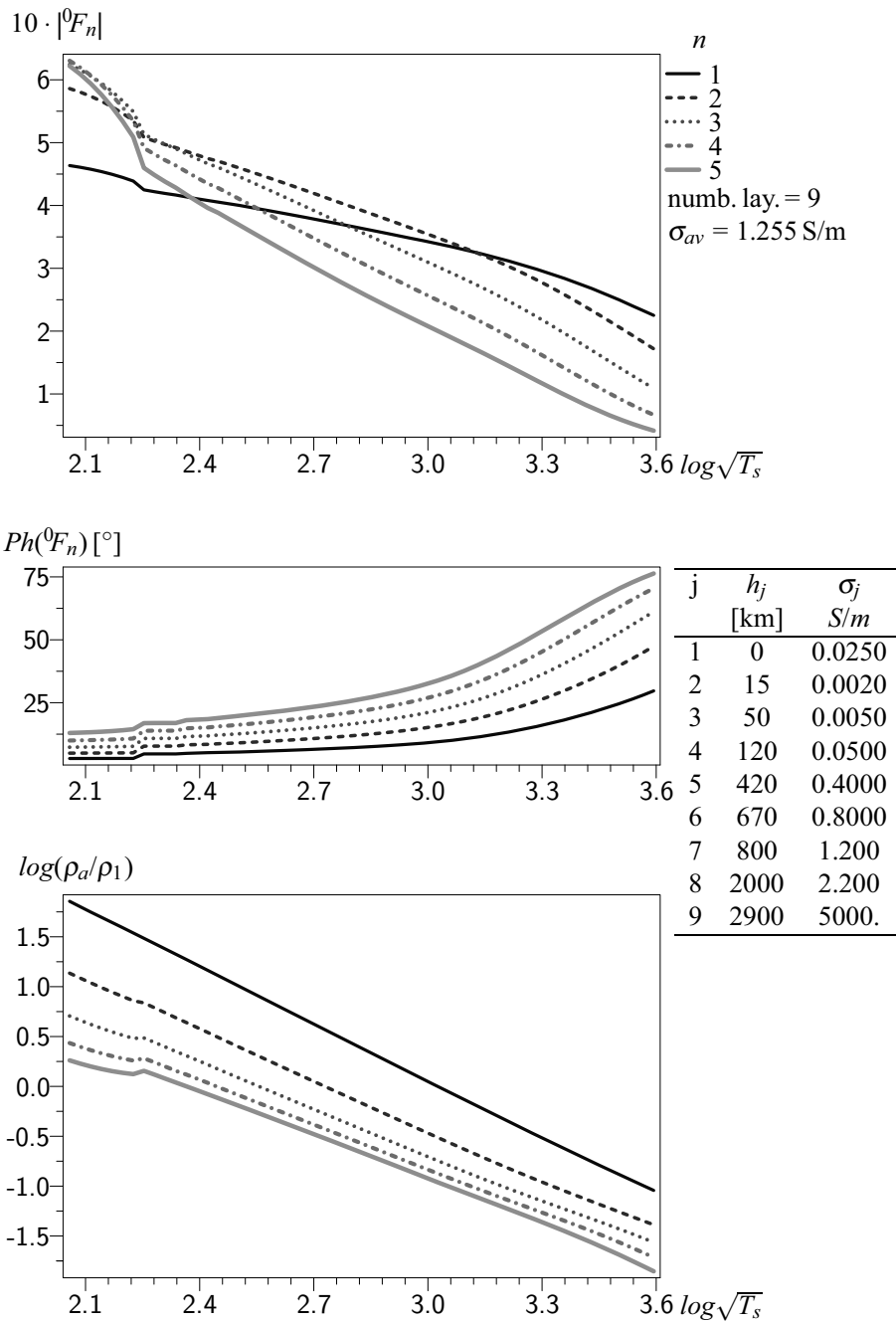


Fig. 3b. The graphs of calculated courses of EM response coefficients 0F_n , their phases and course of $\log(\rho_a/\rho_1)$ in dependence on $\log \sqrt{T_s}$, where T_s is period T in seconds. The pertinent values of depths boundaries h_j and conductivity values σ_j are given in the table, σ_{av} average conductivity for each model (considering depths to 2900 km).

Magnetic field due to some current belts

The calculations of the primary field coefficients $\tilde{A}_n$ were performed according to formula (13). These $\tilde{A}_n$ we multiply by spherical harmonics for necessary degree numbers n and $m = 0, 1, 2, \dots, 5$ and then using formula (17) we calculate coefficients C_{nm} for the exciting potential. By summation with respect to n and m we obtain the magnetic field components. In order to use common local geomagnetic horizontal and vertical components on the surface of the Earth, we put:

$$B_x = -B_\theta, \quad B_y = B_\lambda, \quad B_z = -B_r. \quad (55)$$

The results of numerical calculations we present for the “continental” Earth conductivity model and three types of current belt: a) auroral, b) equatorial, c) Sq current model.

For belts a) and b) we put angular width 20° around central circle α , it means $\alpha_1 = \alpha - 10^\circ$, $\alpha_2 = \alpha + 10^\circ$. The axis of the belt we put at spherical coordinates $\theta_0 = 10^\circ$, $\phi_0 = -110^\circ$, which corresponds to North geomagnetic pole. The net intensity current we put $I = 10^7$ A, we suppose it as distributed uniformly on the width of the belt i.e. $J(\Theta') = I/(\alpha_2 - \alpha_1)$. The field components B_x, B_y, B_z we normed by common norming value $B_n = \mu_0 I/(2a_c)$, where a_c is the radius of the belt supporting sphere ($\equiv a$) in theoretical formulae.

For the **auroral belt** we chose $\alpha = 20^\circ$ and small distance from the Earth surface so $a_c/R_e = 1.1$ which means that this belt is in the height $h_c = 636$ km and then $B_n = \mu_0 I/(2a_c) = 895.58$ nT. The theoretical time courses of the geomagnetic field components were calculated for the observation points: $(\theta = 30^\circ, \lambda = 0^\circ)$, $(\theta = 42^\circ, \lambda = 0^\circ)$, on the rotating Earth, which correspond to high-latitude (e.g. Nurmijarvi) or mid-latitude geomagnetic observatory, e.g. Hurbanovo. In Figs 4a,b there is shown the time course of the summary field (exciting + induced) during 46 hours. In both figures we can see dominant 24 h variation in all three components. The time course of horizontal components (B_x, B_y) resembles repeated geomagnetic variations with prevailing period 24 h. The time course of the exciting magnetic field on the rotating earth for $\theta = 42^\circ, \lambda = 0^\circ$ is presented in Fig. 4c. When comparing with Fig. 4b we can see that tangential components (B_x, B_y) are amplified due to induction, but vertical component B_z is attenuated. Theoretically we can see it in formula (53), where the n -th harmonics of tangential components on the surface are given by the terms $(1 + {}^0F_n)E_{nm}$, but in the radial component we have terms $[n - (n + 1){}^0F_n]E_{nm}$.

For the **equatorial belt** we chose $\alpha = 90^\circ$ and large distance from the Earth surface so $a_c/R_e = 3.0$, so this belt is in the height $h_c = 12755$ km and then $B_n = \mu_0 I/(2a_c) = 328.38$ nT. The theoretical time courses of the geomagnetic field components were calculated for the same observation point (θ, λ) as in previous case. This equatorial belt we consider as a plausible model for the ring current.

In Fig. 5a there is shown time course of exciting (external) field and in Fig. 5b the summary field (exciting + induced) during 46 hours. In both figures we can see also dominant 24 h variation in all three components, but the amplitudes of diurnal waves are smaller in comparison with auroral belt. Numerical calculations proved that EM induction amplifies both horizontal components about 20% and strongly attenuates the vertical component, but the time variations on $\theta = 42^\circ$ are almost the same as for $\theta = 30^\circ$. The amplitudes of stationary primary field due to equatorial current belt along the meridian $\phi = 0^\circ$ are presented in Fig. 5c. We can see that in this field there is dominant spherical harmonics $n = 1$, so B_x is proportional to $\sin \theta$, while B_z is proportional to $-\cos \theta$. Let us note, that in our calculations we consider the direction of stationary current as positive (Eastward), while in the quiet real magnetospheric current the direction is opposite (Westward). The same holds true also for the disturbed Dst ring current. In some magnetograms of very strong geomagnetic storms as recorded at geomagnetic observatory Hurbanovo (e.g. during days 07–11 November, 2004) there is clearly present some part of the disturbed field as repeating with period one day.

Approximation of Sq current system on the northern hemisphere was considered as the current belt with axis of symmetry in the pole $\theta_0 = 45^\circ, \phi_0 = 180^\circ$ in order to meet knowledge given in textbooks *Parkinson, 1983* or *Campbell, 1989*. Very important feature in the time course of Sq geomagnetic variations is their uniform harmonic dependence on the local time and non-uniform distribution on co-latitude θ . There exist also differences for continents and seasons of the year, but these are not so pronounced. The time variations in local time t^* for various meridians λ we can consider as almost the same as along the meridian $\lambda = 0^\circ$ where we use time t as UT. Simple calculation, using 1 hour as a unit for time, will show: $t^* = t + \lambda/(15^\circ/\text{h})$, [h], since the angular speed Ω of the Earth's rotation is: $\Omega = 360^\circ/(24\text{h}) = 15^\circ/\text{h}$. Then we will have for $\lambda - t$ expressions of the magnetic field: $m[\lambda - \phi_0 + \Omega t] = m[\Omega t^* - \phi_0]$. According to *Parkinson, 1983* the focus of the Sq current system in the ionosphere occurs at noon $t^* = 12\text{h}$ (LT), so we must put $\phi_0 = 180^\circ$,

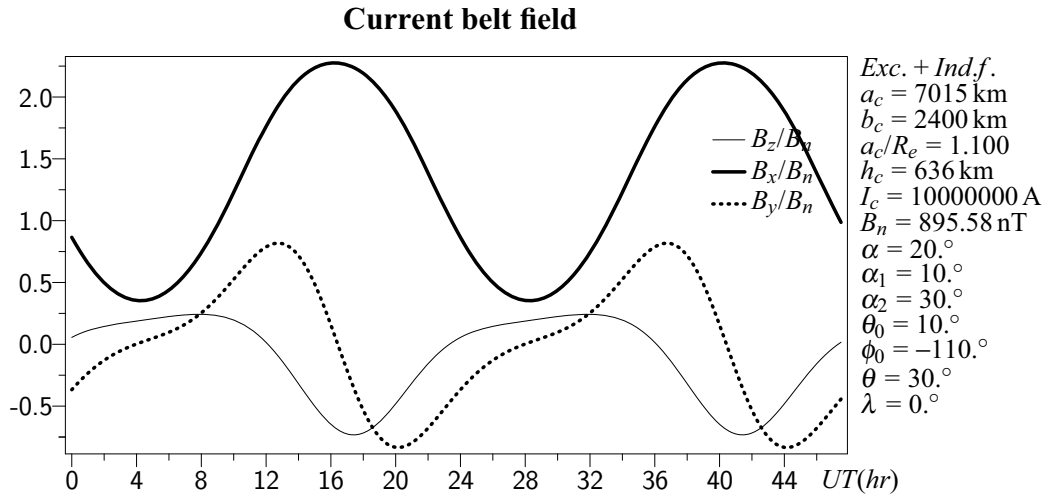


Fig. 4a. Time variations of summary (exciting + induced) surface magnetic field for the auroral current belt around central circle $\alpha = 20^\circ$ near the North magnetic pole (θ_0, ϕ_0). The point observation is $\theta = 30^\circ, \lambda = 0^\circ$ on the rotating Earth.

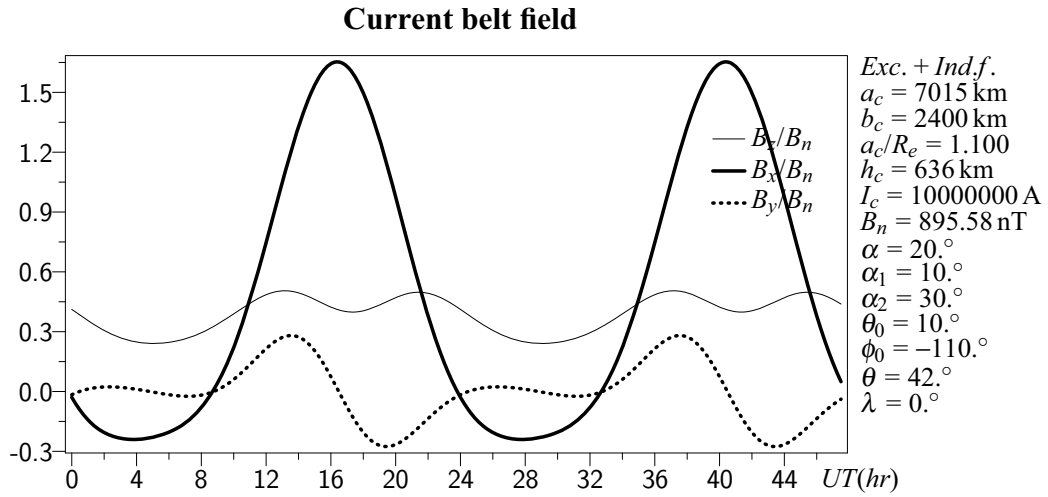


Fig. 4b. The same as in Fig. 4a, but for co-latitude $\theta = 42^\circ$.

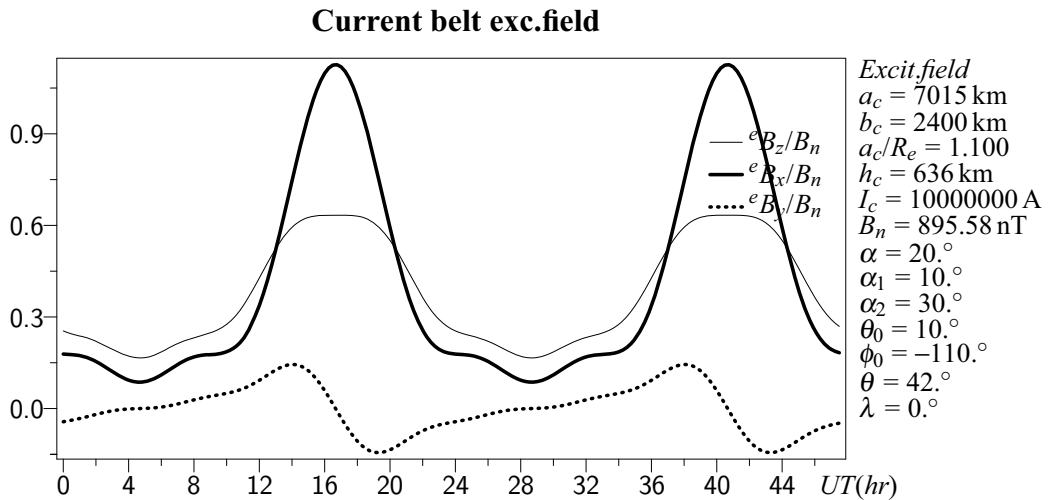


Fig. 4c. Time variations of the exciting field due to stationary auroral current belt ($\alpha_1 = 10^\circ, \alpha_2 = 30^\circ$) at the observatory $\theta = 42^\circ$ meridian $\lambda = 0^\circ$ on the rotating Earth.

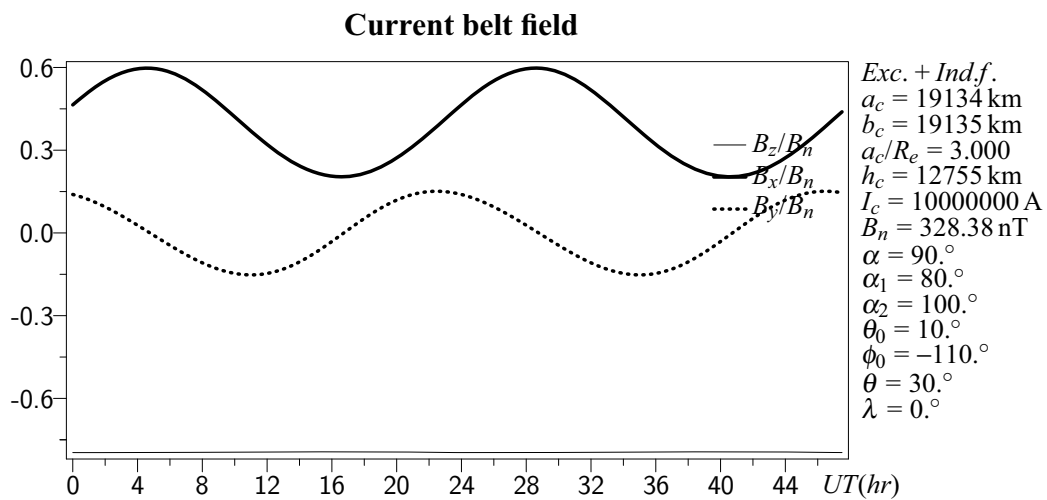


Fig. 5a. Time variations of summary (exciting + induced) surface magnetic field for the equatorial current belt around central circle $\alpha = 90^\circ$ with central axis near the North magnetic pole (θ_0, ϕ_0). The point observation is $\theta = 30^\circ, \lambda = 0^\circ$ on the rotating Earth.

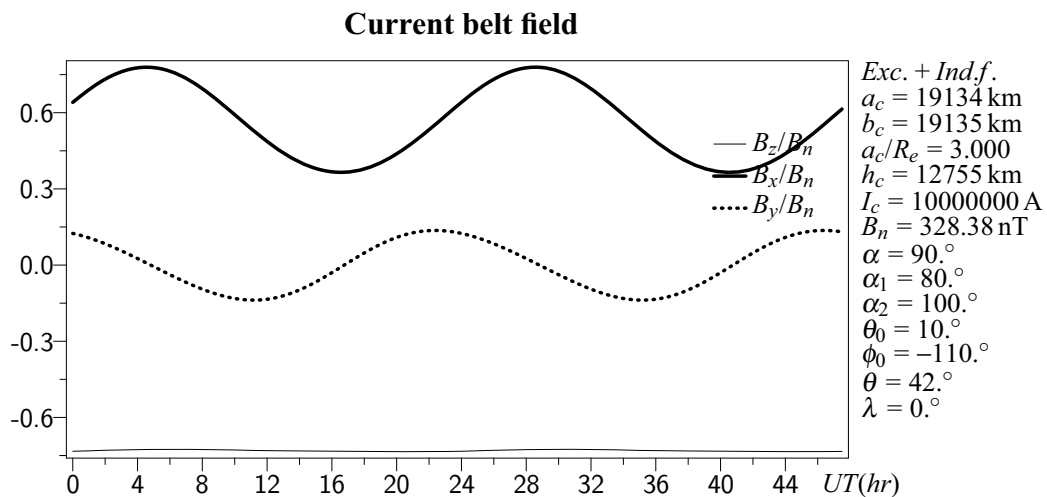


Fig. 5b. The same as in Fig. 5a, but for co-latitude $\theta = 42^\circ$.

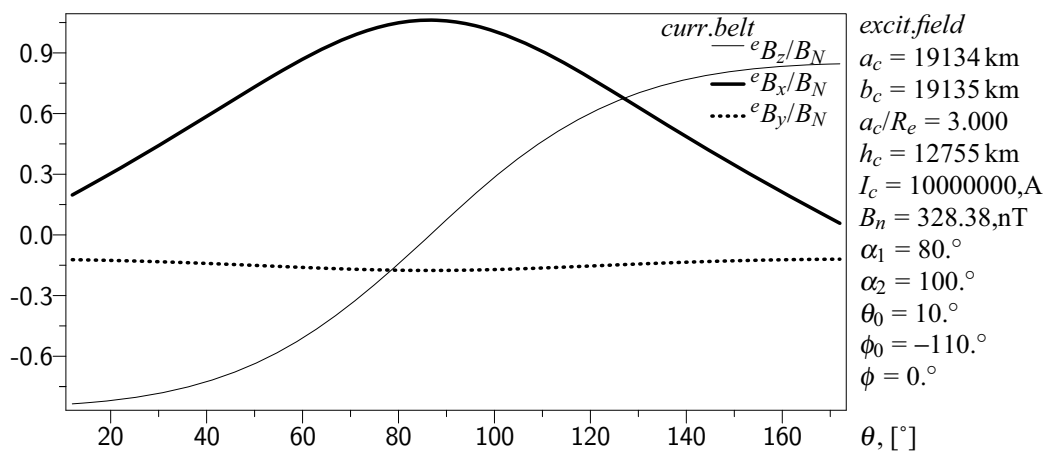


Fig. 5c. Amplitudes of the stationary exciting field due to equatorial current belt along the meridian $\phi = 0^\circ$ (in the non-rotating co-ordinate system).

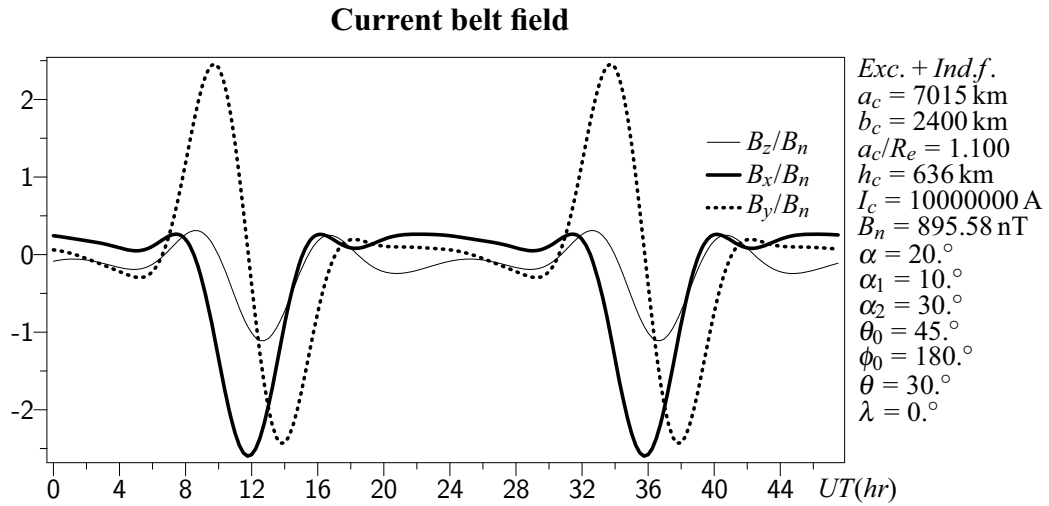


Fig. 6a. Time variations of summary (exciting + induced) surface magnetic field for the Sq current belt around central circle $\alpha = 20^\circ$ with central axis at $(\theta_0 = 45^\circ, \phi_0 = 180^\circ)$. The point observation is $\theta = 30^\circ, \lambda = 0^\circ$ on the rotating Earth.

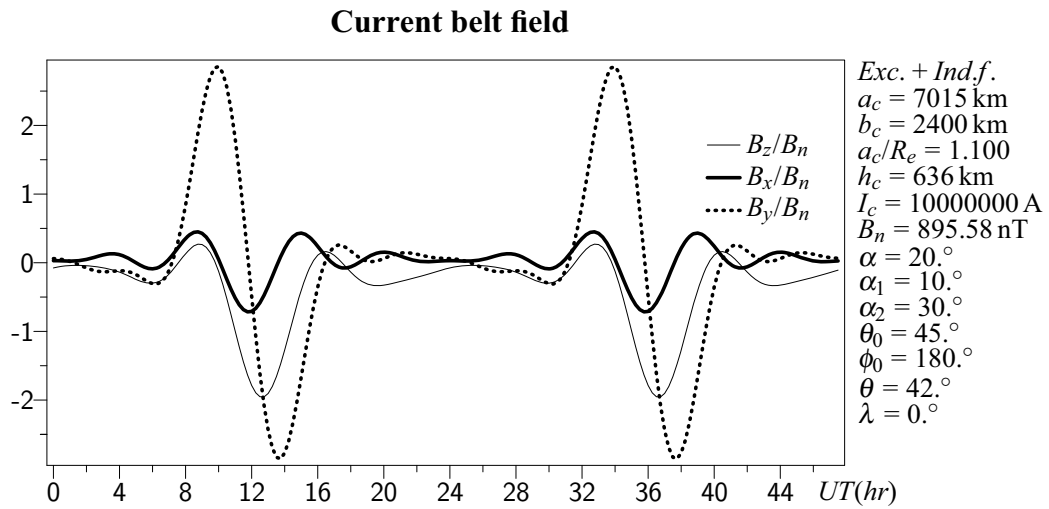


Fig. 6b. The same as in Fig. 6a, but for co-latitude $\theta = 42^\circ$.

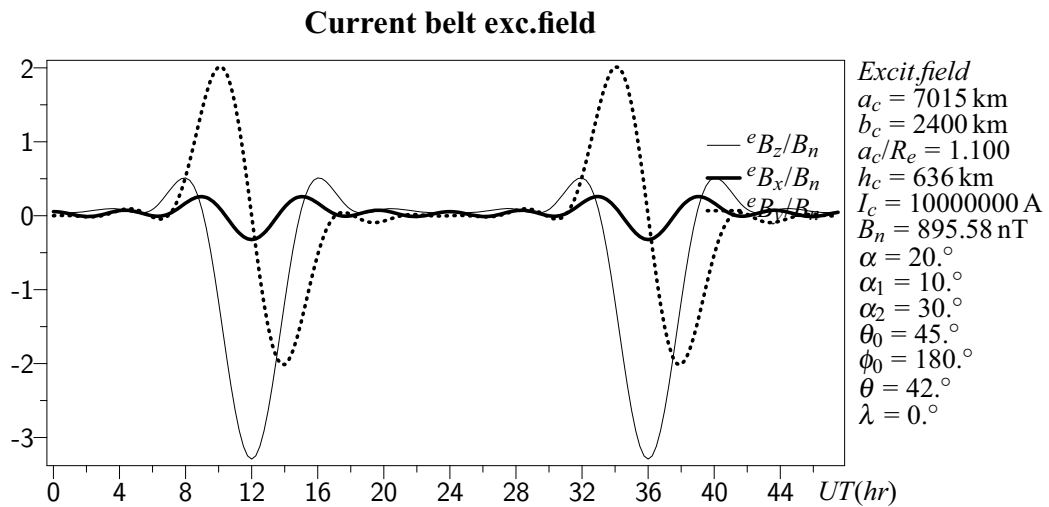


Fig. 6c. Time variations of the exciting field due to stationary Sq current belt with parameters given in Fig. 6a at the observatory $\theta = 42^\circ$ meridian $\lambda = 0^\circ$ on the rotating Earth.

the radius of the sphere with current we put $a_c = 1.1R_e$. In Figs 6a,b we present time course of the geomagnetic components for co-latitudes $\theta = 30^\circ, 42^\circ$, respectively and in Fig. 6c the time variations of the exciting field on the rotating Earth for $\theta = 42^\circ$. Comparing Figs 6b,c we can see also the amplification of tangential components and attenuation of B_z . The time courses in Figs 6a,b are in good agreement with general features of Sq geomagnetic variations observed in polar and mid latitude geomagnetic observatories (see *Parkinson, 1983*).

Acknowledgments

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MT Robust Remote Reference Processing revisited

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1 Introduction

Magnetotelluric (MT) data is often heavily affected by cultural noise, particularly when measurements are carried out close to inhabited areas. The geothermal test site Groß Schönebeck is located only 25 km north of the outskirts of Berlin and consequently the MT data acquired in 2006/2007 is contaminated by noise. Figure 1 reveals obvious artificial features in the apparent resistivities and phase curves at periods of 10 s and 100 s.

To overcome or at least reduce the influence of noise several techniques can be applied:

- Robust Statistics to minimize the influence of outliers
- Remote reference processing to minimize the impact of local noise

The robust processing code based on Junge (1993) and Ritter *et al.* (1998) has been used successfully to process single site data but it is not straight forward to adopt the scheme for remote reference processing.

2 Remote Reference

For MT data processing bivariate equations of the following type have to be solved:

$$Z_i = a \cdot X_i + b \cdot Y_i + r_i \quad (1)$$

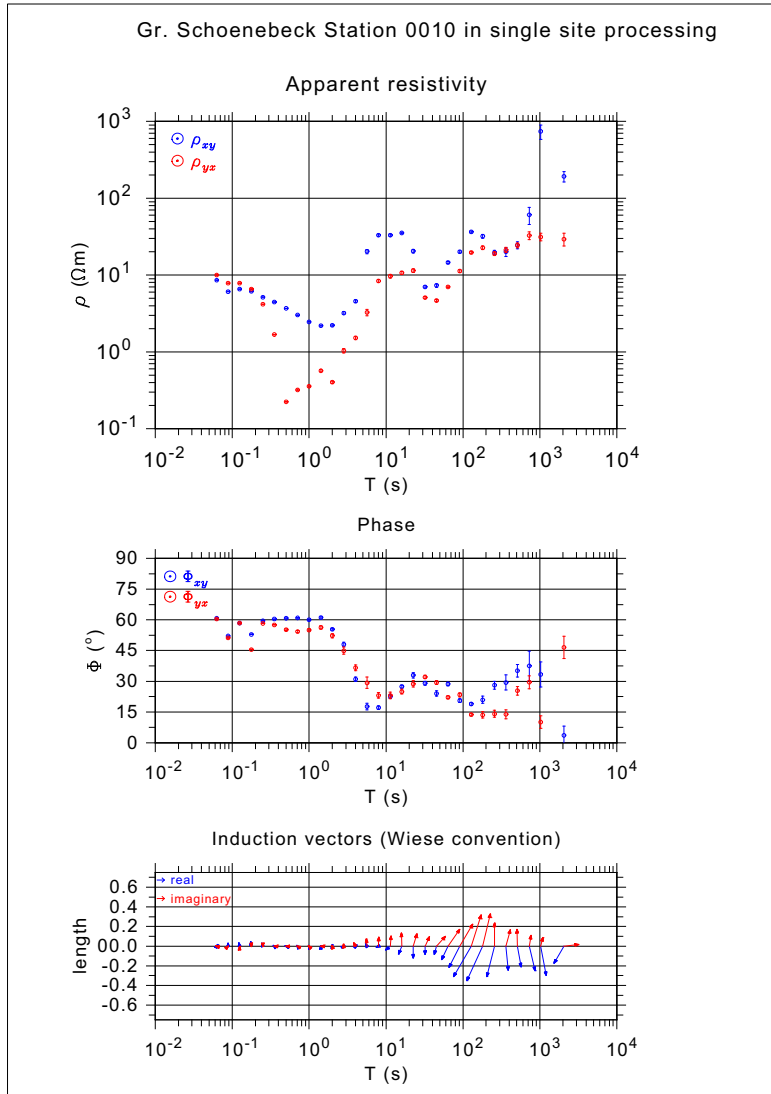


Figure 1: Typical single site processing results for a station from the 2007 I-GET magnetotelluric survey (see also Muñoz *et al.*, this volume), showing apparent resistivities, phases and induction arrows.

where a and b denote the frequency dependent, complex valued transfer functions which do not depend on time. X_i , Y_i and Z_i refer to observations of electromagnetic field components recorded at different time segments i . X usually refers to the magnetic field component in x-direction B_x , Y to the magnetic field in y-direction B_y and Z to either one of the horizontal electric field components E_x or E_y , or the vertical magnetic field B_z . The noise term r_i is attributed to Z_i . To solve for the transfer function a a least squares approach is used:

$$a = \frac{\langle YY^* \rangle \langle ZX^* \rangle - \langle YX^* \rangle \langle ZY^* \rangle}{\langle XX^* \rangle \langle YY^* \rangle - \langle XY^* \rangle \langle YX^* \rangle} \quad (2)$$

where $\langle ZX^* \rangle \equiv \sum_i (Z_i X_i^*)$. In the remote reference formulation auto spectra of the local fields are replaced by cross spectra between local and remote (index R) fields:

$$a_R = \frac{\langle YY_R^* \rangle \langle ZX_R^* \rangle - \langle YX_R^* \rangle \langle ZY_R^* \rangle}{\langle XX_R^* \rangle \langle YY_R^* \rangle - \langle XY_R^* \rangle \langle YX_R^* \rangle} \quad (3)$$

Any uncorrelated noise between local and remote station cancels out.

3 Robust Statistics

Robust statistics is based on the stability theory of statistical methods and examines the influence of deviations (outliers) from a parametric model. Basic concepts include the breakdown point of a robust method as an indicator for its stability (robustness) against outliers. The ordinary least squares (OLS) approach is an optimum solution for Normal distributed data but tends to fail for long tailed distributions, i.e. distributions with an increased probability for outliers. Because ordinary least squares methods are well established and easy to compute, we use a weighted least squares approach (WLS) with weights w_i based on Huber's M-estimates (Huber, 1964). The actual weights are calculated to minimize the residuals r_i , by down-weighting outliers:

$$OLS : \sum_i r_i^2 \rightarrow \min \quad (4)$$

$$WLS : \sum_i w_i r_i^2 \rightarrow \min \quad (5)$$

The squared residual is given by the following expression:

$$r_i^2 = |Z_i - a \cdot X_i - b \cdot Y_i|^2 \quad (6)$$

4 Modifications and Results

In the remote reference case it is not straightforward to define the residual r_i . Obviously the remote reference transfer functions a_R and b_R have to be taken into consideration. Since these transfer functions are calculated using cross spectra between local and remote fields (see eq. 3) they do not resemble the residual defined in equation 6, which only depends on local spectra. To obtain a suitable expression the residual can be redefined *ad hoc* to include also the horizontal magnetic spectra from the remote site:

$$\begin{aligned}
 |r_{RR,i}|^2 &\approx \frac{|\langle rX^{R,*} \rangle| |\langle rY^{R,*} \rangle|}{|\langle X^R Y^{R,*} \rangle|} \\
 &= \frac{|\langle ZX^{R,*} - a_R \cdot XX^{R,*} - b_R \cdot YX^{R,*} \rangle| |\langle ZY^{R,*} - a_R \cdot XY^{R,*} - b_R \cdot YY^{R,*} \rangle|}{|\langle X^R Y^{R,*} \rangle|}
 \end{aligned}
 \tag{7}$$

Figure 2 shows processing results before these modifications were applied to the code. Obviously the results are severely distorted. The usage of the new definition of the residual is a major improvement (see Figure 3). However, the results are still not quite as good as desirable, because apparent resistivities and phases are not well resolved in the so-called dead band around 10 s.

Obviously the robust scheme is not able to identify all of the “bad” data. To obtain a better understanding of the actual weighting process Figure 4 shows the transfer functions a_R (upper left quadrant) and b_R (upper right quadrant) for the case $Z = E_x$ at a period of $T = 8$ s. For this period range the processing cannot reconstruct reliable values for apparent resistivities and phases. The picture also shows the residuals, i.e. the misfit for each data event (lower left quadrant) and the corresponding weights (lower right quadrant). The robust scheme consists of two algorithms, which are applied sequentially. Where the first robust algorithm calculates the residuals by evaluating the transfer functions for each time segment separately, the second algorithm computes global transfer functions which are averaged over all time segments. The black dots indicate an event with a weighting factor above 0.7. Red dots mark events that were down-weighted by the first robust algorithm by a factor of 0.7 or less. Events that were not already down-weighted by the first algorithm but eventually by the second algorithm (by a factor less than 0.7) are coloured in green.

When examining the distribution of the transfer functions, it is obvious that some data points located far away from the centre of the distribution are not down-weighted. The a_R is more affected than the b_R component. Other events, although being strongly biased by noise, are not at all recognized

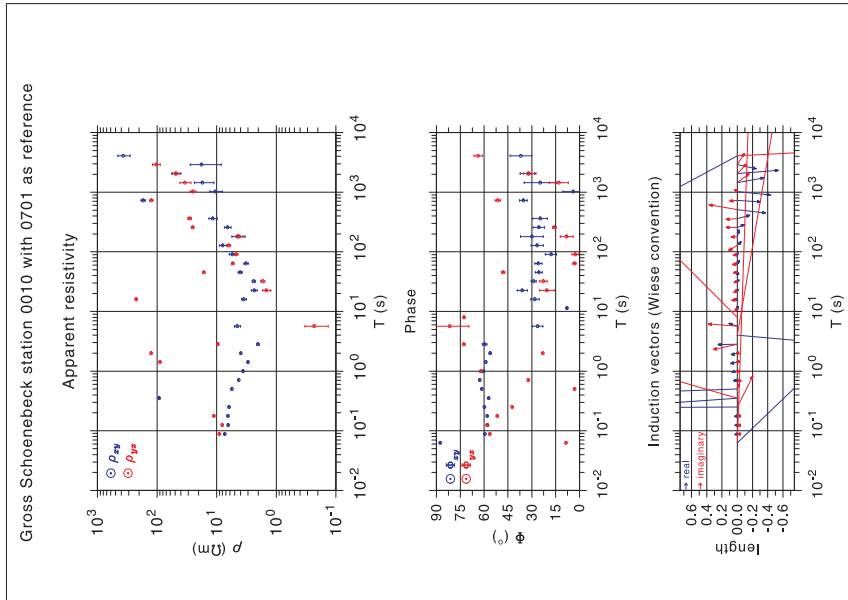


Figure 2: Using the old definition for the residuals leads to strongly distorted results for the remote reference processing.

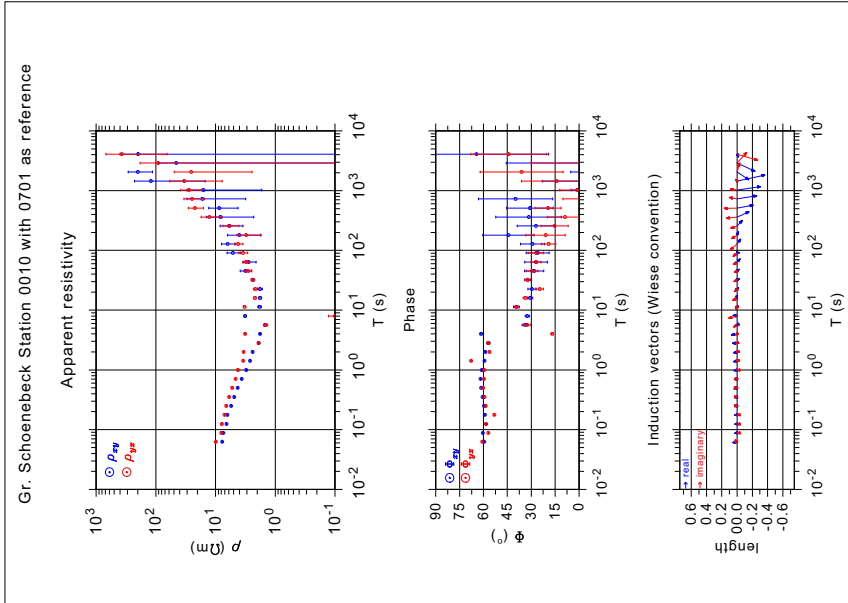


Figure 3: Robust remote reference processing result using the new definition of the residuals in equation 7.

by the weighting scheme. The reason could be that the calculation of the residual is based on the simultaneous estimation of both transfer functions. This means that if one transfer function is well determined and the other one poorly, the residual can still be below the threshold for down-weighting. Thereby, huge outliers in either one of the two connected transfer functions can be masked if one of the transfer functions is well determined.

The picture in the lower left quadrant shows that most of the large residuals are already recognized by the first algorithm, whereas the second mainly works as a refinement to this “preselection”. The graphical display of the weights confirms, that the first algorithm is responsible for most of the down-weighting. There is a transition zone between supposedly “good” data (weighting factor above 0.7) and “bad” data (weighting factor below 0.7). However, hardly any weights are between 0.7 and 0.95. The causes for this abrupt transition are unclear.

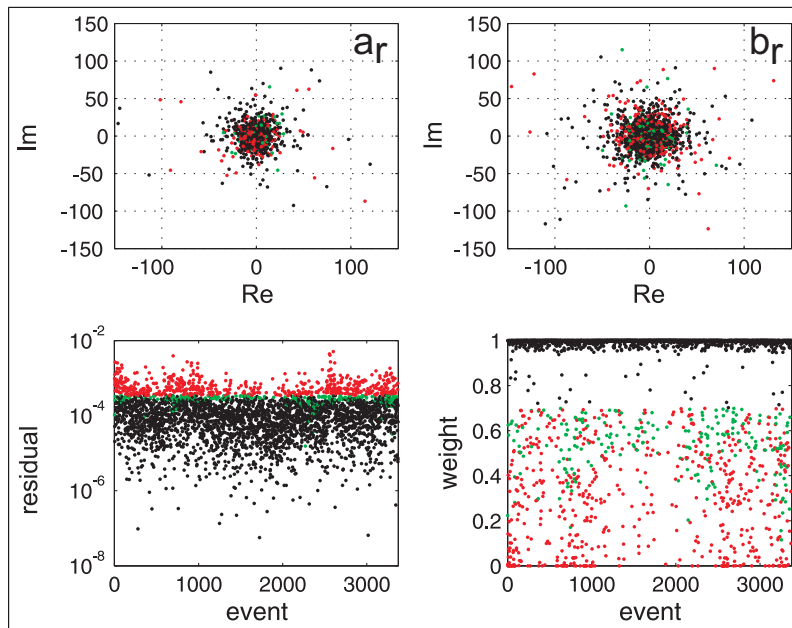


Figure 4: Transfer functions a_R , b_R , residuals and weights for the period of $T = 8$ s from Groß Schönebeck experiment (station 10).

5 Conclusions

The definition of a new residual (along with other modifications to the code) significantly improved the robust remote reference scheme which is now ca-

pable of efficiently suppressing noise in the data efficiently. However, there are still problems to achieve good data quality of very disturbed datasets particularly in the period range of the dead band (around 1 s).

A closer inspection of the weighting process revealed that many outliers remain undetected. This could be due to the fact that we solve a bivariate equation, where one well estimated transfer function can mask an associated severely distorted transfer function.

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Stochastic Sampling for Mantle Conductivity Models

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Abstract

We present a bayesian Monte Carlo analysis of geomagnetic induction data for the mantle conductivity distribution. We use a modified version of the Monte Carlo method with Markov chains based on an effective, data adaptive Metropolis sampling approach, and simulate samples from the posterior probability distribution of the resistivities in the mantle for four recently published global, regional, and satellite induction data sets. Stochastic sampling provides comprehensive maps of the parameter space based on fairly ranking the models according to their ability to explain the experimental data, as well as on respecting the prior information on the model parameters. From four generally formulated and tested priors for the mantle resistivities, the non-informative distribution on strictly increasing conductance is the most non-restricting prior that, at the same, avoids the non-likely high-resistivity tails from the marginal resistivity distributions. A prediction power of the MCMC sampling approach is demonstrated by a comparison of published maximum likelihood bounds on average conductivities in specific mantle zones with those produced simply by computing the average conductivities from the Markov chain of models.

1 Introduction

The electrical conductivity in the Earth's mantle is a particularly important parameter for studies of the temperature distribution, mineralogical composition and fluid content in the mantle. Carrying on the classical works by Banks (1969), Roberts (1984), Schultz and Larsen (1987), and others, new induction responses have been obtained in recent decades based either on largely extended data series and new methods and processing techniques (e.g., Olsen, 1998, 1999a; Honkura and Matsushima, 1998; Semenov, 1998; Schmucker, 1999; Semenov and Jóźwiak, 1999; Praus *et al.*, 2004), or on thoroughly re-assessed responses from earlier mantle conductivity studies (e.g., Constable, 1993; Medin *et al.*, 2007). New geomagnetic data from satellites have further extended the range of induction responses of the Earth both as regards the frequency spectrum and spatial coverage of the Earth (e.g., Olsen, 1999b; Constable and Constable, 2004; Kuvshinov and Olsen, 2006; Velínský *et al.*, 2006).

Based on the derived induction responses a number of mantle conductivity models have been suggested that revealed and refined the basic features of the electrical conductivity distribution in the Earth's mantle. Extremal D^+ models (Parker, 1980) have been studied to test the compatibility of the induction data with the 1-D hypothesis on the conductivity distribution, as well as to estimate the smallest misfit achievable within the 1-D model approximation (e.g., Constable, 1993; Olsen, 1999a). D^+ models are useful theoretical tools, but are non-physical by nature. Models with few uniform thick layers (e.g., Banks, 1969; Larsen, 1975; Olsen, 1998) are an option if it is reasonable to assume that a piecewise constant conductivity distribution with depth is an acceptable approximation to the true conductivity structure. In the Earth's mantle, however, the exponential conductivity vs. temperature dependence rather suggests to employ smoothly varying conductivity vs. depth models, even in those parts of the mantle that are assumed uniform as to their mineral composition.

Regularized Occam approach (Constable *et al.*, 1987) aims at restoring the simplest, minimum structure that is still compatible with the experimental data. The simplicity can be defined in various ways, which may accent some particular features of the restored models, the most common being the minimum norm that minimizes the distance from a pre-defined model, or the minimum gradient or minimum Laplacian norms that produce the smoothest models fitting the data (e.g., Constable, 1993; Olsen 1998, 1999a; Semenov, 1998; Semenov and Jóźwiak, 1999). Other structural penalties have been suggested as well aimed at, e.g., minimizing the total variation of the conductivity profile, the spatial extent of anomalous domains (minimum support norm), or the number of conductivity jumps within the model (minimum gradient support norm, Portniaguine and Zhdanov, 1999).

Stochastic inversion methods provide alternatives to conventional non-linear inverse procedures. They scan the model parameter space and identify multiple models that match a given dataset, thus providing additional

information on parameter uncertainty. Grandis *et al.* (1999) suggested using a bayesian Markov chain Monte Carlo (MCMC) approach to the solution of a 1-D magnetotelluric inverse problem, with the model parametrization similar to that used in traditional linearized Occam inversions. Bayesian approach not only produces maps of data matching models in the parameter space, it also ranks the models and calculates posterior model probabilities conditioned on the observed data. While neither analytical, nor direct search, nor random shooting approaches are feasible strategies to restoring the probability distributions in a multi-parametric magnetotelluric inverse problem, the MCMC is a technique that allows simulating a sample from the posterior probability distribution numerically in a computationally feasible way.

The aim of this paper is to demonstrate and discuss results of application of the bayesian MCMC to the inversion of global geomagnetic induction data for mantle electrical conductivity. We will especially address two aspects of this problem, specifically (i) the impact of the data quality on the probabilistic inversion, and, (ii) the role prior information on the model parameters plays in the inversion results. The structure of the paper is as follows: In Section 2, we give a brief outline of the bayesian MCMC approach as we apply it in this study. Section 3 characterizes four global and regional induction data sets, previously published, which we have used in the MCMC experiments. In Section 4, we present the MCMC sampling results for the mantle conductivity obtained for the four data sets under various common prior assumptions as to the model parameters in the Earth. Section 5 concludes the study by comparing our MCMC results with conductivity models presently available.

2 Bayesian Stochastic Sampling

In a bayesian approach, both the parameter estimation and the assessment of the parameter uncertainties are treated as problems of determining the posterior probability of parameters of the conductivity model conditioned on the observed induction response data. If a model class is specified, say M , then our knowledge on the model parameters, $\mathbf{m}$, prior to the experiment, formalized as a prior probability $\text{Prob}(\mathbf{m}|M)$, can be updated by employing the experimental data, $\mathbf{d}^{\text{exp}}$, according to the Bayes rule,

$$\text{Prob}(\mathbf{m}|\mathbf{d}^{\text{exp}}, M) = \frac{\text{Prob}(\mathbf{d}^{\text{exp}}|\mathbf{m}, M) \text{Prob}(\mathbf{m}|M)}{\text{Prob}(\mathbf{d}^{\text{exp}}, M)}, \quad (1)$$

(see, e.g., Gelman *et al.*, 2004). Here, the probability function $\text{Prob}(\mathbf{d}^{\text{exp}}|\mathbf{m}, M)$ is the likelihood, and specifies how likely a model with parameters $\mathbf{m}$ is to produce the particular set of observed data $\mathbf{d}^{\text{exp}}$. The posterior probability density function $\text{Prob}(\mathbf{m}|\mathbf{d}^{\text{exp}}, M)$ is considered a solution to the inverse problem, and can be further used in assessing the parameters, evaluating point estimates, confidence intervals, etc.

Provided the noise in the experimental data is normal Gaussian with standard deviations $\delta \mathbf{d}^{\text{exp}}$ and the data items are mutually independent, the likelihood in (1) can be written

$$\text{Prob}(\mathbf{d}^{\text{exp}}|\mathbf{m}, M) \propto \exp \left[- \sum_{j=1}^{N_D} \left(\frac{d_j^{\text{exp}} - d_j^{\text{mod}}(\mathbf{m})}{\delta d_j^{\text{exp}}} \right)^2 \right], \quad (2)$$

where N_D is the number of individual data items, and $\mathbf{d}^{\text{mod}}(\mathbf{m})$ is a solution to the direct problem for the parameters $\mathbf{m}$. The nominator $\text{Prob}(\mathbf{d}^{\text{exp}}, M)$ in (1) is a normalization constant independent of $\mathbf{m}$ and need not be considered explicitly in cases where only model ranking is required.

In this study, the model class M is defined by a spherically symmetric model with a conductive mantle reaching from the surface down to the depth of 2900 km, and with a perfectly conducting core between the bottom of the mantle and the centre of the Earth at $R_E = 6371$ km. The crust and mantle portion of the model consists of a uniform crust, thickness $h_c = 30$ km, and a suite of thin layers with fixed, log-uniform thicknesses throughout the mantle. The variables for the inversion are the logarithm of the resistivity of the crust, $\varrho_c \equiv \varrho_1$, and logarithms of the resistivities of the individual mantle layers, $\varrho_2, \dots, \varrho_L$, where L is the total number of layers. Typically, we have used $L = 61$ in most of our experiments. To solve the direct problem for the finely stratified mantle, we employed the standard 1-D direct magnetotelluric algorithm for stratified conductors and further made use of the Weidelt's flattening transformation in our computations to account for the sphericity of the Earth (Weidelt, 1972; see also Olsen, 1999a).

The prior probability in (1), $\text{Prob}(\mathbf{m}|M)$, describes the available knowledge about the layer resistivities prior to the data being observed, and this may be one of the most sensitive issues of a bayesian analysis, especially if no or only very limited information is available apriori.

A bayesian analysis requires the priors on the parameters to be specified explicitly in (1). As we wanted to understand the role the priors play in modulating the mantle conductivity models we have tested four sufficiently general priors for the layer resistivities in the mantle:

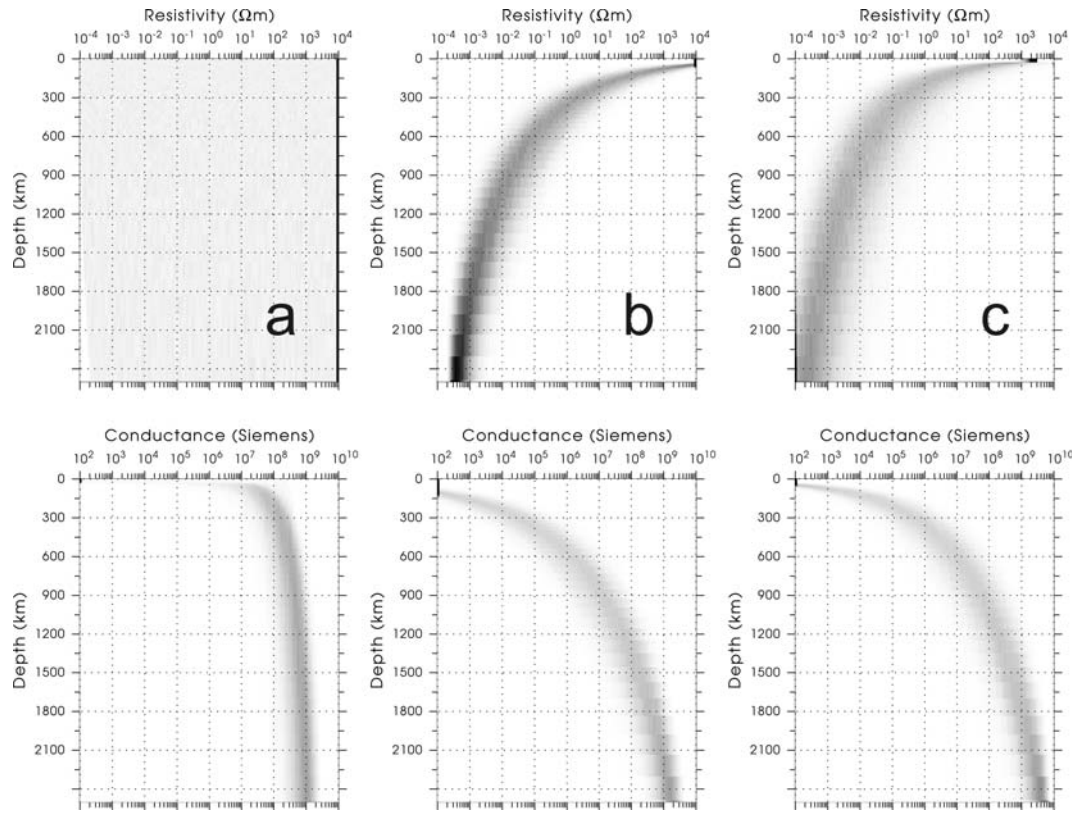


Figure 1: Gray shade maps of prior marginal probability density functions for the resistivities of the mantle layers in the $\log \rho - z$ frames (top panes) and for conductances at the bottom of the mantle layers in the $\log S - z$ frames (bottom panes). The gray shade maps represent converted relative histograms approximating the marginal priors for the resistivities/conductances for each individual mantle layer. For the gray shade scale, white is for no occurrence and black for the occurrence in 10% or more cases for resistivities, and in 30% or more cases for conductances. a—Non-informative Jeffrey's prior on each layer resistivity, resistivities considered mutually independent (Pi in text), b—Prior for the case of the resistivities distributed uniformly within individual mantle layers, but non-increasing resistivity is required throughout the mantle (Piii in text), c—Prior for the case of the conductances at the bottom of individual mantle layers distributed uniformly, strictly increasing conductance is required throughout the mantle (Piv in text).

Pi) Without having carried out any induction experiments, we could not anticipate any specific mantle resistivity features, except perhaps for some loose resistivity bounds dictated by general physics. The prior that expresses total ignorance about a positive parameter, $\varrho_i > 0$, $\varrho^{\min} < \varrho_i < \varrho^{\max}$, is the Jeffrey's prior, $\ln \varrho_i \sim U(\ln \varrho^{\min}, \ln \varrho^{\max})$, $i = 1, \dots, L$, where $U(a, b)$ is for a uniform distribution on the interval (a, b) (e.g., Ulrych *et al.*, 2001). If no relations between the layer resistivities are assumed apriori, the prior pdf on $\mathbf{m} = (\varrho_1, \dots, \varrho_L)$ is constant and given by $\text{Prob}(\mathbf{m}|M) = \ln^{-L} \varrho^{\max} / \varrho^{\min}$. The top panel in Fig. 1a shows, as histograms converted into gray scale maps, marginal priors for the individual resistivities ϱ_i , $i = 1, \dots, L$, in the $\log \varrho$ vs. depth frame. The particular choice of the resistivity bounds was $\varrho^{\min} = 10^{-4} \Omega\text{m}$, $\varrho^{\max} = 10^5 \Omega\text{m}$ here. It is interesting that the prior for conductance, $S_i = \sum_{j=1}^i h_j / \varrho_j$, seems to prefer models with rapidly increasing conductance in the shallow part of the mantle in this case (Fig. 1a, bottom pane).

Pii) Even without any induction measurements, other geophysical parameters in the mantle suggest that the assumed independence between the layer resistivities is a weak point of the previous prior Pi. Specifying a correlation function between prior uncertainties can be used to enforce proper smoothing into the conductivity model, but that information is not available in practice. We use a simple prior by Haario *et al.* (2004) that is based on the assumption that the neighbouring discretized values of a resistivity profile cannot be too different, which can be formalized as, e.g., $\ln \varrho_{i+1} - \ln \varrho_i \sim N(0, \nu^2)$, where $N(\mu, \sigma^2)$ is for a normal pdf

with the mean μ and variance σ . Alternatively, second or higher order differences of $\ln \varrho$ can be used as well. By using in (1) this prior along with the likelihood (2), we have

$$\text{Prob}(\mathbf{m}|\mathbf{d}^{\text{exp}}, M) \propto \exp \left[- \sum_{j=1}^{N_D} \left(\frac{d_j^{\text{exp}} - d_j^{\text{mod}}(\mathbf{m})}{\delta d_j^{\text{exp}}} \right)^2 - \frac{1}{2\nu^2} \sum_{k=1}^{L-1} (\ln \varrho_{k+1} - \ln \varrho_k)^2 \right], \quad (3)$$

which formally gives the Occam first derivative smoothed solution, with the structure penalty weight $\lambda = 1/2\nu^2$, as a maximum a posteriori probability (MAP) estimate. This smoothing prior does not allow a simple visualization and is therefore omitted in Fig. 1.

Piii) The third prior is identical with the non-informative Pi above except that strictly non-decreasing resistivity is required throughout the mantle, $\varrho_{i+1} \leq \varrho_i$, $i = 1, \dots, L-1$, an assumption used as an option by Medin *et al.* (2007) in their study on average conductivities in selected mantle domains. This prior (Fig. 1b) shows rather a strong regularizing effect. The marginals in Fig. 1b can be made more uniform and their maxima less pronounced if the resistivity bounds ($\varrho^{\min}$, $\varrho^{\max}$) are broadened. This, however, has technical disadvantages since it increases the volume of the parameter space, and necessarily also the computation demands of the MCMC sampling.

Piv) The fourth prior is similar to the previous one, but is formulated for the conductances at the bottom of the mantle layers, $S_i = \sum_{j=1}^i h_j / \varrho_j$, $i = 1, \dots, L$. Here, we assume that $\ln S_i \sim U(\ln S^{\min}, \ln S^{\max})$ and $S_{i+1} > S_i$ for any i (see Fig. 1c for the conductance bounds $10 < S_i < 10^{10}$, in siemens).

Grandis *et al.* (1999) used the MCMC method with a standard Gibbs sampler to simulate samples from the posterior pdf (1). In this approach, the domain under study is scanned successively in a component-by-component way, and an ergodic Markov chain is designed with the invariant distribution equal to the posterior pdf (1). After a large enough sample from the posterior probability density function has been obtained, basic bayesian integrals for the means, covariance matrices, etc., can be easily evaluated from the posterior sample and used to assess the resistivity models as well as their uncertainties.

We have used a slightly less demanding sampling procedure based on the single component adaptive Metropolis procedure (SCAM, Haario *et al.*, 2003) which proceeds in similar cycles as the Gibbs sampler except that the updates to the individual components are generated by a data adaptive Metropolis rule. As in this algorithm a longer history of the chain is used for adapting the variances for the Metropolis step, the chain is evidently not Markovian any more. Nonetheless, Haario *et al.* (2003) have proved its convergence to the target posterior under sufficiently general conditions. More details on a general theory as well as on particular aspects of our MCMC implementation can be found, e.g., in (Gelman *et al.*, 2004; Grandis *et al.*, 1999; Haario *et al.*, 2003; Červet *et al.*, 2005).

3 Data Sets for Mantle Conductivity Studies

To compare the effect of the stochastic sampling approach in the inversion of induction responses of different origin and different quality, we have applied the bayesian MCMC sampling technique to four different sets of induction responses published in recent years. In particular, we have dealt with the following data sets:

Summarized Representative Responses by Medin et al. (2007)

Medin *et al.* (2007) used a 34 admittance set of data that is a combination of Constable's (1993) global responses averaged and statistically re-assessed from previous data by Roberts (1984) and Schultz and Larsen (1987), and of data by Olsen (1999a) from land observatories in Europe. Prior to the modeling, the data were checked for a 1-D compatibility by assessing misfit of the extremal D^+ solution. Data that contributed most to the violation of the 1-D hypothesis have been removed from the set. The thorough analysis and selection of individual data items into this data set justify our taking this admittance collection as a reference data set.

Standard Global Responses for Sq and Long Period Variations by Praus et al. (2004)

Praus *et al.* (2004) presented a standard set of 13 global induction response data for two period ranges. Specifically, they studied the Sq daily variation data from 88 to 94 land observatories over the globe for the IQSY year 1965. Additionally, they processed data for long period variations, within the period range of 4 days up to 1 year, from 40 to 48 world observatories independently in three 3-years segments with largely different solar and geomagnetic activities (1958-60, 1963-65, 1967-69) by the standard Z/H -method. The data were supplemented by the 11-year response estimate by Harwood and Malin (1977). The global non-robust processing approach used in the study by

Praus *et al.* (2004) necessarily produced data with larger variance as compared to other data sets used. Moreover, the data collection is discontinuous as it does not involve data from the range of Dst transients.

Magnetic Variation Response for North-Central Europe by Semenov (1998)

Semenov (1998) published local impedances from magnetic variation studies over several regions all over the world. We consider here his averaged induction response for the region of North-Central Europe, roughly involving the geologically intricate territory of a broader vicinity of the Trans-European Suture Zone. The data set consists of 16 complex impedances within the period range of about 4 hours to 3.7 years. At the high frequency end, the data are supplemented by 3 impedance values from long period magnetotelluric studies, extending thus the period range down to 20 minutes. At the lowest frequencies, this data set has also been supplemented by the 11-year response estimate by Harwood and Malin (1977).

Satellite Geomagnetic Induction Responses by Kuvshinov and Olsen (2006)

Kuvshinov and Olsen (2006) processed five years (2001-2005) of scalar and vector magnetic field measurements from the Ørsted and CHAMP satellites, and scalar magnetic field measurements from the SAC-C satellite. They published a set of 17 complex values of the C -response function within the period range of about 14 hours to 143 days. As the inhomogeneous, highly conducting ocean may affect the induction data considerably, the authors developed and applied a scheme for correcting for the effect of induction in the non-uniform oceans. The main distorting effect of the ocean occurs at periods up to 7 days, and after the correction has been made, the satellite responses suggest upper mantle conductivities similar to those obtained from ground-based data. Here, we have used only the corrected response set by Kuvshinov and Olsen (2006).

4 Mantle Conductivity Models via Stochastic Sampling

For each of the data sets listed in the previous section, we have carried out several MCMC sampling runs with the priors suggested in Section 2. A typical setting of an MCMC (SCAM) run was: 1 million iteration steps; from those the first 100000 steps were considered a burn-in phase and discarded from the analysis. According to a typical behavior of the parameter autocorrelations along the chain, we only considered each 100-500th model from the chain to reduce the dependence between the sample elements. It is in general difficult to assess convergence of the MCMC sampling, though theoretically it is guaranteed. We have relied on two diagnostic criteria in this respect, the stabilization of the chain over a large number of iteration steps and, in a few experiments with several parallel chains running, similarity of the inter-chain and within-the-chain parameter variances.

In an analogous way as for the prior pdf's in Fig. 1, we also present the histograms, converted into gray scale maps, approximating the posterior marginal pdf's for the resistivities of the individual mantle layers as well as the conductances at their bottoms. The histograms are constructed from the sample parameters generated by the SCAM algorithm applied to the posterior (1) with the Jeffrey's prior on entirely non-correlated layer resistivities considered. As expected, the histograms show considerable occurrence of the resistivity values in the high-resistivity sections of the maps, indicating that many of the models in the sample may oscillate wildly, with practically no limit on the resistivity from above. From below, the resistivities are limited by about 10 Ωm between 100 to about 400 km in models Fig. 2a, b and d (global observatory data by Medin *et al.*, 2007, and by Praus *et al.*, 2004, and satellite data by Kuvshinov and Olsen, 2006, respectively), while the regional data by Semenov (1998) shift the lower resistivity envelope towards 2-3 Ωm between 200 and 400 km, and suggest a more conductive structure to exist at those depths (Fig. 2c). Below 400 km the minimum resistivity envelope decreases rapidly in all models, to about 1 Ωm at 600 km and to values around 0.1 Ωm at 800 km. Within the depth range of 400 to 900 km, several marked spots of increased probability are observed, especially well developed in the maps based on the high-quality data by Medin *et al.* (2007) and on the satellite data by Kuvshinov and Olsen (2006) in Figs. 2a and d, respectively. Those spots seem to partly correspond to the δ -peaks of the respective D^+ solutions overlaid on the probability maps in Fig. 2. It seems to be a logical relation, as an inversion for unconstrained 1-D resistivity should tend to produce a D^+ -like solution. Below 900 km, an almost constant minimum resistivity profile is indicated by Fig. 2a, with one more spot of increased probability between about 1150 and 1350 km, which cannot be distinguished in any other of the models presented. The slow decrease of the minimum resistivity in the lower mantle in Figs. 2b, c may already indicate insufficient resolution of the data, and increased attraction of the models to the prior distribution deep in the mantle, which then seems to take almost a full control over the resistivities below the depth of about 2000 km, and even earlier, at about 1500 km, for the satellite data.

To assess the adequacy of the posterior probability samples generated by the SCAM process to the experimental data, we may compare the pdf's of the model responses from the chain with the stochastic model of the experimental data. Though the responses of the models from the SCAM chain should replicate the pdf's of the experimental data after a sufficiently long run, and exact tests for the goodness-of-fit could be arranged, we will

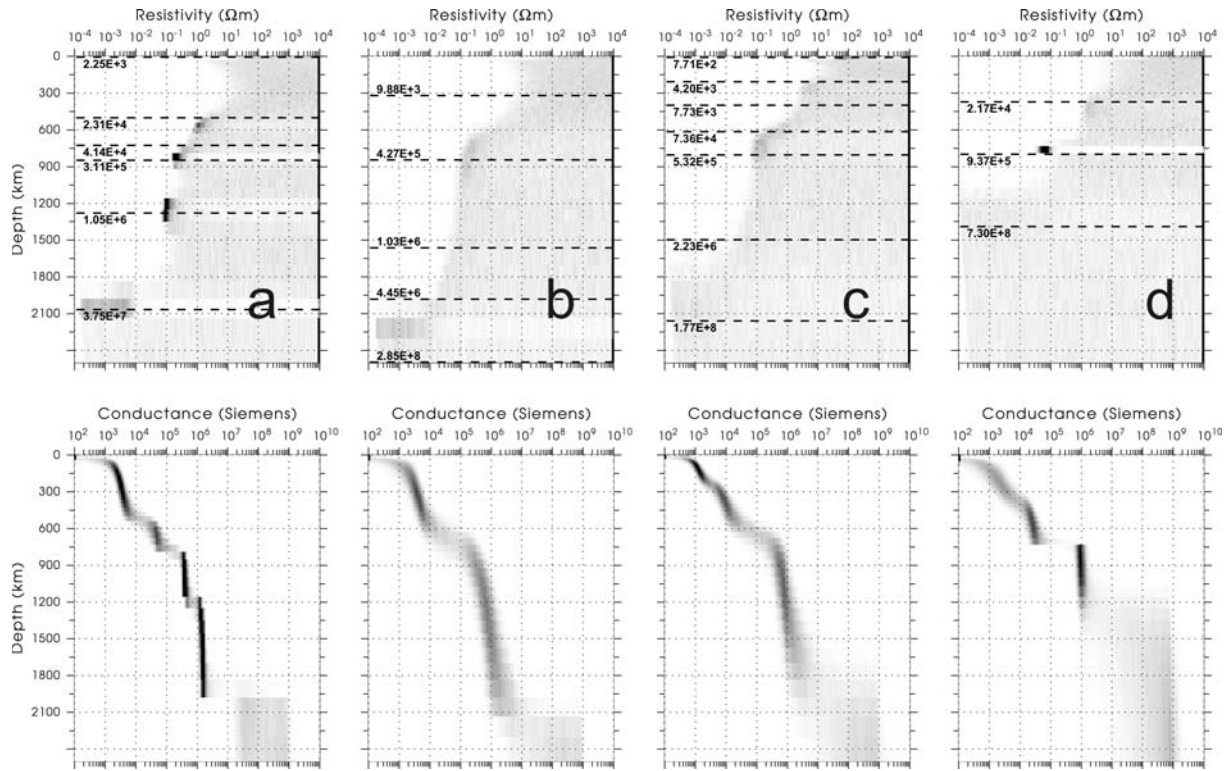


Figure 2: Gray shade maps of marginal probabilities of the resistivities of the mantle layers from a SCAM sample generated from the non-informative prior P_i (Section 2) and the four data sets listed in Section 3, specifically, for data by a—Medin *et al.* (2007), b—Praus *et al.* (2004), c—Semenov (1998), d—Kuvshinov and Olsen (2006). The gray shade maps represent converted relative histograms approximating the marginal posteriors for the resistivities/conductances for each individual mantle layer. For the gray shade scale, white is for no occurrence, and black for the occurrence in 10% or more cases for resistivities (top panes), and in 30% or more cases for conductances (bottom panes). The dashed lines in the resistivity maps show the positions of the δ -peaks of the D^+ solution, and are labeled with the corresponding sheet conductances in siemens. RMS bounds for 90 per cent of all models in the SCAM chain, for model a: 1.254–1.335 (1.211), b: 0.550–0.853 (0.508), c: 0.609–0.859 (0.433), d: 1.354–1.508 (1.290). In the brackets, the RMS for the D^+ solution is given.

assess the model-to-data fit only qualitatively here. For this to do, we have added to the figure caption of Fig. 2, and later also to Figs. 3, 4, and 5, intervals of the RMS parameters (normalized by the data variances) for 90 per cent of all the models generated during the stabilized phase (after burn-in) of the chain.

Fig. 3 shows the posterior marginals for the resistivities in models generated by the SCAM algorithm under the smoothing prior P_{ii} (see Section 2). In this case, we have to specify, in terms of the parameter ν in (3), how tense the link should be between neighboring layer resistivities. Though more sophisticated approaches might be suggested, we have taken the simplest way here by choosing ν apriori from the standard L -curve approach within a linearized Occam inversion run applied to the best data set by Medin *et al.* (2007). We have kept this particular value of ν also for the other data sets analyzed in order to guarantee that the same structural constraint is applied in all cases.

With the smoothing prior employed, the resistivity marginals simplify considerably, showing a steady resistivity decrease from about 100–500 Ωm at 150 km to 0.7–2 Ωm at 800 km, followed by a very slow resistivity decrease to about 0.3–0.6 Ωm at 1500 km. Beneath about 1300–1500 km, Fig. 3a, and to a less obvious degree also Fig. 3c, indicate a faster resistivity decrease in the lower mantle. This feature is not resolved by the other two data sets.

An almost one order of magnitude increase of the resistivity is suggested within the shallowest, 100–150 km thick section by models with the (first derivative) smoothing prior in Figs. 3a and c. Moreover, in the regional model from the Semenov's (1998) data in Fig. 3c, the separate conducting layer between about 200 and 300 km persists, which has been already indicated by sampling for the 'non-regularized' resistivities. Poorer short period data for Fig. 3b do not resolve any shallow features. Neither is the resistivity increase in the lithosphere indicated by the satellite data in Fig. 3d, but here the correction introduced by Kuvshinov and Olsen (2006) to eliminate the effect of the oceans might have affected that part of the induction response which is sensitive to shallow depths.

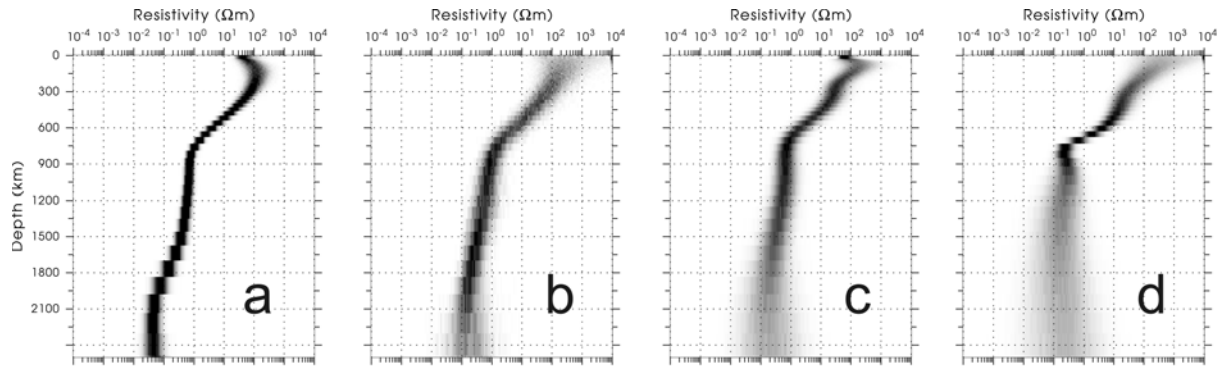


Figure 3: Gray shade maps of marginal probabilities of the resistivities of the mantle layers from a SCAM sample generated from the smoothing prior Pii (Section 2) and the four data sets listed in Section 3. RMS bounds for 90 per cent of all models in the SCAM chain, for model a: 1.369–1.475, b: 0.616–0.888, c: 0.645–0.872, d: 1.464–1.599. For the plot attributes, see caption to Fig. 2.

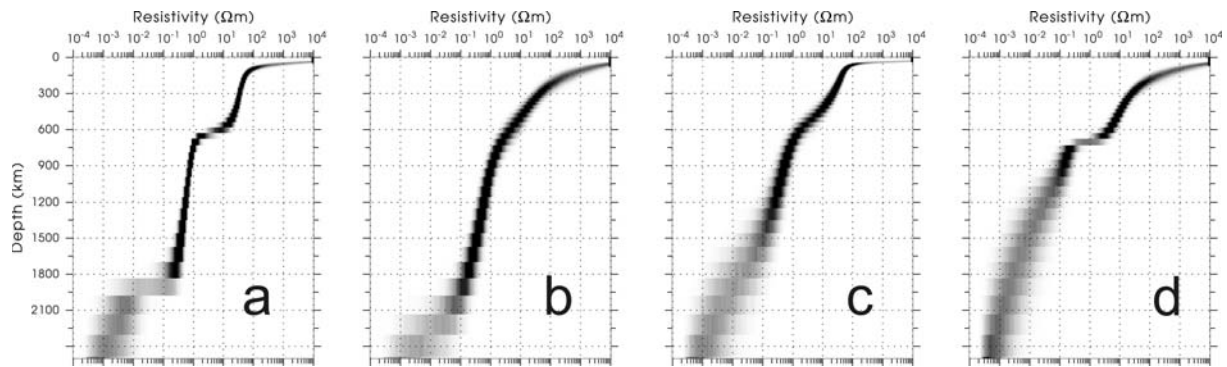


Figure 4: Gray shade maps of marginal probabilities of the resistivities of the mantle layers from a SCAM sample generated from the prior Piii requiring non-increasing resistivity throughout the mantle (Section 2) and the four data sets listed in Section 3. RMS bounds for 90 per cent of all models in the SCAM chain, for model a: 1.448–1.575, b: 0.653–0.913, c: 1.117–1.390, d: 1.481–1.607. For the plot attributes, see caption to Fig. 2.

The satellite model also suggests anomalously low resistivities, of the order of $0.1 \Omega\text{m}$, at mid-mantle depths, which contradicts the Occam model presented by Kuvshinov and Olsen (2006). This may be related to the particular choice of the smoothing parameter ν here. For the models in Fig. 3d, the RMS is between 1.464 and 1.599 for 90 per cent of the SCAM generated models, while Kuvshinov and Olsen (2006) report a misfit value which would correspond to the RMS of 1.78. In our MCMC experiments, ν would have to be decreased by a factor of 3 to 5 to achieve the same RMS range, which would flatten the model considerably in its mid-mantle section.

Requiring the resistivity to be a non-increasing function of depth throughout the mantle (prior Piii in Section 2) seems to be a strong regularizing factor, as can be seen from marginal resistivity histograms in Fig. 4. The models show only the main feature of a sharp resistivity decrease at mid-mantle depths of 600–700 km. We had difficulties with fitting properly the regional data set by Semenov (1998) under this prior, as systematic discrepancies between the model and experimental data occurred regularly at the shortest periods up to 2×10^4 s. It may indicate that a distinct conducting layer in the upper mantle is required by this particular data set, but some inconsistencies originating from merging the long period tensor magnetotelluric data with the scalar geomagnetic induction response in this period range cannot be excluded either. The shallow high resistivities for all models are required by the data under the Piii prior, and restricting the resistivity by a decreased $\varrho^{\max}$ increases the misfit (e.g., from 1.278–1.964 RMS for 90% of models at $\varrho^{\max} = 10^5 \Omega\text{m}$ to 1.905–2.189 at $\varrho^{\max} = 300 \Omega\text{m}$ for the data set by Medin *et al.*, 2007).

The prior Piv (Section 2), requiring a strictly increasing conductance throughout the mantle, may seem somewhat unproductive, since any conductance has to be an increasing function of depth, whatever the underlying resistivity distribution. But combined with the Jeffrey’s uniformity condition on each $\ln S_i$, $i = 1, \dots, L$, this prior seems to better conform our apriori belief on the mantle resistivity distribution in that it eliminates both the low and high-resistivity extreme domains from the resistivity pdf’s without restricting the structure too much.

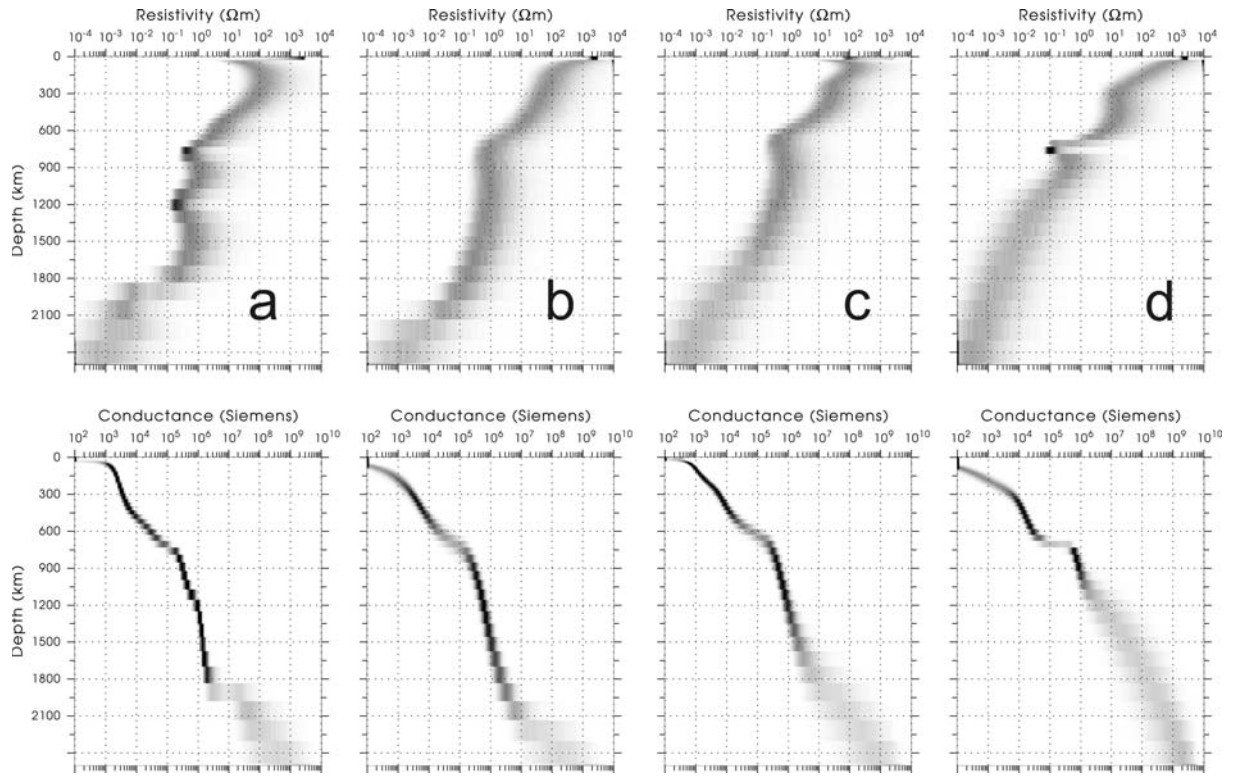


Figure 5: Gray shade maps of marginal probabilities of the resistivities of the mantle layers from a SCAM sample generated from the conductance prior Piv (Section 2) and the four data sets listed in Section 3. RMS bounds for 90 per cent of all models in the SCAM chain, for model a: 1.278–1.964, b: 0.518–0.734, c: 0.590–0.769, d: 1.380–1.513. For the plot attributes, see caption to Fig. 2.

Marginals of the resistivities and conductances from models sampled with this prior employed are shown in Fig. 5. They principally reproduce the conductivity features observed in Fig. 2 for independent layer resistivities, but are more effectively channelized, and do not show the long, most likely spurious, high-resistivity tails.

5 Conclusion

The numerical MCMC experiments with a 1-D inversion of geomagnetic induction responses for the mantle electrical conductivity have shown that the stochastic sampling can be considered a suitable tool for this kind of geophysical inverse problems. A relative simplicity of the underlying direct problem allows us to generate sufficiently representative samples from the posterior pdf and obtain representative marginal posterior pdf's for the resistivities in the mantle. As compared to standard linearized inverse approaches, stochastic sampling provides a more comprehensive map of the parameter space based on fairly ranking the models according to their likelihoods (i.e., according to the model-to-data fit), and with regard to the prior information available.

We have demonstrated the role of the prior pdf's by testing four sufficiently general prior types that may reflect our limited knowledge of the deep electrical structure of the Earth. We have applied the MCMC sampling to four different data sets, ranging from global ground-based data of different quality (Medin *et al.*, 2007; Praus *et al.*, 2004), through regional induction responses by Semenov (1998), through the latest satellite data by Kuvshinov and Olsen (2006). The averaged and thoroughly pre-assessed data set by Medin *et al.* (2007) has proved superior as to the resolution of the mantle conductivity structure. As regards the prior distributions, they show rather a strong effect on the final model parameter distributions. In particular, the smoothing and non-increasing resistivity priors, Pii and Piii in Section 2, show a strong regularizing effect, while the strictly-increasing-conductance prior, Piv in Section 2, may be employed if no extra smoothing/correlation is welcome and the high-resistivity tails of the resistivity marginals, most likely spurious, are to be avoided.

Bayesian analysis, and the MCMC as an efficient tool to carry it out practically, is an effective approach to predicting specific or derived features of models, as well as in cases when heterogeneous information is to be interpreted jointly. For the latter case, the study by Khan *et al.* (2006) on constraining the composition and thermal state of the mantle beneath Europe from inversion of long period electromagnetic sounding data is an excellent

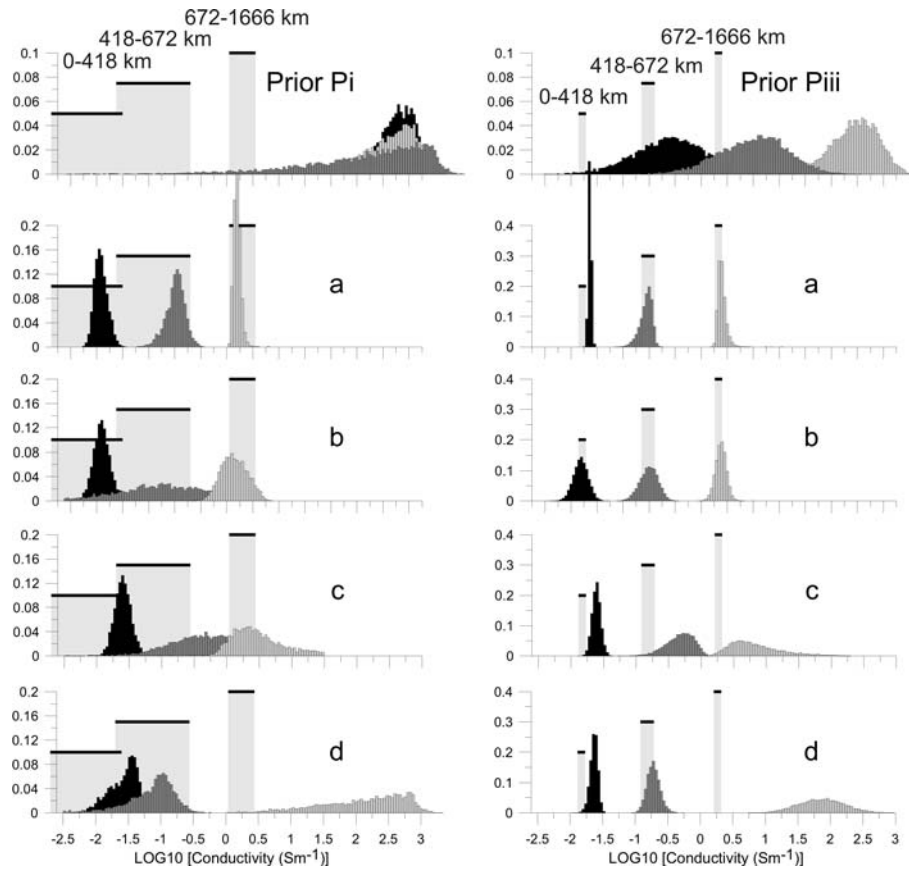


Figure 6: Comparison of ML estimates of average conductivities in three mantle domains, 0–418 km, 418–672 km, and 672–1666 km from Medin *et al.* (2007) with those predicted from the SCAM samples. Left column compares ML ranges of 90% χ^2 bounds on the average conductivity (light gray boxes) with histograms of the SCAM generated values under the non-informative prior Pi. Right column compares ML results obtained under the condition $d\sigma(z)/dz \geq 0$ with histograms from SCAM simulations under the non-increasing-resistivity prior Piii. The model labels, a–d, correspond to those in Fig. 2.

example. We present here, in Fig. 6, only a simple comparison of MCMC estimates of average conductivities in specific domains of the mantle with those obtained by a direct maximum likelihood (ML) inversion by Medin *et al.* (2007). The direct ML and MCMC ranges compare well for most of the models at the upper bound of the average conductivities, provided the mantle domain considered is sensed by the data. The difference in the predicted lower bounds for the case of the non-informative prior is mainly due to the fixed $\varrho^{\max} = 10^5 \Omega\text{m}$, which rules out all acceptable models with extremely high-resistivity layers beyond that limit from the MCMC sample. Nonetheless, this example demonstrates that successful predictions are possible from the MCMC simulations, without much additional costs over and above those that are necessary for the basic MCMC runs.

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XML, HTML and SQL

Human-Machine Dialogue

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Introduction

Data storage, handling and interchange has become a major problem in geophysics. ASCII files such as calibration data or the EDI format are still in use but they do not provide the capabilities of automated data storage and access. Binary formats such as NetCDF perform well in mass data storage but proprietary software is needed to access the data. The XML specification can solve most of these problems.

ASCII, Binary and XML Storage

The XML data format has become popular in the recent years. It solves the data storage and data access problem in an easy manner.

An simple example illustrates this:

```
>HEAD
  DATAID=079
  ACQBY=UNKNOWN
  LAT=. .52:18:34
  LONG= 12:16:54
>=DEFINEMEAS
  MAXCHAN=7
  MAXRUN=999
  MAXMEAS=9999
```

The **ASCII** storage format uses cryptic separators to define keys and value. Without any knowledge one can not differentiate between main sections, subsections (>, >=, = ??).

In addition, ASCII files are read and written by “hand-made” parsers which lead to slight changes (spaces, tabulators, line feed) over the time and make the data file unreadable over long terms.

```
struct status {
    bool on;
    int gain;
};

-----

struct status {
    bool on;
    double voltage;
    int gain;
};
```

The most efficient and compact format is the **binary** format.

The programmer is forced to use *exactly* the same structure as proposed. The binary format is used for mass data storage but the data description is generally incomplete. To change the data description the binary structure must be extended and the software must be re-written and re-compiled in order to read the new files.

This leads either to a minor acceptance or to a split of data description and data storage.

```
<caldata chopper="on">
  <c1 unit="Hz">1.00000000e-01</c1>
  <c2 unit="V/ (nT*Hz)">2.05e-01</c2>
  <c3 unit="deg">8.85960000e+01</c3>
</caldata>
<caldata chopper="on">
  <c1 unit="Hz">1.25890000e-01</c1>
  <c2 unit="V/ (nT*Hz)">2.027e-01</c2>
  <c3 unit="deg">8.82250000e+01</c3>
</caldata>
```

The **XML** format solves most of the problems mentioned above.

The `<key>value</key>` syntax solves the data storage as well as the data organization into sections and subsections.

A validating parser checks the consistency of the file automatically.

The outstanding feature of XML parsers is that the file can be extended any time (for example `<c4>value</c4>`) – the parser will

always read and write the *complete* file – even though it may contain new sections which were not used before.

Therefore XML is used for extensible data storage where users or institutes can use the base functionality of XML files and software but can integrate their own specific information additionally.

Data-Mining

Instead of writing a manual, the XML specification allows the user to write a declaration of the data being used.

```
<DataDescription>
  .....
  <DataField name="c2"
    optype="continuous"
    dataType="double">
    <Unit unit="V/ (nT*Hz)" />
    <Unit unit="V/nT" />
    <Unit unit="V" />
    <Unit unit="re" />
  </DataField> .....
</DataDescription>
```

Inside the DataDescription section we can define

- a) a valid data type (here double)
- b) the option (here continuous for repeating entries)
- c) and valid units / strings used for the attributes of the c2 field (e.g. V, re)

With this additional information the validity of the keys can be proofed and during data input the data can be transformed into valid units.

In this example a software developer will prepare his code for four different units in column c2.

Data Access

```
$xml = simplexml_load_file($file);
}
echo "cal chopper on\n";
foreach ($xml->caldata as $data) {
    if ("on" == $data['chopper']) {
        echo
            $data->c1 . " "
            . $data->c2 . " "
            . $data->c3 . "\n";
    }
}
```

With the XML libraries – supplied for most of the programming languages – the data access becomes easy. To read the calibration data from the example above a few lines of code are only needed.

The data read and data write functionality is stable and complete and the data interchange becomes easy.

Data Storage

id	ci	ci_serial_number	ci_date	caldata
1	MFS-06	12	2007-08-08	<?xml version="1.0" encoding="utf-8"?> <?xml-style...
8	MFS-06	122	2007-09-10	<?xml version="1.0" encoding="utf-8"?> <?xml-styl...

Storing XML data on the file system leads to the problem of the remote access and, to a greater extend, of the indexing of these data files.

XML data can however be stored in SQL databases; a subset of information is used as an index (here the calibrated item, serial number and calibration date). Instead of scanning the XML data the SQL database selects the data by your pre-defined search criteria much faster. Hence that the column headers must have the same name as the keys inside the XML file.

Visualization



Together with a corresponding CSS (cascading style sheet) XML data can be visualized with standard browsers.

No proprietary software needs to be installed in this case.

In addition the CSS can be easily translated into other languages without changing the XML keys.

Conclusion

Even though XML standards are still not available in geophysics, more and more groups are in the developing phase.

Well designed XML data files with a few lines of data description can be easily accessed and the interchange is safe.

The complete specification of the Metronix calibration files can be found at www.metronix.de in the geophysics section.

Recent and on-going MT studies of the San Andreas Fault zone in Central California

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Introduction

Numerous models have been proposed for the lithospheric structure of the San Andreas Fault (SAF) but it remains controversial if and how large transform faults penetrate the mantle lithosphere. Fluids, however, seem to be ultimately linked with many fault related processes. Fluids can be released in mineral reactions, by crushing and overprinting of existing fabric in high strain zones. Veins and fractures of a fault system can also provide the means for transport of fluids. Directly accessing the role of fluids within the core of the SAF is one of the major goals of the San Andreas Fault Observatory at Depth (SAFOD), a major earth science initiative to study the in-situ physical and chemical conditions of a large transform fault (Hickman et al., 2004).

Among the most intriguing and mysterious phenomena of fault zone related activity along the San Andreas Fault are observations of deep, non-volcanic seismic tremors (NVT) (Nadeau & Dolenc, 2005). These deep (> 30 km) tremors have only been observed in an area approximately 40 km SE of the SAFOD. The source mechanism of NVTs is to date not well understood. Recently, Shelly et al. (2007) suggested that tremors beneath Shikoku, Japan, are a different manifestation of so-called low-frequency-earthquakes (LFEs). They interpret tremors as a swarm of LFEs shaking the earth for hours, days or even weeks at a time. LFEs were previously inferred to represent fluid-enabled shear-slip on the fault plane (Shelly et al., 2006; Ide et al., 2007) instead of direct flow-induced oscillation as suggested by Katsumata and Kamaya (2003).

The presence or absence of NVT along the SAF appears to coincide with the transition from being locked (SW of Cholame) to intermediate creep (NW of SAFOD) and could reflect significant structural changes affecting the deep hydraulic system along this portion of the SAF which in turn could be detectable with MT. The *DeepRoot* magnetotelluric (MT) experiment near the SAFOD revealed a steeply dipping upper crustal high electrical conductivity zone which could represent a deep-rooted channel for crustal and/or mantle fluid ascent (Becken et al., 2008). In the scope of the projects *ELSAF* and *TremorMT*, we continue our efforts to image an entire segment of an active plate boundary with a network of MT stations, from the Pacific into the Great Valley, crossing the NVT region beneath the SAF near Cholame. Our present research activities onshore will be extended offshore with the *Deep San Andreas Fault Boundary Structure from Marine MT* experiment conducted by Scripps Institution of Oceanography, UCSD, in 2008.

This paper summarizes the most important results of the DeepRoot project give details about the objectives of the TremorMT measurements (completed in fall 2007) and the upcoming ELSAF experiments (planned for spring 2008).

Results from DeepRoot

MT data along a 45 km long profile across the SAF near the SAFOD (Fig. 1) was used to derive a crustal electrical resistivity model. Data analysis revealed that the majority of the data are consistent with 2D modelling assumptions. The dominant geoelectrical strike direction is N42.5°E in agreement with the strike of the SAF and other major geological units in the region.

A 2D resistivity cross-section was obtained from minimum structure inversion. The final resistivity model (Fig. 2) reveals an image of a heterogeneous upper crust with a number of conductive and resistive anomalies that are related to the sedimentary sequences, the Franciscan subduction complex and blocks of Salinian crust forming the basement to the NE and SW of the SAF, respectively. The most intriguing feature of the regional electrical resistivity model is an approximately 5-8 km wide, sub-vertical corridor of high electrical conductivity (EC) in the upper crust that widens in the lower crust (LCZ). The zone of high conductivity reaches the near-surface 5-10 km NE of the SAF and is sub-horizontally connected with the FZC at the SAF. Low resistivities of less than 5 Ohm-m are imaged within the EC to a depth of 8-10 km adjacent to the SAF. From there, the anomaly appears to link with a lower crustal high conductivity zone (LCZ) at non-seismogenic depths >12-15 km. Constrained inversions show that neither the upper branch of the high-conductivity zone (EC) nor its lower crustal root (LCZ) can be removed without significantly increasing data misfit.

A joint interpretation of the resistivity structure and the geochemical data from the SAFOD and nearby water wells suggests that the crust near the SAF provides pathways for crustal and upper mantle fluids, while the eastern fault block represents a trap of fluids. This interpretation is supported by (i) increasing He³/He⁴ ratios within the SAFOD (up to 11% mantle derived) to the NE of the SAF (Wiersberg, 2007) (ii) high He³/He⁴ ratios farther to the NE (up to 25% mantle), observed within the Varian-Philipps (VP) and Middle Mountain oil wells (Kennedy et al., 2007) (iii) an electrically conductive channel linking the upper crustal eastern conductor (EC) with a lower crustal conductive zone and a conductive anomaly within the upper mantle. Furthermore, super-hydrostatic fluid pressures within the SAFOD to the NE of the SAF (Zoback et al., 2006) and within the VP well (Johnson et al., 95) suggest that the EC represents a region of overpressured fluids trapped between an impermeable SAF, the WCF and below a surface seal. Elevated fluid pressures in this region could be related to continuous or episodic fluid supply of deep rooted fluids.

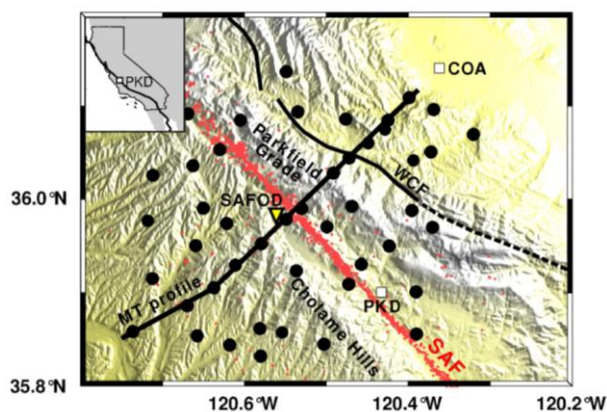


Fig. 1. Site map of DeepRoot experiment. Emphasis for this paper is on the MT data recorded at 66 sites along a 45 km long profile (solid black line), centred on the SAFOD site (yellow triangle). This profile includes 11 sites from a previous MT survey (Unsworth et al., 2000) in its central part. Solid black circles indicate site locations of additional long-period/broad-band recordings in array configuration. Red dots indicate the seismicity along the SAF (Thurber et al., 2006). PKD Parkfield; COA Coalinga; WCF Waltham Canyon Fault; inset shows a map of California, the SAF and the location of Parkfield.

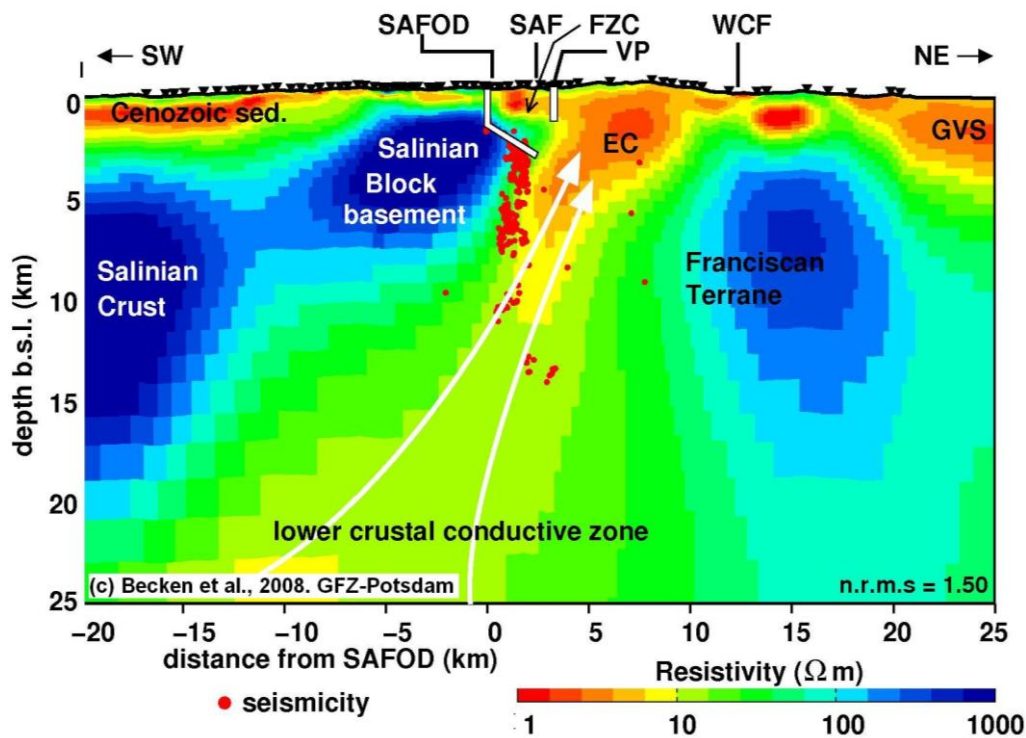


Fig. 2. MT resistivity model along the 45 km profile across the SAF near the SAFOD site. Superimposed on the models are the SAFOD main hole and the Varian-Philippis well (VP); red dots indicate the seismicity (Thurber et al., 2006) within 3 km distance from the profile. We speculate that a deep-reaching sub-vertical corridor of high conductivity, linking the upper crustal eastern conductor (EC) with a lower crustal conductivity zone (LCZ) and further with a mantle conductor, images a channel for deep crustal or mantle fluids (sketched with white arrows). This channel is enclosed by the resistive Salinian Block basement and Salinian Crust to the SW and a resistive block in the NE at mid-crustal levels (labelled Franciscan Terrane). At the SW end of the profile, above the resistive Salinian Crust, the model indicates a 2 km thick conductive layer near-surface, that coincides with low seismic p-wave velocities (1.5-3.5 km/s) (Hole et al., 2006) and can be attributed to presumably Pliocene and Miocene marine sediments (Diblee, 1972). The model recovers also the shallow fault-zone conductor (FZC) below Middle Mountain (e.g. Unsworth et al., 2000). The sediments of the Great Valley Sequence (GVS) could be responsible for the high conductivity imaged in the upper crust of the NE part of the profile.

Within the upper crust, these fluids do not migrate through the seismically-defined SAF. Instead, upper crustal migration pathways are provided within the high-conductivity EC sandwiched between the SAF and a SW-dipping segment of the WCF, interpreted to represent a thrust-fault penetrating at least to the bottom of the seismogenic zone (Carena, 2006). Seismically active as well as blind thrust-faults in this region could also provide the means for fracture-related fluid-flow, in agreement with the modelling assumptions of Miller (1996).

High lower crustal conductivities within a 25 km wide zone around the SAF indicate anomalously high fluid content. Constrained inversion shows that a lower crustal barrier for fluid flow is incompatible with the MT data. The resistivity model strongly supports deep fluid circulation and migration through the lower crust. The exact geometry of these deep pathways cannot be resolved. However, inversion models containing a narrow, steeply dipping conductive zone as an a priori conductive structure achieve the best data fit, and seem to support the existence of a narrow pathway.

We speculate that the source region for the fluids is related to a mantle conductivity anomaly below the Pacific plate (beyond the axes limits of Fig. 2). This mantle anomaly could be related to fluids released from a subducted slab of oceanic crust, which may exist in this

region (Zoback et al., 2002). However, a longer MT profile extending to the coast and continued off-shore is required to constrain this feature. Further geochemical sampling of water wells along the MT profile would be necessary to test if elevated mantle gas content exists between the SAF and the WCF as predicted by the MT model. Farther to the NE, mantle derived fluids should be minor constituents to the fluid composition, similar to the observations to the SW of the SAF.

TremorMT and ELSAF

Analysis of triggered event data from the borehole High Resolution Seismic Network (HRSN) at Parkfield, California, revealed tremor-like signals originating to the south within the Cholame Valley, approximately 40 km SE of Parkfield. Their locations indicate that, within the search radius, the tremors are confined to a ~25-km segment of the SAF and occur at depths of between approximately 20 and 40 km. Nadeau & Dolenc (2005) suggested that either fluids are not important for the SAF tremors or an alternative fluid source (when compared with subduction zones) exists below the seismogenic zone in this area. Ellsworth et al. (2005) confirmed the observation of non-volcanic tremors in May 2005 during the deployment of a multi-level borehole seismic array in the SAFOD main hole. An apparent correlation between tremor and local micro-earthquake rates at Cholame (Nadeau and Dolenc, 2005) suggests that deep deformation associated with the Cholame tremors may be stressing the shallower seismogenic zone in this area. Further evidence for stress-coupling between the deep tremor zone and the seismogenic SAF is observed in the correlation between tremor and the 2004, M6 Parkfield earthquake, approximately 10 km NW of Cholame.

Near Cholame, earlier MT work found evidence for a resistive crust beneath the SAF (Park & Biasi, 1991) which could be indicative of a dry zone capable of trapping fluids in the lower crust and/or the upper mantle. This hypothesis would be consistent with low mantle derived He content in the Jack-Ranch Highway-46 Well (Kennedy et al., 1997) near Cholame and in support of a locally well-confined source region for the non-volcanic tremors (Nadeau & Dolenc, 2005). It would mean, however, that the geological and / or rheological situation near Cholame is markedly different from Parkfield, where the resistivity model and the fluid chemistry (Kennedy et al., 1997; Wiersberg & Erzinger, 2007) suggest a pathway for fluids into the brittle regime of the SAF system.

All of the above observations suggest that fluids play the key role in the generation of non-volcanic tremors, and that tremors (and associated fluids) appear to be closely linked to fundamental processes governing both the deep roots and the seismogenic zone of large fault zones. The presence or absence of NVT appears to coincide with the transition of the SAF from being locked (Cholame) to intermediate creep (SAFOD) and could reflect significant structural changes affecting the deep hydraulic system along this portion of the SAF which in turn could be detectable with MT. Tests based on constrained inversions of the DeepRoot MT data across the SAFOD clearly show that a resistive lower crust is inconsistent with the data (Becken et al., 2008). This also means however, that we could resolve a resistive lower crust if it should exist beneath the Cholame segment of the SAF. Furthermore, if migration of fluids from the lower into the upper crust is blocked by an impermeable seal, the upper crust should be more resistive. In fact, the eastern conductor (EC) which we interpret as the upper crustal branch of the fluid channel near the SAFOD appears to be absent in preliminary inversion models of the southernmost short profile of Unsworth et al. (unpublished), located

just 5 km NW of Cholame.

To address these questions we have been continuing our research activities with the TremorMT (GFZ-funded) and ELSAF (DFG+GFZ funding) projects to image an entire segment of the SAF with a network of MT stations, deployed from the Pacific Ocean into the Great Valley, crossing the SAFOD near Parkfield and the NVT source region beneath the SAF near Cholame. In autumn 2007, we measured MT data along a 130 km long profile across the

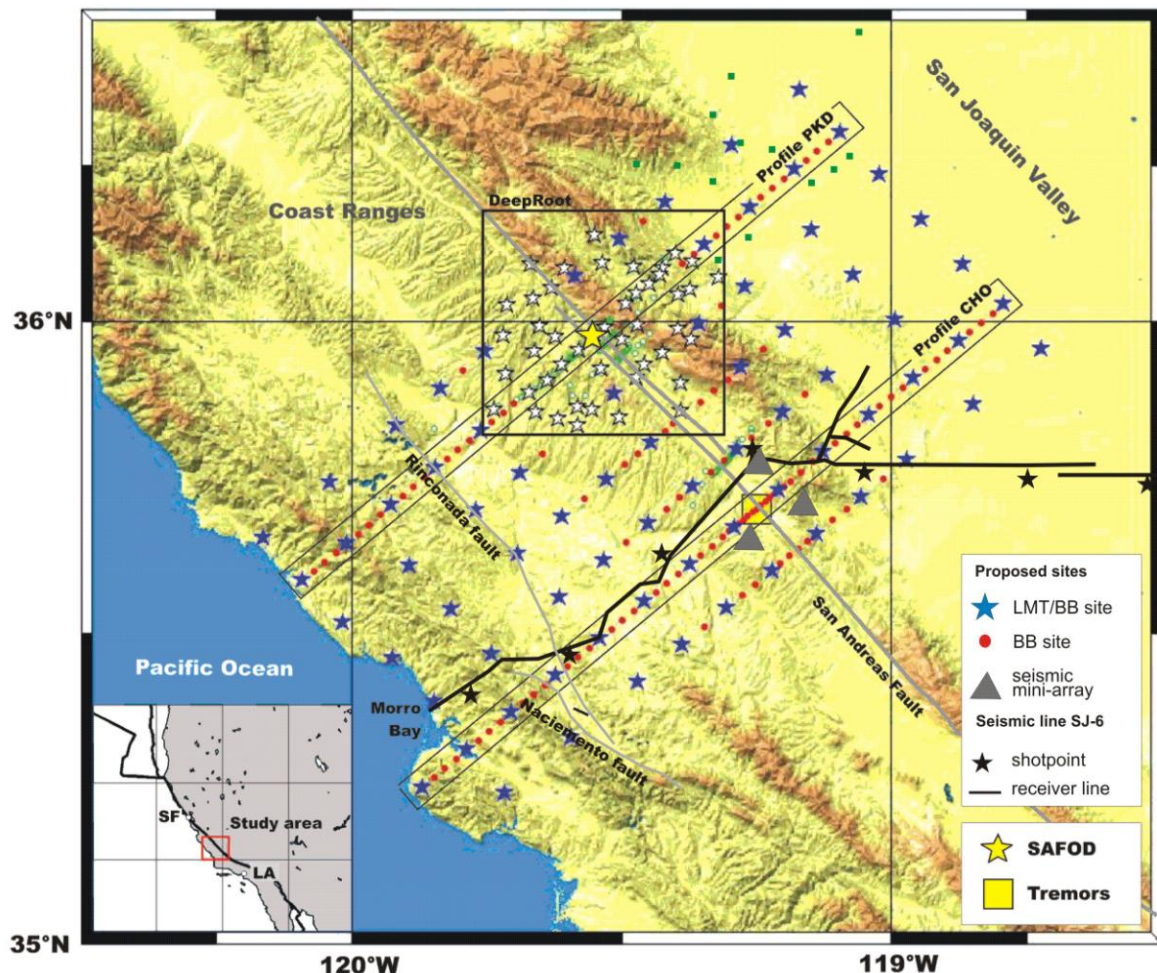


Fig. 3: Proposed and existing MT sites in the Cholame-Parkfield area in Central California. Blue asterisks and red dots indicate the proposed new combined long-period(LMT)/broad-band(BB) and BB-only sites, respectively, white asterisks and green dots indicate existing MT sites, acquired by the GFZ Potsdam and the UC Riverside in 2005/6 and by Unsworth et al. (1997). Additional MT data recently gathered in the NE part by S. Park (collaborator in DeepRoot) are shown as green squares. The SAFOD site near Parkfield is marked with a yellow star and the region of the non-volcanic tremors near Cholame is indicated by a yellow rectangle. Phase I of the project (TremorMT, GFZ funded) was successfully completed in fall 2007 with data acquisition along the 130 km long profile (CHO) and extending the existing 50 km long MT/seismic profile of the DeepRoot project from the Pacific coast into the San Joaquin Valley (profile PKD). In phase II of the project (ICDP, ELSAF) profiles CHO and PKD will be connected spatially with an array of LMT/BB and BB magnetotelluric sites, as indicated with blue asterisks and red dots. Gray triangles indicate the locations of the seismic mini-arrays (phase I); the solid black line and black asterisks indicate the location of the existing seismic refraction/reflection line SJ-6 (Murphy & Walter, 1984).

Coast Ranges and centred above the source region of non-volcanic tremors near Cholame (project TremorMT). We extended the existing DeepRoot profile to a length of 130 km to better constrain lower crustal and upper mantle conductivity structure. Furthermore, four

small-aperture seismic arrays (SSAA) were deployed by Ryberg & Haberland (GFZ) in cooperation with the USGS in the vicinity of Cholame to test if the location accuracy of the NVT-events (in particular the depth estimate) could be improved. Preliminary results of the SSAA work, which was carried out in cooperation with W. Ellsworth from the USGS, are very promising as we observed numerous tremor-type signals in a 6 week period which are currently being analyzed. With ELSAF we will continue to collect MT data in spring 2008 with an array of MT sites connecting the high-resolution profiles across the SAFOD and the Cholame Valley. With the 3D array of MT sites we can resolve along-strike variations between the Cholame and Parkfield segments of the SAF (see Fig. 3 for existing and planned MT sites).

Furthermore, our research activities onshore will be extended offshore in a collaborative research effort with our colleagues from Scripps Institution of Oceanography, UCSD. Brent Wheelock, Kerry Key, and Steven Constable will be extending our land profiles with their Scripps funded 'Deep San Andreas Fault Boundary Structure from Marine MT' experiment. The offshore data will be collected in autumn/winter 2008. The combination of onshore and offshore data will help us to see the whole picture as modelling shows that important parts of the San Andreas Fault structure, e.g. a deep rooted source of fluids in the upper mantle, can only be fully imaged by extending the MT array offshore.

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Inversion of the geomagnetic induction data from EMTESZ experiments in NW Poland by stochastic MCMC and linearized thin sheet inversion

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Abstract

72 long period induction arrows obtained during EMTESZ experiments in north Poland were inverted by global optimization Monte Carlo Markov chain stochastic method and unimodal thin sheet inversion. From the stochastic inversion with the thin sheet at the surface of the Earth we obtained histograms for unknown conductances and the most probable model was constructed. The results of MCMC are compared with the results of linearized unimodal thin sheet inversion where the thin sheet is placed in 3 km depth. The resulting models are compared with surface distribution of the conductance of the sedimentary cover.

1. Introduction

The main task of the international project EMTESZ is to achieve on the basis of electromagnetic methods an essential progress in understanding the structure, tectonic position and role of the Trans-European Suture Zone (TESZ) as a fundamental lithospheric boundary separating south-western and north-eastern Europe, which is supposed to have played a key role in the evolution of the Paleozoic orogens across the whole of Europe (*Pharaoh et al., 1996*). TESZ is crossing NW-SE through the continent and exceeding 2000 km in length. It is a broad and deeply seated structurally complex zone of Palaeozoic deformation where younger terranes were sutured to the Precambrian Baltic Shield and East European Craton (EEC) during the formation of Pangea. At the surface the TESZ is largely masked by sedimentary basins of the Polish Trough, while at depth it marks the increase in crustal thickness from about 30 km under the western Europe to about 45 km beneath the EEC. Various hypotheses for the tectonic evolution of the TESZ are debated today, ranging from mobile to more static scenarios (*Pharaoh et al., 1996*). Understanding the contrasting signatures of the European deep lithosphere requires a detailed analysis of its tectonic history and correlation of its physical parameters through various geophysical experiments.

The EMTESZ experiment has been organised as an international collaboration project that aims at constructing an electrical image of the entire lithosphere along a profile running from the Polish Basin to the East European Craton, crossing the TESZ as a first-order tectonic boundary. Within this study, several research teams from Poland, Czech, Germany, Finland, Russia, Sweden and Ukraine participate in the field measurements, data processing, analysis and interpretation.

The international teams measured in NW Poland at first along two seismic profiles P2 and LT7 and later some small profiles and many individual points were added. At present more than 100 MT and AMT points form a geoelectrical array suitable for various interpretations.

Electric and magnetic field was measured at each station in a broad period range and geoelectrical characteristics were obtained. Various geoelectrical characteristics were

interpreted by different teams. We concentrated on the interpretation of the geomagnetic transfer functions which express a relation between vertical and horizontal magnetic fields and are represented by induction arrows or vectors.

2. Inversion MCMC method

We used global optimization Monte Carlo Markov chain method (MCMC) to solve the Bayesian magnetovariational inverse problem in 2D domain representing superficial thin sheet (*Grandis, H. et al. 2002*). Forward problem solution is based on Weidelt's algorithm (*Vasseur, G. and Weidelt, P., 1977*).

The thin sheet at the surface is divided into homogeneous square cells, and the model parameters are the conductivity of each cell. In our experiments we used a mesh with 40x22 cells; it means we had 40x22 unknown conductances. There are some practical problems which we meet, if we use this approximation. At first we must give a guess of "normal structure" around calculated area. We must guess the normal 1D structure and the normal surrounding conductance. From this reason we tested several values of the normal conductance from 600 S to 1800 S which correspond to estimates obtained by *Varentsov et al., 2004*. Because we use for inversion experimental data obtained for long period 1024 s, our "thin" sheet should in fact contain information from first few km. The original experimental data must be also transformed to the rectangular mesh suitable for thin sheet calculation. There is problem with relatively large areas without any experimental data. From such locality we obtain only very vague information.

In the inversion procedure we are using Gibbs sampler. Starting from the latest state of Markov chain with parameter $\mathbf{d}$, the Gibbs sampler loops through all the components of the vector $\mathbf{d}$ and updated each individual component of $\mathbf{d}$. After all components of the parameter vector have been updated in this way, one iteration step of the Gibbs sampler is completed (*Menke, W., 1989*). The inversion procedure gives a series of models which should be probabilistically distributed in accord with the experimental data. We used 11 possible values of conductance from 400 S to 12000 S. In MCMC inversion we obtain a whole *histogram for each cell*, which shows how much likely the particular conductance values are at the cell considered. We can estimate the uncertainty of the conductance by observing the flatness or peakiness of the histogram. As a result of our inversion, we obtain aposteriori probabilities for the discrete conductances within individual sheet cells.

At first we concentrated on induction data from the vicinity of two measured seismic profiles P2 and LT7 located in NW Poland. Average conductance model obtained by MCMC inversion for period 1024 s is presented in Fig. 1. The mean conductance is computed from the conductance histogram at each cell.

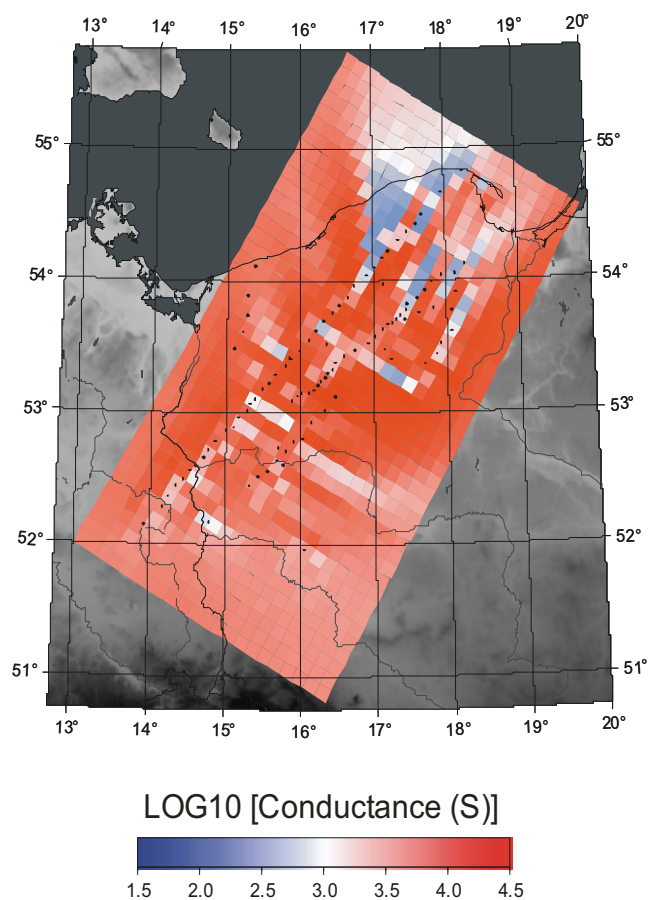


Fig. 1: Average conductance model from MCMC. T=1024 s

Comparison of the real parts of the induction arrows $\mathbf{W}$ for this model and the experiment again for period 1024 s is in Fig. 2. There is very a good agreement between a model and observed induction vectors. Also the agreement for the imaginary part of the induction vector is good.

3. Inversion with new data from NW Poland and with new geoelectrical characteristics

We also tried to interpret the data from broader area. We interpreted old experimental data with several new stations and we inverted two long periods 1024 s and 2048 s simultaneously. The computer program was adopted also for simultaneous interpretation of induction arrows and horizontal magnetic transfer functions with respect to a reference station (*Brasse et al., 2006*)

$$H_Z = W_X H_X + W_Y H_Y$$

$$H_X = M_{XX} H_X^R + M_{XY} H_Y^R$$

$$H_Y = M_{YX} H_X^R + M_{YY} H_Y^R$$

and we tried to find the solution with minimum distance from predefined reference model with homogeneous conductance of 1500 S. Inter-station transfer functions were related to the site with coordinates (54.0488° N, 18.1183° E) in the north-east part of the studied area. The horizontal inter-station magnetic tensor has been suggested as a parameter with relatively weak sensitivity to the spatio-temporal variations in the excitation field, as it relates magnetic field components over a distance that is short in comparison with a dimension of the source field. Thus it can help in eliminating the external non-stationary and non-uniform source effects on the recorded data.

In this case we restricted to the mesh of 22x22 cells and we used 18 values of conductance for the Gibbs sampler from 200 S to 10000 S. In Fig. 3 conductance model is presented

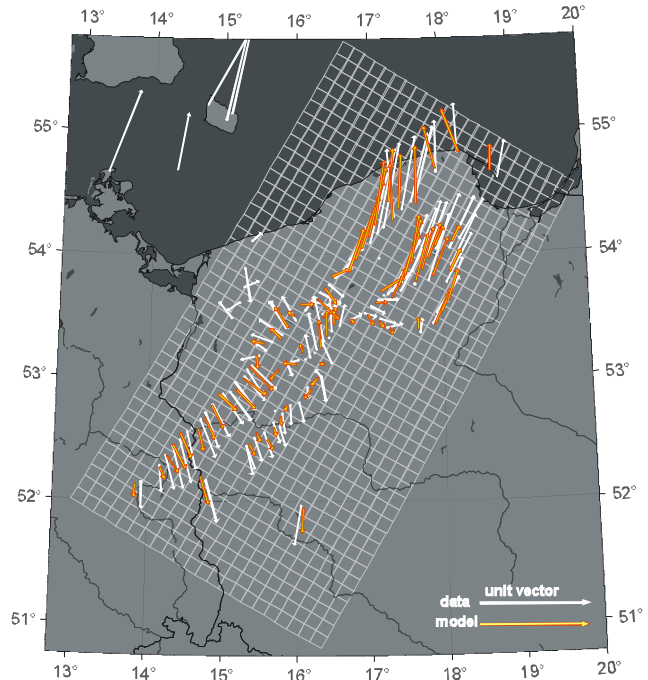


Fig. 2: Comparison of model (orange) and observed (white) $\text{Re}\mathbf{W}$ for $T=1024$ s.

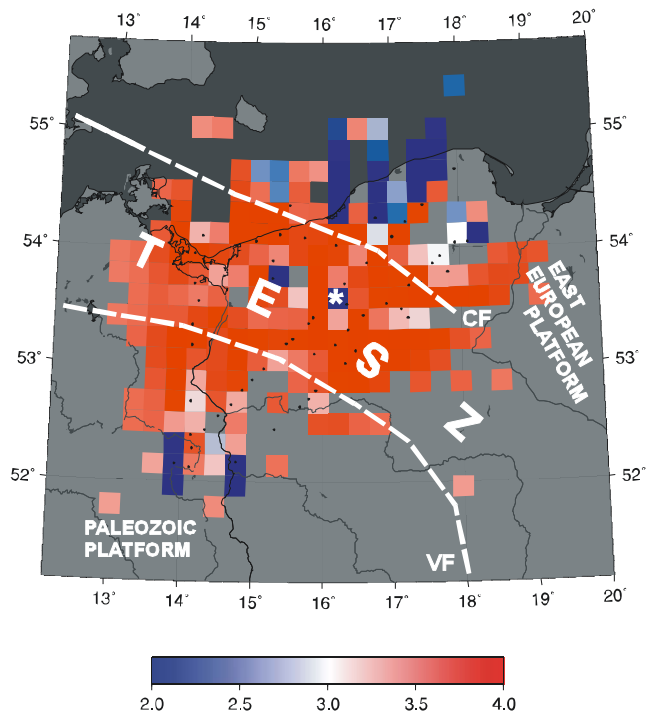


Fig.3: MCMC solution for $\mathbf{W}$ with cells where 90% of conductance fall within 1/2 of decade. 1024 and 2048 s periods considered. VF - Variscan Front; CF - Caledonian Front; * - Czaplinek block

obtained from the MCMC inversion only for induction vectors W only. The cells for which 90 percent of the conductances from stochastic samplings fall within one half of a decade are there displayed.

In Fig. 4 conductance model is presented obtained for the simultaneous inversion of the induction arrows W and horizontal magnetic transfer functions M . The cells for which 90 percent of the conductances from stochastic samplings fall within *one third of a decade* are there displayed. By means of adding horizontal magnetic transfer functions to the inversion we obtain higher resolution of the results.

4. Linearised inversion

Results of the MCMC are compared with the results of the linearized thin sheet inversion (Fig. 5). For the linearized inversion the unimodal thin sheet approach was applied, where the effect of only the horizontal current components within the thin sheet was considered and the anomaly source is replaced by an equivalent current system in this sheet.

Forward problem solution is based on the approximation of the Earth's anomalous structures by a thin horizontally inhomogeneous sheet buried at a specified depth in a generally layered Earth (Wang, 1988) and the integrated conductivity (conductance) within a conducting thin sheet is calculated from the recorded geomagnetic transfer functions at the Earth's surface. Both single-station vertical transfer functions (components of the induction vectors) and the horizontal inter-station transfer functions were employed in the inversion procedure.

Regularization was solved using the minimum gradient support focusing (Portniaguine and Zhdanov,

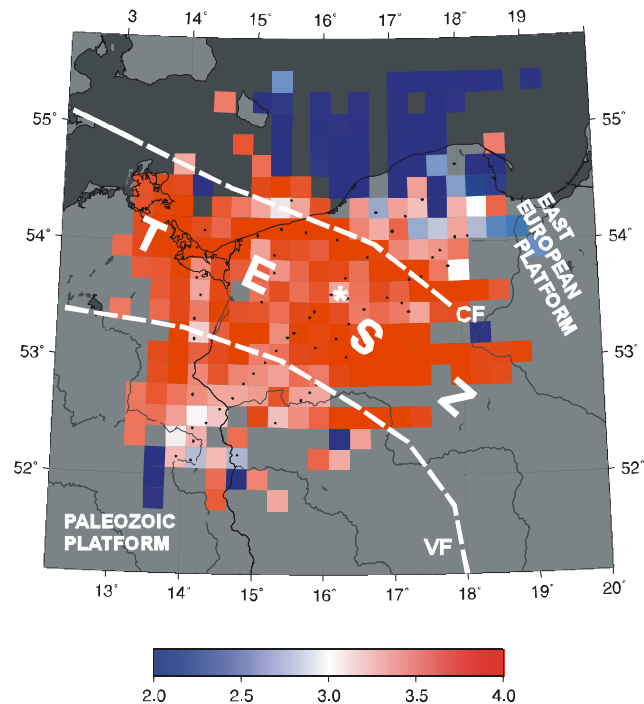


Fig. 4: MCMC solution for $W+M$ with cells where 90% of conductance fall within 1/3 of decade. 1024 and 2048 s periods considered. VF - Variscan Front; CF - Caledonian Front; * - Czaplinek block

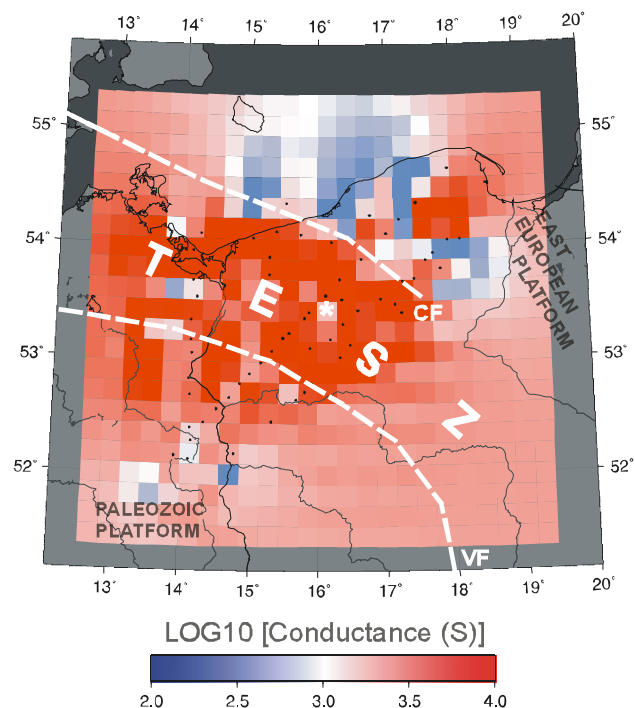


Fig. 5. Linearized solution . Conductance model for $W+M$, T=1024 s; VF - Variscan Front; CF - Caledonian Front; * - Czaplinek block

1999), which makes it possible to approximate specific features of the geomagnetic induction anomalies generated by sharp tectonic boundaries and blocky geological structures.

The linearised inversion was solved by minimizing the Tikhonov parametric functional with the weighted norm of difference between the observed data and the model data as functions of the model conductance (Kováčiková *et al.*, 2005). The minimization of the parametric functional was solved by an iterative procedure using the re-weighted conjugate gradient method (Portniaguine and Zhdanov, 1999).

Inversion was carried out for the periods 1024 s and 2048 s. The thin sheet is buried at the depth of 3 km in a medium with the resistivity of 100 ohmm and a halfspace at a depth of 100 km with resistivity 50 ohmm. Normal conductance at thin sheet margins is 2000 S. Model of conductance consists of 31x31 cells with cell dimension 20 km. The thin sheet is located at the depth of 3 km, although the situation within the region is much more complicated. Nevertheless, this is not the real depth of the anomaly source, this depth just mark an upper boundary of the anomaly (Banks, 1986).

5. Conclusion

The results of both inversion procedures, the MCMC and the linearised inversion, show similar features. Conductance within the anomalous zone reaches 10 000 S. Contrary to complicated "mosaic" model generated from only the vertical induction vectors (Fig. 1) application of the horizontal transfer functions leads to the more expressive image. The non-conducting area of the East European Craton at the north-east as well as more conductive Paleozoic terranes at the south-west part of the models divided by a well-seen conductive belt corresponding to the TESZ can be found in the model conductance distribution (Figs. 4 and 5). The non-conducting cell in the center of the model corresponds to the resistive Czaplinek area. The block with high conductance in the north-east part of the model in Fig. 5 corresponds to the reference station and does not mark any real feature.

The anomalous zone of TESZ is the prominent tectonic feature of this area, indicated by various geophysical methods - seismic, gravity and geoelectromagnetic. There are various theories concerning the origin of the anomalous conductivity within this zone. The high conductance values in Pomerania are connected with extremely thick Paleozoic to Cenozoic sediments with thickness reaching more than 10 km. The area is also characterized by occurrence of salt pillows and diapirs (Brasse *et al.* 2006, Resak *et al.* 2007). Nevertheless this work itself can give an image about distribution of the conductivity within the studied area and about the shape of the anomalous zone but it can neither determine the depth of the anomaly source, nor explain the origin of the anomaly. This problem requires more detailed 2D and 3D modeling and cooperation of various geophysical methods.

Acknowledgements

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ADVANCED METHODS FOR JOINT MT/MV PROFILE STUDIES OF ACTIVE OROGENS: THE EXPERIENCE FROM THE CENTRAL TIEN SHAN

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1. Introduction

Areas of active orogenesis are places where the multi-disciplinary geological-geophysical studies can substantially extend understanding of the fundamental geodynamic processes. The geoelectromagnetic branch of these investigations may and has to be an important source of information on the crustal tectonics, petrophysics, thermal and fluid regimes of the interiors. However the complex structure of orogens introduces rigorous requirements to the methods of study, including reasonably detailed observation grids and precise processing, effective analysis of the data, defining the dimensionality of the interpretational model, and finally, high resolution and stable inversion tools.

The geoelectric investigations of Tien Shan region held by the “Naryn” Working Group (the authors of the paper) during last years were concentrated on the development of rational joint system of magnetotelluric (MT) and geomagnetic (MV) soundings, which could satisfy the above mentioned demands and reliably reconstruct the heterogeneous conductivity structure of the orogen, still being formed since Cenozoic activation.

The main efforts were done to make our methods more robust to the principle difficulties of the geoelectric investigations in the mountains, which include irregular observation grids, the influence of the topography, strong near surface heterogeneity of conductivity distribution; multi-level anomalous geoelectric structure of crust and upper mantle of orogen. The prominent principles of the elaborated complex of MT/MV sounding in such areas are the following:

- synchronous observations;
- integration of MT/MV data sets with “synchronization” of data of different field campaigns;
- multi-site processing with attention to accuracy of additional component and long periods estimates of transfer function (TF);
- modern schemes of invariant analysis robust to galvanic distortions;
- effective inversion procedure with detailed model parameterization, providing adequate reconstruction of both smooth and sharp conductivity boundaries and accounting for the topography;
- using all the variety of local and inter-station TFs for the inversion.

In a case of elongated orogens they should be extended by:

- construction of quasi-2D ensembles of data for profile inversion with accounting for 3D distortions;
- specific inversion strategy based on successive partial and multi-component profile inversions considering different sensitivity to targets and “immunity” to 3D distortions of the different data components.

The paper describes application of this approach to the analysis and interpretation of MT/MV data at the 700-km “Naryn” transect providing the geoelectric cross-section of Central Tien Shan along 76°E line.

2. Integration of the data ensembles along regional transect “Naryn”

20 years of data acquisition of RS RAS (Research Station of Russian Academy of Sciences) in Tien Shan region have brought the data set of about 800 soundings (Rybin et al., 2001, Fig.1a), with tens of broad-band and long-period MT sites (CES2, MT-PIK, MT-24 and LIMS equipment) being spread, in particular,

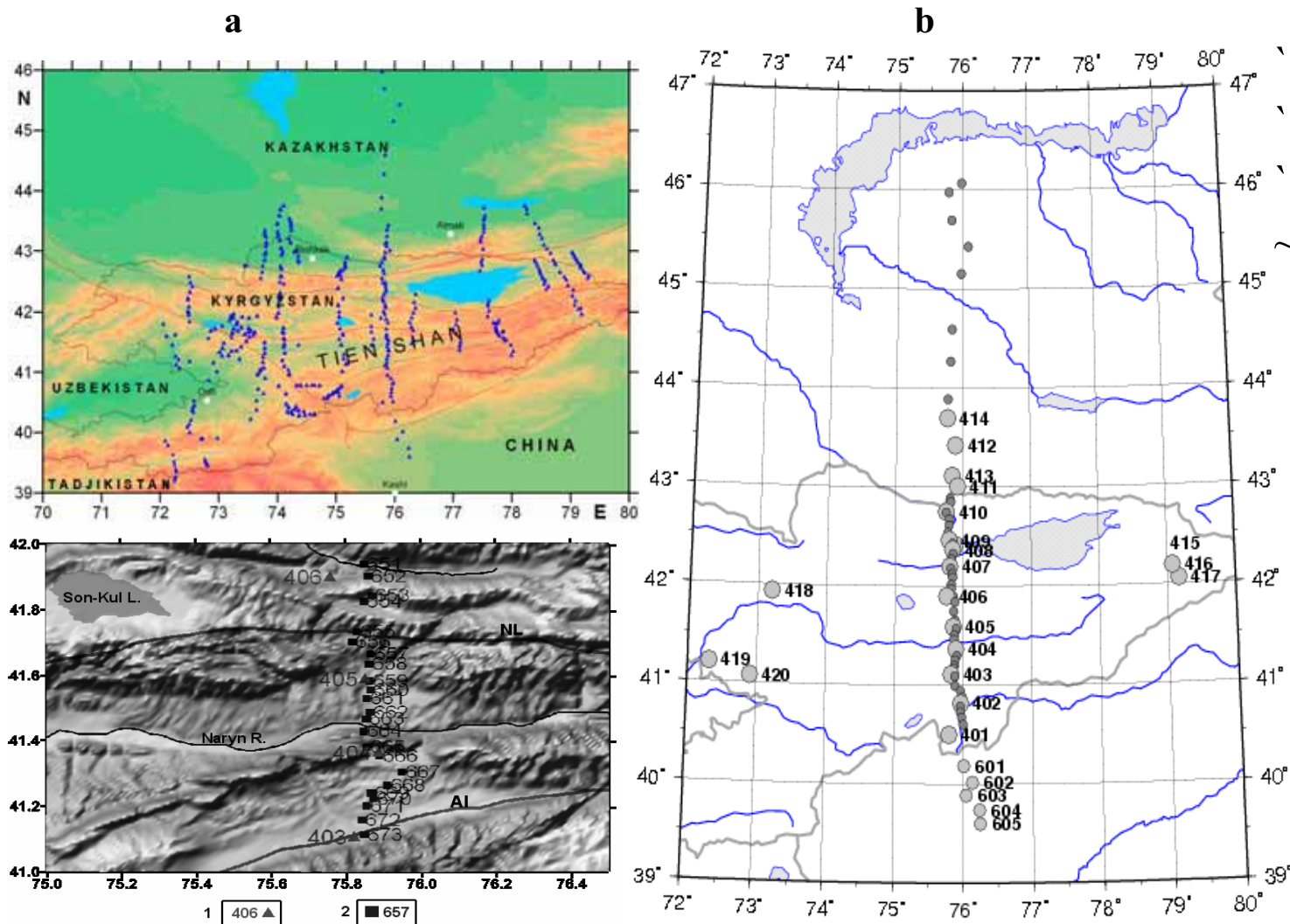


Fig. 1. The schemes of MT/MV sites locations at the territory of the Tien Shan region conducted by staff of Research Station of Russian Academy of Sciences (Bishkek, Kyrgyzstan): (a) the overall sketch of prospecting and long-period soundings with different MT instruments (CES-2; MT-PIK and LIMS) in 1986-2003; (b) MT/MV soundings along “NARYN” transect (dark grey circles – prospecting soundings with CES-2 and MT-PIK; big light grey sites with numbers – long-period soundings with LIMS); (c) the profile segment of detailed prospecting soundings with “Phoenix”-MTU-5 in 2005. Sites: 1 – LIMS; 2 – MTU-5. Major fault zones: LN - Nikolaev Line, AI – Atbashi-Inilchek.

on the regional transect “Naryn” (Fig.1b). The first simultaneous observations for 19 long-period MT/MV soundings were done on this profile with LIMS equipment in 1999 – 2001 in collaboration with geophysicists from Californian University of Riverside, presenting the results of their interpretation in (Bielinski et al., 2003). The broad-band observations of NS RAS in 2005 with “Phoenix”-MTU-5 stations working out in the details the segment of the profile around Nikolaev Line fault zone (Naryn basin) were also held synchronously (Fig.1c).

LIMS and “Phoenix” soundings were processed by “Naryn” WG with the modern tools supplying the estimates of both local and inter-station TFs. These tools (Varentsov et al., 2003; Varentsov et al, 2005) are based on the new remote reference (RR) and multi-RR schemes, which sort the partial TF estimates in the final averaging with the account for several criteria of the homogeneity of the horizontal magnetic field. This technique, mRRMC scheme– multi Remote Reference with Magnetic Control, was originated and has benefited in the frames of the EMTESZ-Pomerania project (Sokolova et al, 2005b). The progress was made also at “Naryn” profile: the estimates of the additional impedance components were improved and long-period responses stabilized for LIMS data (Sokolova et al, 2005a) as well as for broad-band “Phoenix” ones (especially for the estimates in so-called “dead” period range).

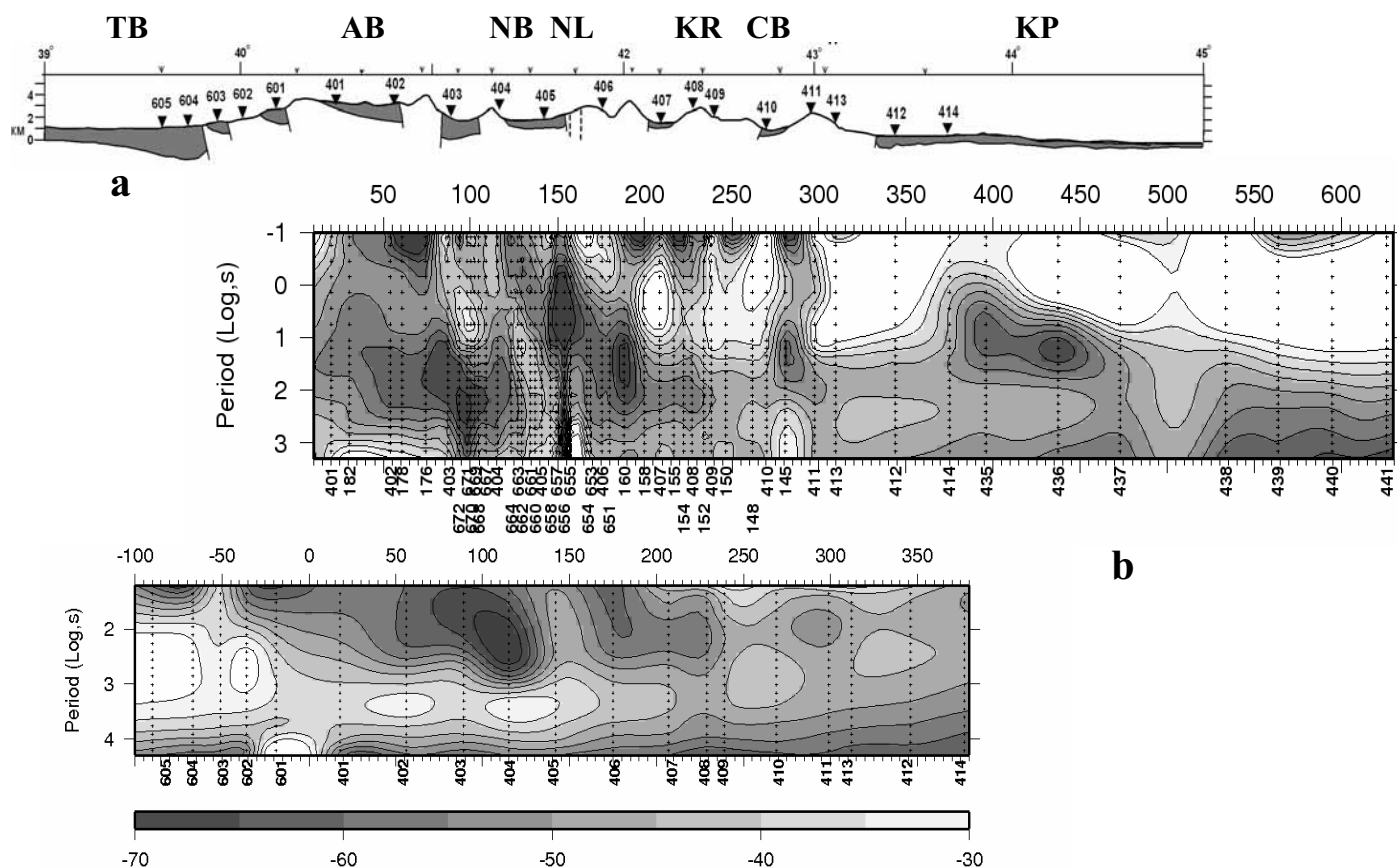


Fig. 2. Pseudosections of Z_{yx} (“longitudinal” impedance) phase along “Naryn” line and relief profile with sedimentary basins and abbreviators of the main geomorphological features of Central Tien-Shan and surroundings (TB – Tarim basin, AB – Atbashi basin, NB – Naryn basin, NL – Nikolaev Line fault zone, KR – Kyrgyz range, CB – Chu basin, KP – Kazakh plate): (a) - **MT** ensemble (CES-2, MT-PIK, MT-24+ LIMS and «Phoenix» soundings); (b) - **LMT** ensemble (LIMS soundings). Coordinates along profile are given from $x=0$ (Kyrgyz-China border) in km and phase scale palette – in degrees.

The old prospecting data on the profile were revised for the selection of conditional ones and combined with the results of new synchronous MT soundings. The multi-component data set at the heterogeneous grid was compiled from the impedances (Z), tippers (W_z) and first estimated inter-station horizontal magnetic tensors (M) in the overall period range 0.01-20000 s. The M - estimates (16-1500, 2000s) connected the horizontal field in 17 sites observed in 1999, 2001 and 2005 via single base in S. 410. As an example, the subsets of this data ensemble are presented in Fig.2 by phases of longitudinal impedances in prospecting (**MT**) and long-period (**LMT**) ranges with indication of sounding sites location, the relief profile, sedimentary basins and abbreviators of the main geomorphological features of Central Tien-Shan. Because of irregularity of observation grids in both period ranges and different coverage by the data of different parts of the profile the further analysis and inversion of the data were held separately for two mentioned subsets.

3. Invariant analysis and construction of quasi-2D data sets

For estimation of strike and dimensionality parameters new schemes of invariant analysis were applied according to *Caldwell et.al, 2004* (CBB scheme, based on phase tensor transformation), *Berdichevsky and Dmitriev, 2008* (complex dimensionality criterion, combining several robust impedance and tipper SKEWs) and *Varentsov 2007a* (tipper and M tensor SKEW and strike estimates).

Due to the application of these effective schemes, robust to galvanic distortions, and high precision estimates of all the components of TFs (including additional ones) reliable strike and dimensionality parameters were obtained. Fig. 3 compares Swift's and angle CBB Skew pseudo section for LMT data ensemble and shows, first, robustness of CBB estimates to static galvanic distortion and, second, several areas with increased CBB values depicting the data 3D-distorted in different degree by near surface or

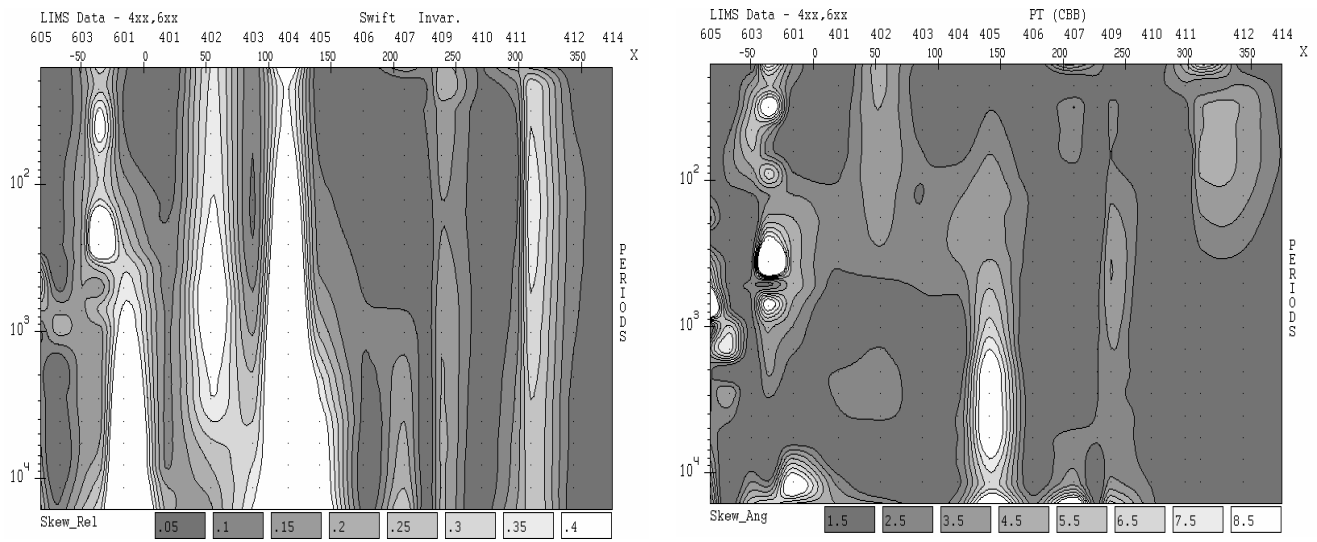


Fig. 3. Pseudosections of Skew estimates for **LMT** data ensemble: amplitude impedance Skew according to *Swift, 1983* (left) and phase tensor angle Skew according to *Caldwell et al., 2004* (right).

crustal inhomogeneties. The most prominent one, marked at rather long periods by Skew values above threshold for 2D structures ($\sim 6^\circ$), is located around S.405 on the border of Naryn basin near the crossing of the Nikolaev Line fault zone (NL, the main structural margin of Tien Shan, separating its Northern, *Caledonian*, and Middle, *Hercinian*, parts, see Fig.2). The described behavior of CBB SKEW is generally corresponds to the spatial-period distribution of **LMT** magnetovariational SKEWs.

In **MT** ensemble the northern part (Kazakh plate) of the profile with mostly small (1-2D) values of impedance SKEWs differs from mountainous one, where local areas of 3D distorted data are often depicted. Here large SKEWs are characteristic for tipper data at periods less then 10s, while for longer ones mosaic picture of small and increased values are common.

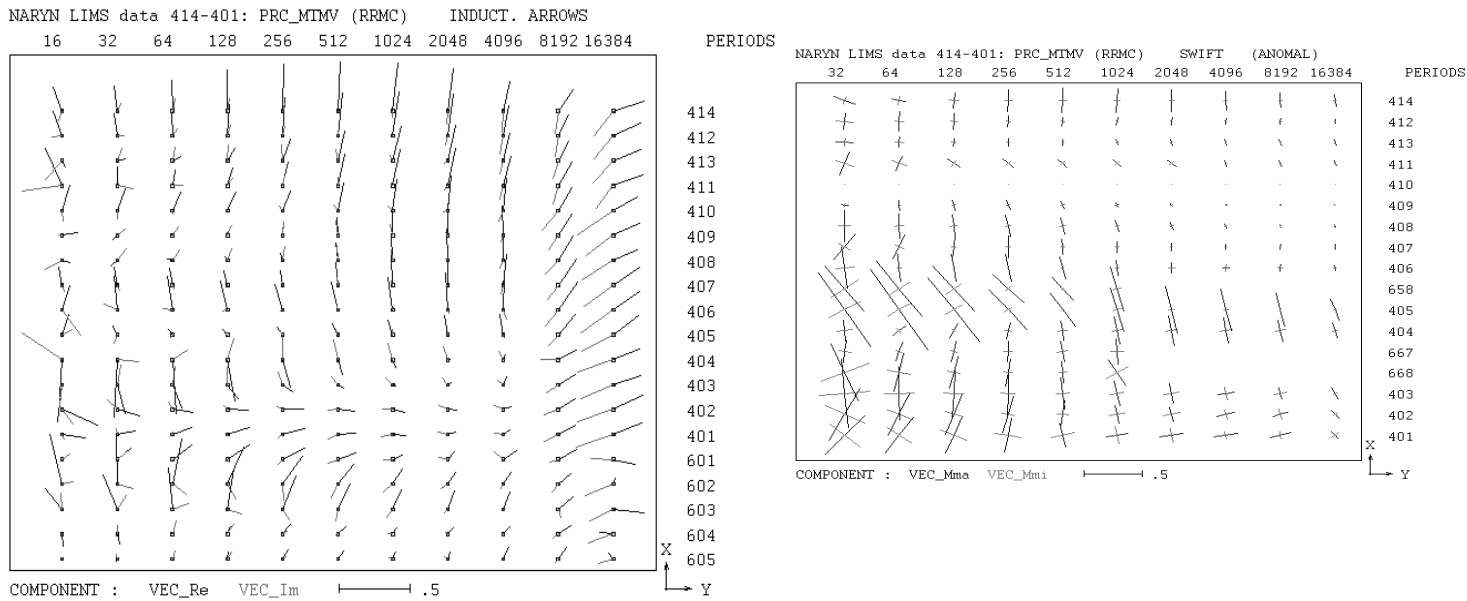


Fig. 4. The induction vectors (in Wiese convention, left panel) and diagrams of main directions of anomalous horizontal magnetic tensors (according to *Varentsov, 2007a*) (right) for LMT ensemble of data at “Naryn” profile. The horizontal axes on the panels give the periods, the vertical ones – the sounding sites with geographical North (X) at the top.

The strike of geoelectric structures was examined with a help of induction vector period-spatial behavior and the analysis of main axes of extreme diagram (“ellipses”) of phase tensors and anomalous horizontal magnetic tensors (according to *Varentsov, 2007a*) (Figs. 4, 5). For the periods more then 256s the sublatitude strike corresponding to the regional tectonics was revealed (with exception of S.405 near NL zone), that is also shown by sector histograms of extremal axes for phase tensors “ellipses” in the range 256-10000s (LIMS data). For shorter periods substantial influence of sedimentary valleys configuration is

reflected in the deviation of these axes and induction vectors from regional strike (compare Fig.5a and 5b as well as corresponding histograms, estimated in 5b only for “Phoenix” data in the range of 10-1500s).

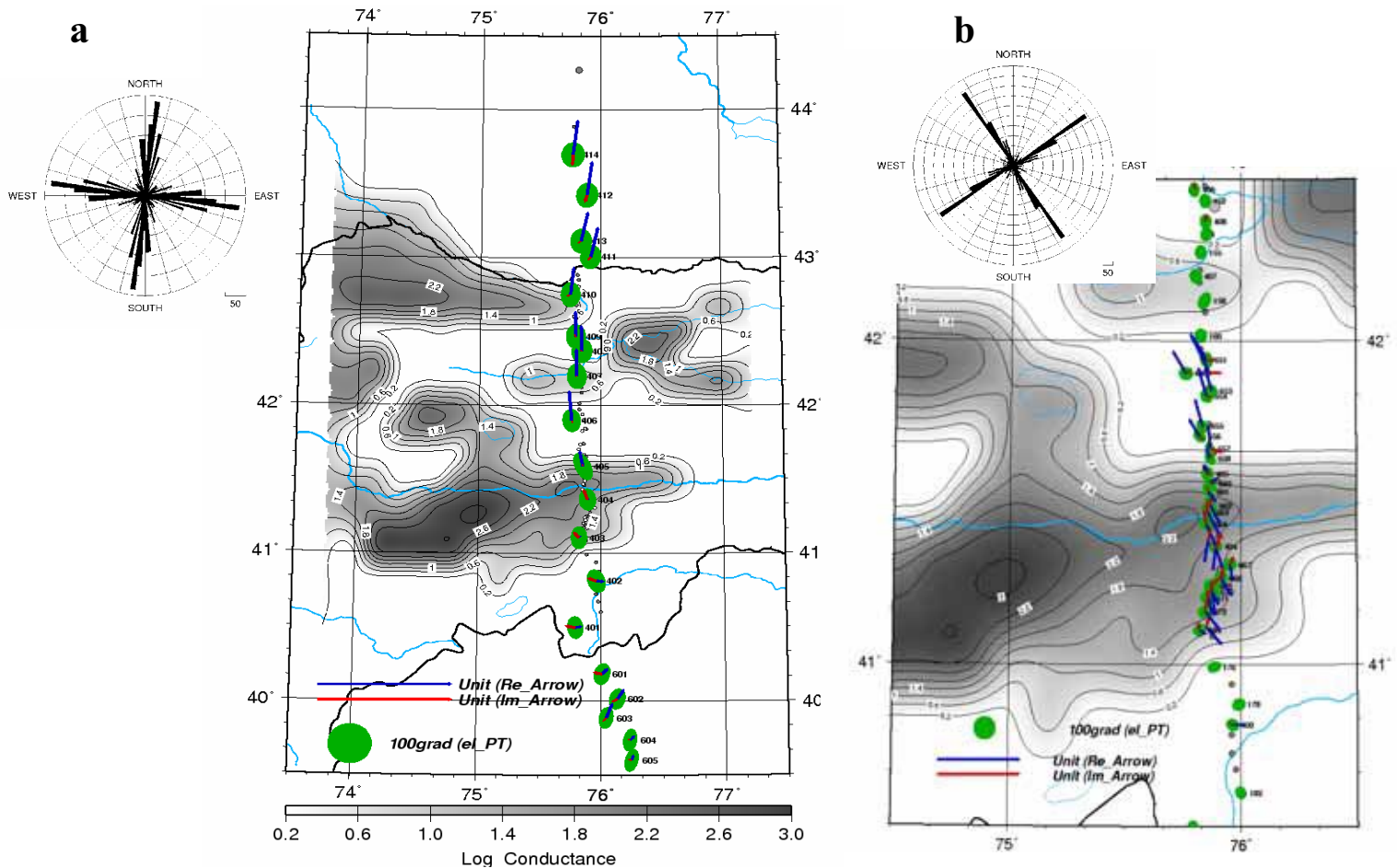


Fig. 5. Induction vectors, extremal direction “ellipses” of phase tensors with corresponding angle sector histograms on the map of total conductance of sedimentary cover (in log Sm, *Melnikova, 1991*): (a) - for LIMS data at T=2048s; (b) – for “Phoenix”, CES-2 and LIMS data at Naryn Basin segment of the profile at T=256s. The scale of induction vectors is shown by unit arrows; the scale of main values of phase tensors – by circles with radii of 100° and scales of histograms – by length of segments in number of samples.

These figures also demonstrate the long period effects of inhomogeneous source in tippers data (Fig. 4) as well as marks of regional 3D distortions of Z and M at long period (Fig. 4.5a) for southern part of the profile, which are probably caused by borders of Tarim Basin in its NW corner or crustal anomaly there.

Despite the fact that the presence of the impedance and tipper distortions, caused by near surface and/or crustal inhomogeneities of limited strike, was revealed in **MT** and **LMT** ensembles, the regional quasi-2D behavior of the profile data sets was generally approved and the validity of 2D approach for the interpretation was confirmed for both data ensembles with exceptions of high (<10s) and long (>8000s) period tippers obviously strongly distorted by near surface or source heterogeneities.

Quasi 2D ensembles of TFs for profile inversion in broad-band (**MT**, 0.1-1500s) and long-period (**LMT**, 16-20000s) ranges were compiled from the impedances (Z), tippers (Wz) and horizontal magnetic tensors (M) estimates (the latter ones are present only in **LMT** range). Each component of the data was rotated according to the regional strike (0°) and supplied with a specific mask of weights, reflecting data accuracy and a quantitative measure of local 3D distortions calculated via invariants SKEWs and Strikes Fig. 6. demonstrates the data (tippers ReWzx and Mxx component of tensor M), corresponding error bars estimated in processing routine and resulted weights just described above. The latter will serve as penalties in the inversion, suppressing the influence of the 3D distorted data and thus concentrating the inversion procedure on the recovering of the regional 2D structures.

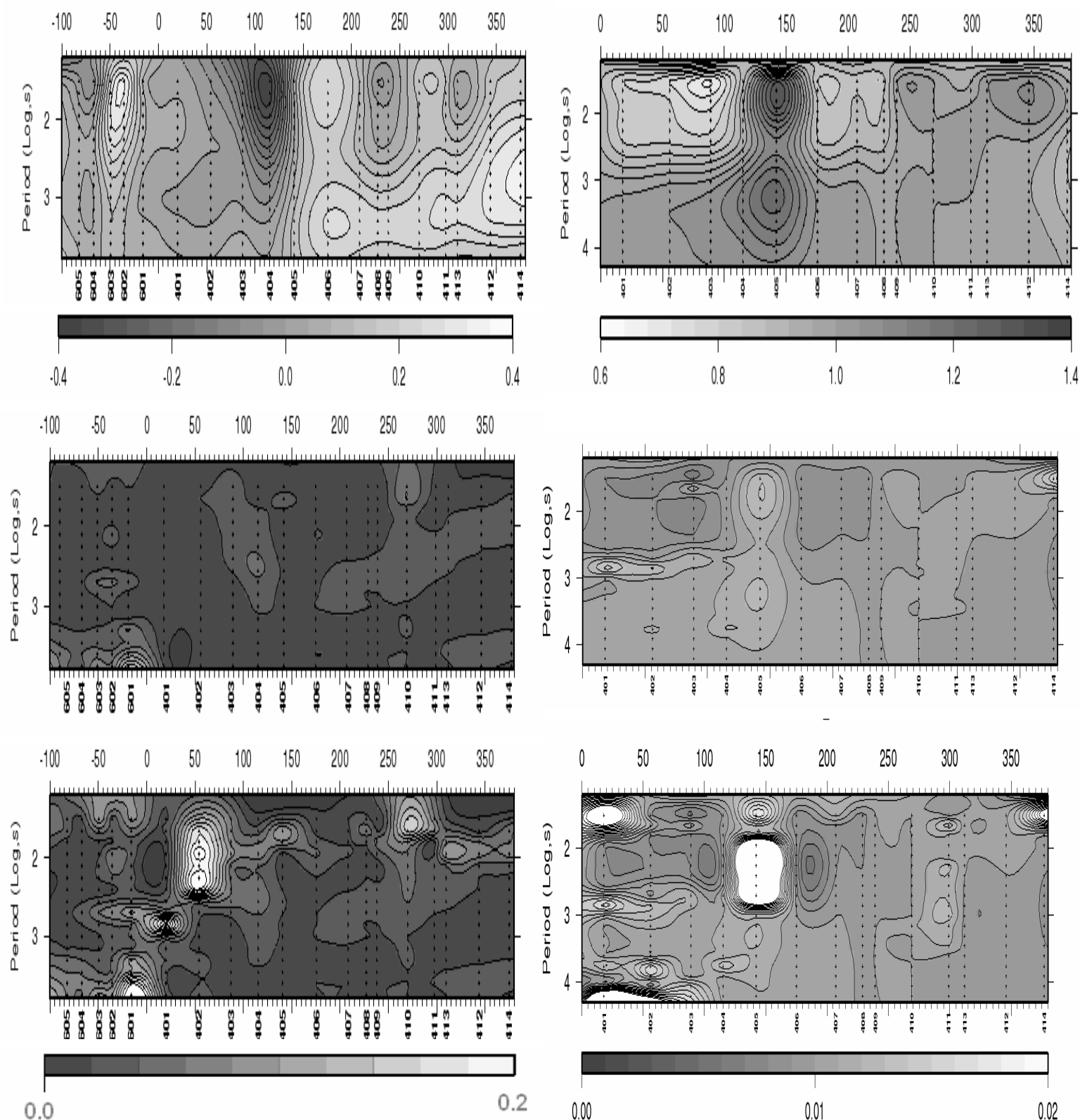


Fig. 6. The pseudosections of LMT ensemble components (top row), estimation error bars (middle row) and 2D inversion penalties, calculated on the results of invariant analysis (bottom). The panels of left column present data for ReWzx and right column - for Mod Mxx. The scale palettes for middle row panels are the same as for the correspondent bottom ones.

4. Methods of profile inversion and results of its minimal constrained round

The strategy of “Naryn” profile data inversion was aimed at the suppression of 3D distortions and focused on the target 2D geoelectric features. It implied reasonable choice of a starting model and the priority of phase and geomagnetic data, introduced by the logicity of partial inversions, application of a’p priori weights and/or calculated penalties defining quasi-2D ensembles. The inversion round combined the successive partial inversions (starting from the geomagnetic data and step by step fixing the parameters for reliably determined conductivity blocks) and, in parallel, simultaneous weighed multi-component bi-modal inversions. The regularized 2D inversion technique (Varentsov, 2007b), which offers adaptive block and piece-wise continuous approximation of conductivity distributions and a variety of resources for the solution stabilization, was the basic tool for interpretation.

To use all the advantages of the extended **MT** data set with dense spacing, the starting model of the extensive grid was used with the windows for the detailed scanning in the areas of expected conductivity anomalies (Fig. 7). This starting model and vector of conductivity parameters were the same for both **MT** and **LMT** subsets of data, which were inverted separately. Normal sections were chosen on the 1D inversion results for **MT** curves at the sites of Kazakh and Tarim platforms. In this model the approximation of the real topography along profile and of sedimentary valleys was incorporated after the modeling studies, which have revealed their significant influence on the observed **MT/MV** responses. Fig.8 shows the responses of starting model (complete grid geometry and conductivity distribution

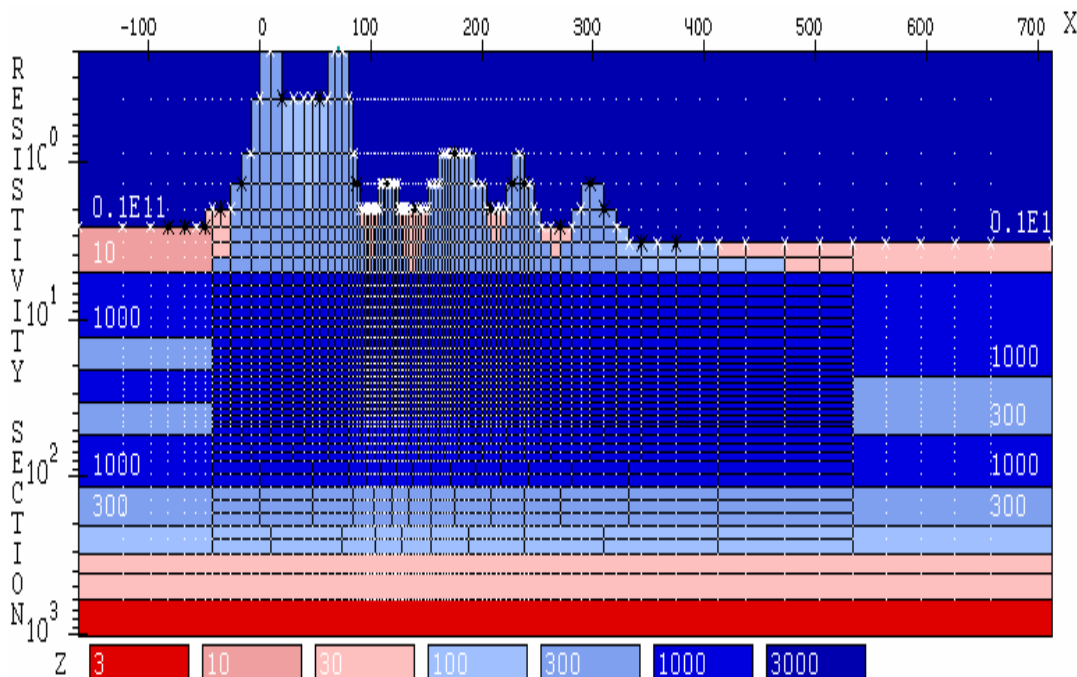


Fig. 7. The starting model of conductivity distribution (in Ohm.m) for profile inversion of **MT** and **LMT** data sets at “Naryn” transect: the grid dimension is 109x53 with ~ 2000 optimized parameters; X - distance in km with X=0 at the Kyrgyzstan-China border, Z - log depth in km with Z=0 at the relief top.

presented in Fig.7.) and of two its modifications with changed central (“anomalous”) parts (lowermost row panels): with flat surface or with topography approximation but without sedimentary valleys. The topography effects calculated for the components of **MT** data reach: 4-5° at periods of 1-2s for phases of longitudinal impedance and value of 0.1 for real part of tipper at first units up to first tens of seconds. The incorporating of “sedimentary valleys” with realistic integral conductance (about 100-150 Sm, see Fig.5) enhances the effects up to 15° or more for phases and up to 0.175 – for tippers as well as shifts them to the longer periods – up to first tens and first hundreds of second, correspondingly. The effects of topography are also significant in all the other components and spread to first second for phases of Z_{xy} , to tens of second for $Im W_{zx}$, being quasi static for amplitudes of Z_{xy} .

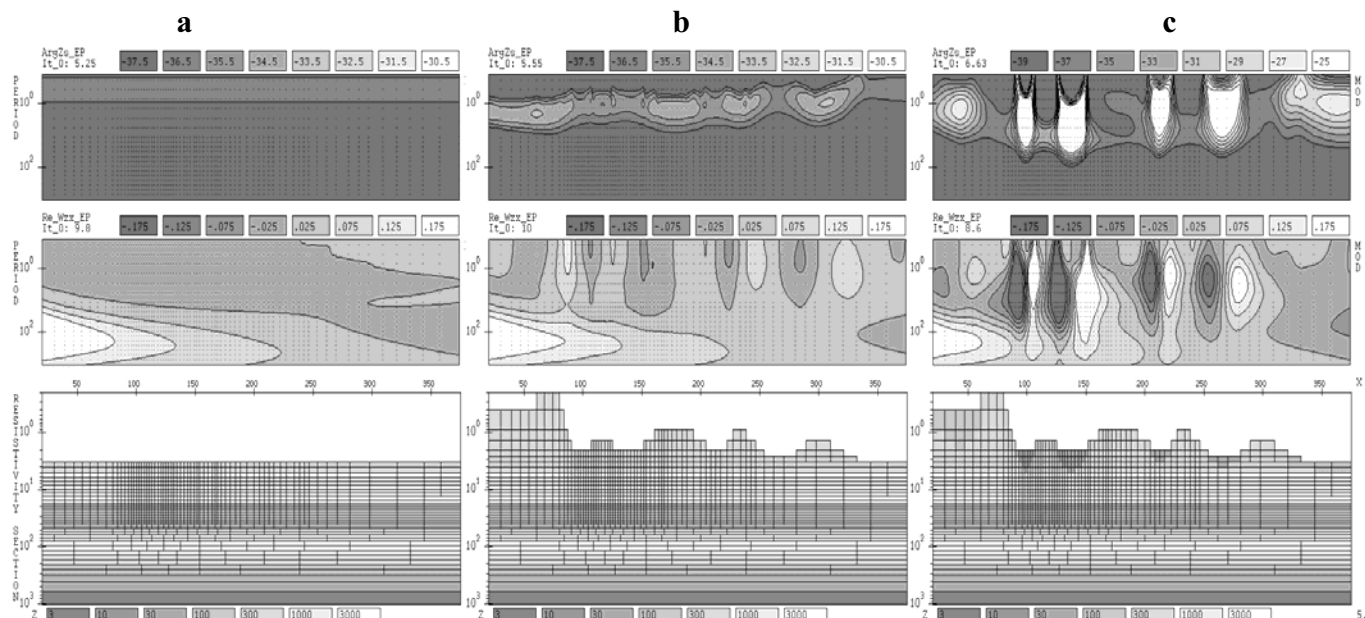


Fig. 8. 2D model studies of the topography and sedimentary cover effects on MT/MV sounding results along “Naryn” profile on the base of starting model, depicted in Fig. 7: (a) - results for “normal” structure (flat topography, different left and right normal sections); (b) – “normal” structure with topography; (c) – starting model (including topography and sedimentary valleys). The lowermost row of the panels presents the central (“anomalous”) part of the models used, the following upper rows present the corresponding pseudosections of Re Wzx (middle panel) and Arg Zyx (top).

The resulting model of this “minimal constrained” round of inversion is constructed as a robust averaging of the successful partial inversion results (in a space of conductivity parameters of each model cell) for both (**MT** and **LMT**) ensembles. It provides a solution in a manner resembling the “boot-strap” approach (Efron, 1979). This model is shown in log scale of depths in Fig. 9 (with example of data fitting for tipper component of **MT** ensemble) and in natural depth scale - in Fig. 10a.

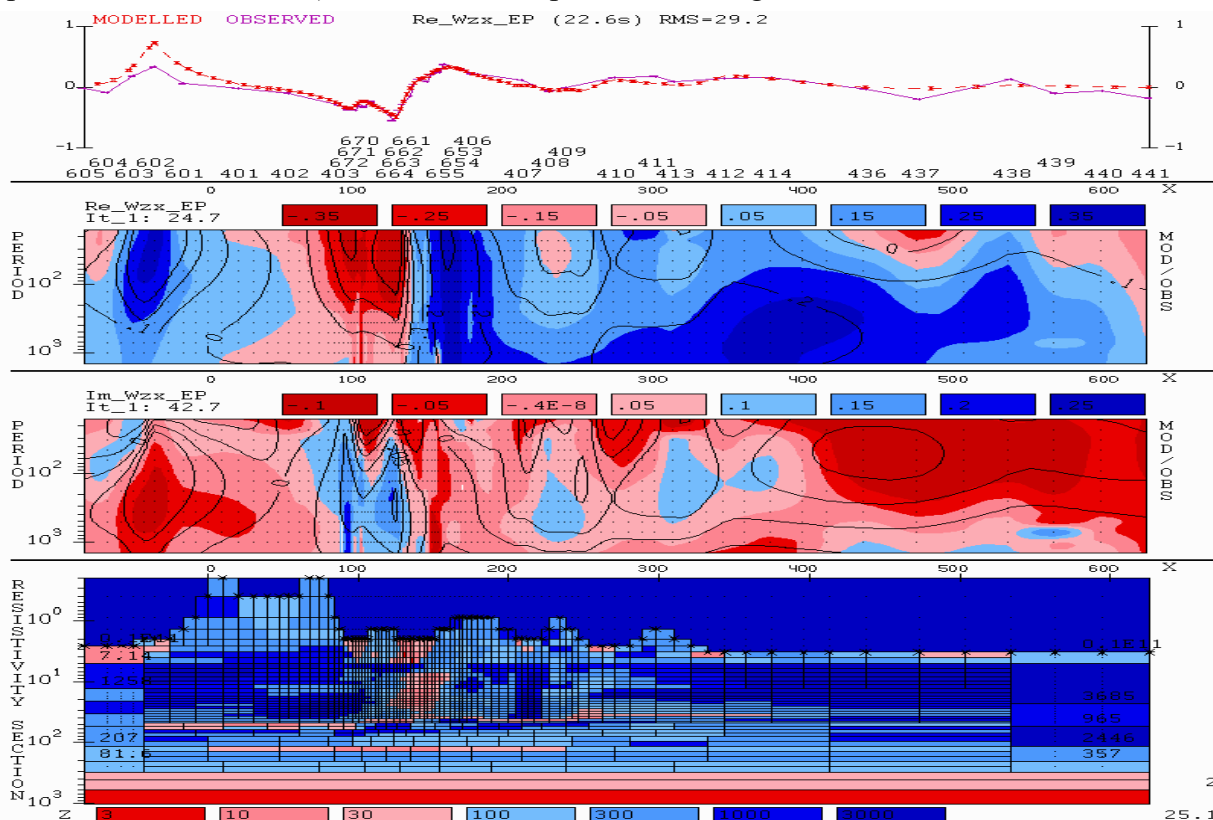


Fig. 9. The geoelectric cross-section of the Central Tian-Shan along “Naryn” profile (in log depth scale), obtained by the averaging of the results of a set of minimal constrained inversions of **MT/LMT** data (a) and data fitting on examples of Re Wzx and Im Wz components of **MT** ensemble.

The important features of resolved geoelectric structures are the low crustal conductive layer on the depth 40-50km uprising to 35km and decreasing integral conductivity in northern direction, sporadically revealed upper crustal one and the sub-vertical large-scale conductive zone in the upper and middle crust under “Nikolaev Line” tectonic unit. The application of the powerful well-focused inversion tools and representative character of data sets have permitted to obtain more resolution of the conductivity distribution along “Naryn” profile in comparison with the earlier studies of *Trapesnikov et al., 1997* as well as more stable solution compared with the results of *Bielinski et al., 2003*.

An interesting correlation of this still preliminary image of geoelectric section of Central Tian Shan with recent results of receiver function tomography, presented in Fig. 10b for the line 76°E according to *Vinnik et al., 2006*, already may be productive for new ideas on complex geophysical model of the orogen. However the main mission of this model is to serve as a base for model sensitivity tests and checking up different hypotheses as well as to be a starting point for the next round of inversion, constrained by a’pory geological-geophysical knowledge. This final stage of interpretation of MT/MV data at “Naryn” profile is under way now as it is necessary for extracting from these data well-grounded geodynamic implications.

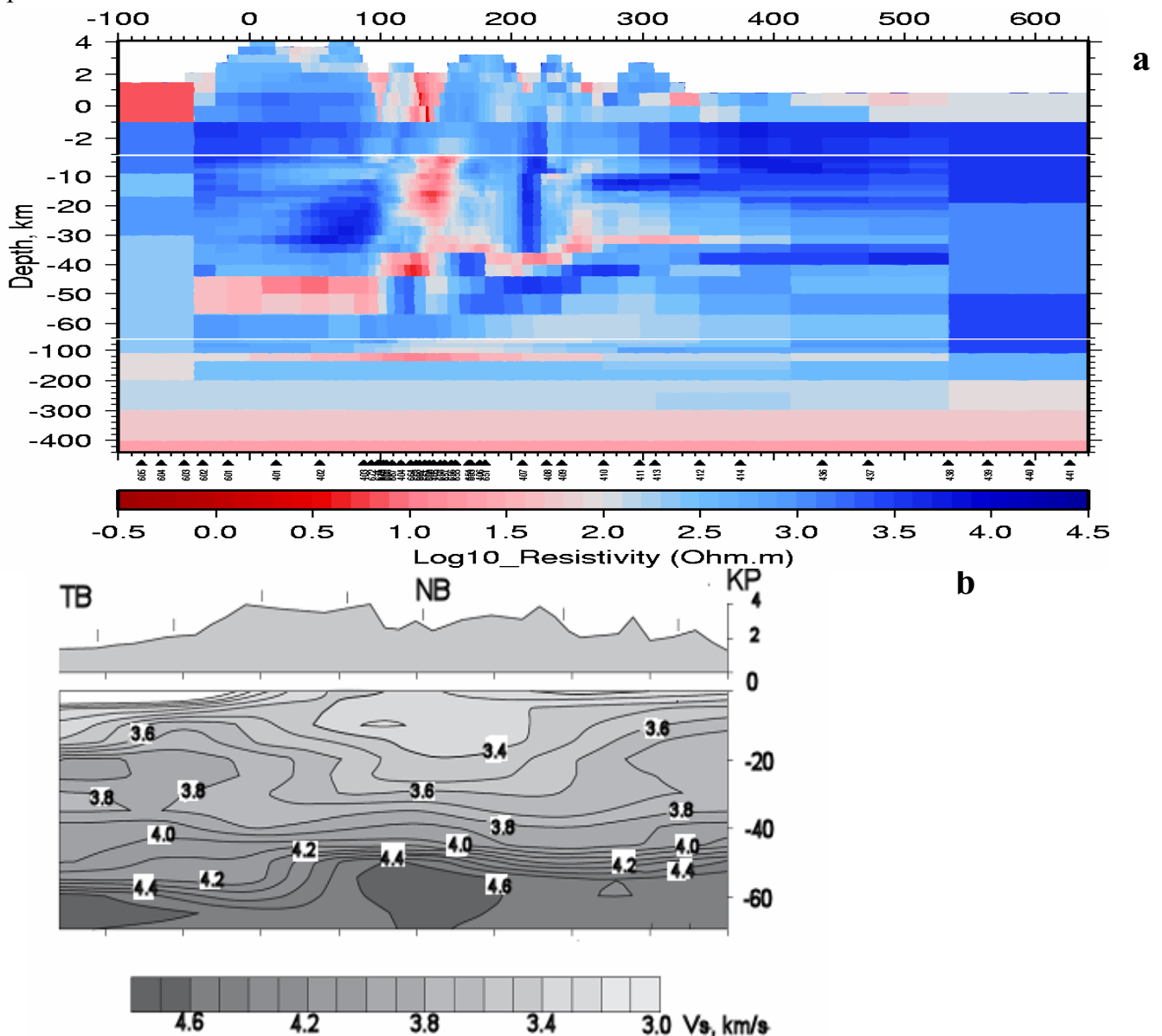


Fig. 10. The goelectric cross-section of the Central Tian-Shan along “Naryn” profile, obtained by the averaging of the results of a set of minimal constrained inversions of MT and LMT data (a) and depth distribution of shear wave speed (V_s) along 76°E given by receiver function tomography (b) (*Vinnik, et al., 2006*).

5. Conclusions

The paper presents recent advances in the methods of geoelectrical investigations in Tien Shan based on the original approaches in the processing, analysis and inversion of the MT/MV sounding data elaborated by the “Naryn” Working Group. We concentrate on the experience of profile studies being more economical and easily implemented in mountain surroundings than array soundings. It was demonstrated that this approach is valid for the data interpretation on the longest regional transect in Tien Shan – profile “Naryn”, where abundant collection of MT/MV data has been gathered by Research Station of Russian Academy of Sciences.

The approaches, which help to overcome the problems of 2D interpretation in real situation of a mountain range (even elongated one) were in the focus. They include attention to geomagnetic data and robust schemes of invariant TF analysis, adaptive parameterization of conductivity distribution, operations with heterogeneous in space and frequency range ensembles of data, construction of quasi-2D ensembles, preventing the inversion procedure from dangerous influence of near-surface and off-profile distortions and, finally, multi-component weighed inversion, considering different sensitivity and “immunity” of data components.

We present the current model of geoelectrical cross-section of the Central Tien Shan with increased resolution, but saved stability of the main features. The model shows general correlation with recent seismic tomography data. Nevertheless it needs checking by model sensitivity tests and next round of inversion, constrained by concurring geodynamic hypotheses, as well as further approval by other geophysical-geological information.

The elaborated methods will be applied for interpretation of the data on the other MT sounding transects crossing Tien Shan before a combination of all the profile inversion results into regional prognostic volume conductivity model of the orogen, valuable for geodynamic studies of this unique area of intracontinental building.

MT studies in the mountains became “a mainstream” of modern geoelectrics and so we believe that our experience, based on the traditions of classical approaches to ill-posed problems solution, might be useful for our colleagues in other regions.

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2D INVERSION RESOLUTION IN THE EMTESZ-POMERANIA PROJECT: DATA SIMULATION APPROACH

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1. Introduction

The EMTESZ-Pomerania array EM sounding experiment was held in 2001-7 to study the resistivity structure of the whole tectonosphere across the Trans-European Suture Zone (TESZ) in NW Poland and NE Germany (Brasse et al., 2006; EMTESZ WG and Smirnov, 2006). The transfer function (TF) data estimated for the EMTESZ-Pomerania array outline the complicated superposition of anomalies related to Polish and N-German sedimentary basins, crustal conductors within the TESZ and upper mantle inhomogeneities. The strike of sedimentary structures ranges around 45°NW, while for deeper structures at central profiles P2 and LT7 (Fig. 1 in Varentsov and EMTESZ-Pomerania WG (2007), this volume) it definitely turns to ~60°NW when estimated with the reduction of galvanic distortions (Varentsov et al., 2005). Different long-period skew responses at these profiles demonstrate minor 3D effects, except relatively local 3D influence of the resistive central block of the TESZ and induction arrows distortions in marginal areas. Thus, there are good grounds at these profiles for 2D inversion of long period TF data rotated to 30°NE.

These data were inverted separately by different participating teams of the EMTESZ-Pomerania project using different tools and to some extent different data ensembles (EMTESZ WG and Smirnov, 2006). Our Troitsk team finally came at both profiles to the inversion of the following 8-component ensembles: H- and E-polarization impedances, Z_{EP} and Z_{HP} (phases and 10 times downweighted apparent resistivities) at periods of 8-16384 s; tippers W_{zx} (Re, Im) at 32-8192 s; and horizontal magnetic inter-station responses M_{xx} (amplitudes and phases, estimated relative to the common base site P8 at NE edge of P2 profile) at 64-32768 s. To reduce the influence of 3D effects, we proportionally extended data error estimates at sites and periods both with large skews and with large strike excursions away from 60°NW. We also substituted original impedance phases with phases of the impedance phase tensor (Caldwell et al., 2004).

The inversion was held with Varentsov's robust code (Varentsov, 2002, 2007a,b). Inversion models included central 2D scanning windows from the surface to upper mantle depths (~300 km) and peripheral 1D normal sections with adjusted resistivities of layers. The total number of estimated parameters ranged at 1500-2000. Inversions were started both from the simplest quasi-1D *a priori* assumptions and from more complicated 1D/2D models summarizing results at previous inversion stages. More details on this inversion strategy are discussed in (Varentsov et al., 2005), while general advantages in the use of horizontal magnetic responses in the joint MT/MV data 2D inversion are presented in (Varentsov, 2007a,b; Varentsov and EMTESZ-Pomerania WG, 2005).

Goelectric models actually obtained at P2 and LT7 profiles (see Fig. 9 in Varentsov and EMTESZ-Pomerania WG (2007), this volume) correlate quite well, but contain a number of fine details, which may be considered as overfit effects and peripheral artefact, taking into account the interference of EM responses from strongly conductive and inhomogeneous sedimentary, crustal and upper mantle structures of the TESZ and 3D data distortions. Nevertheless, several of such fine details are stably preserved in the course of inversion iterations, are invariant with the change of inversion parameters and are repeated at both profiles. A strict objective arises to learn resolution bounds of the applied inversion approach in specific conditions of the EMTESZ-Pomerania project.

2. Method

In this paper the possibilities to resolve complex goelectric structures met in Pomerania are studied within the imitation approach in pure 2D environment. We constructed 2D model (Fig. 1), which generalizes and slightly simplifies real data inversion solutions obtained at P2 profile, and accurately simulated for this model structurally the same (in sites and periods) 8-component data set (Fig. 2) as was considered in the real data inversion. In this synthetic 2D data set we note the normalized horizontal magnetic response simply as H_x .

Both modelling and inversion problems were approximated at a medium-scale (74x57) grid (seen in Fig. 1) with horizontal resolution of 3-4 km and vertical crustal resolution of 1-2 km. The inversion solution was iterated till the convergence break and was monitored at a number of intermediate iterations. We present for each solution a sequence of 2-3 most important intermediate iterations in a column with a “true” model in the bottom and the final inversion model just above it (Fig. 3, 4).

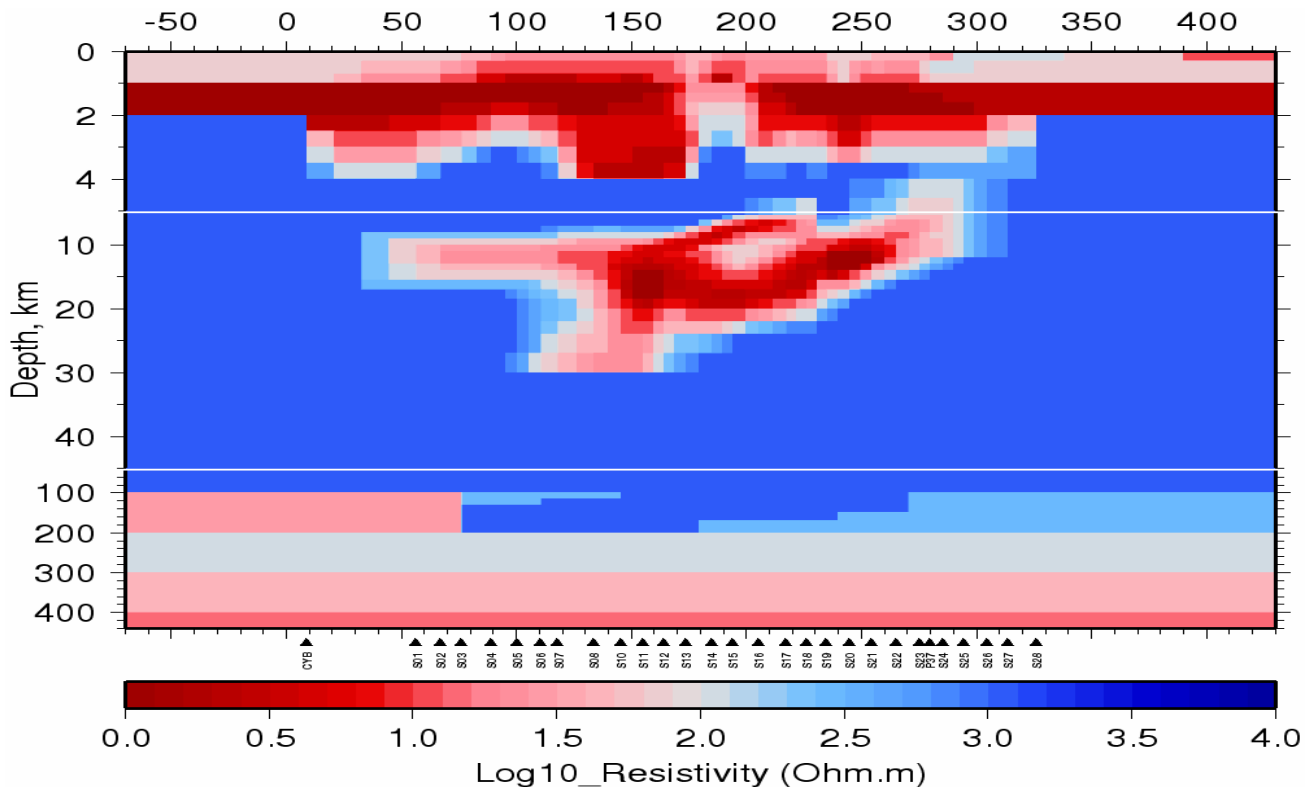


Figure 1. The simplified model of the goelectric structure along P2 profile in the EMTESZ-Pomerania project.

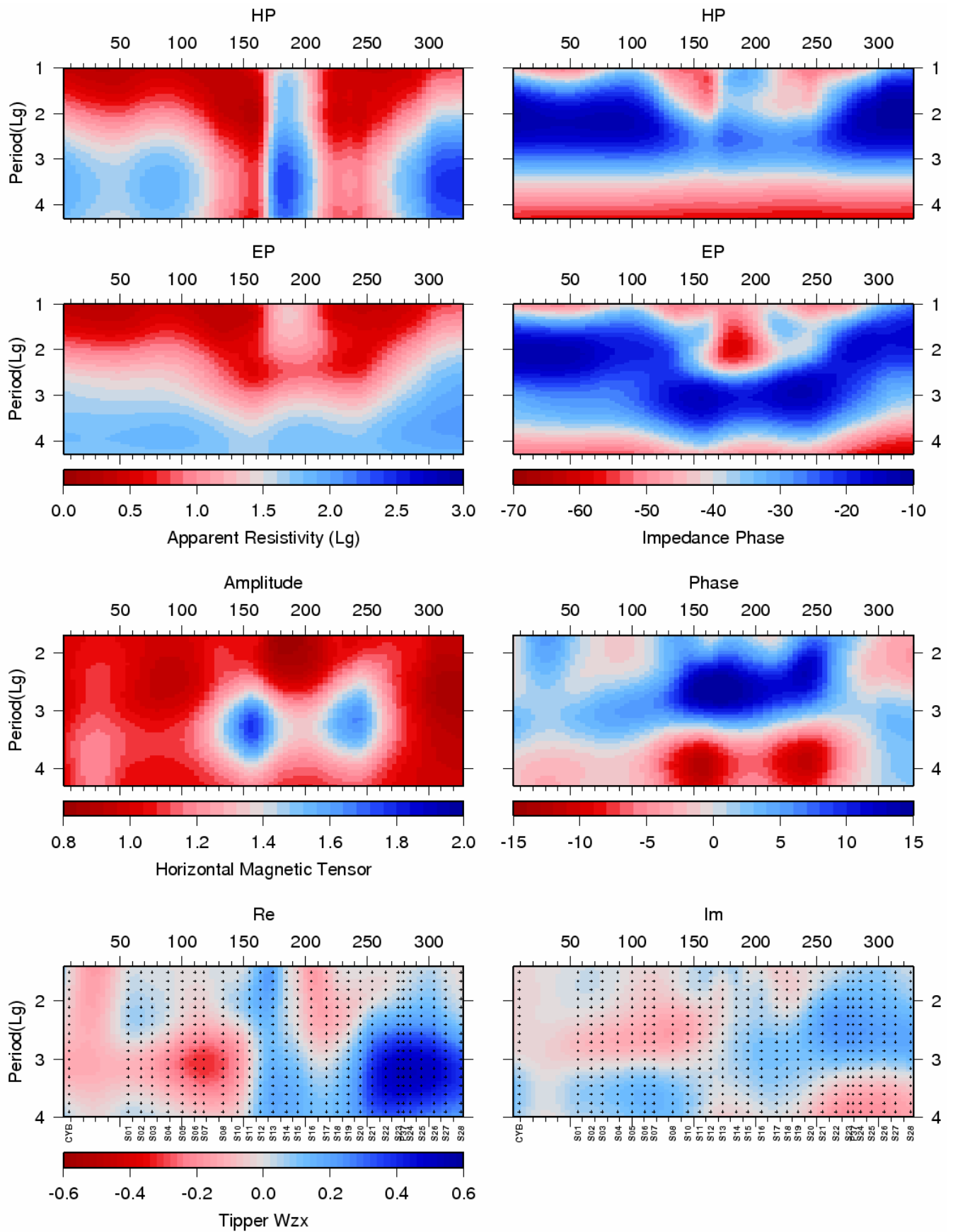


Figure 2. The 8-component data set accurately simulated for the model in Fig. 1 with components, sites and periods being the same as in the real data inversion at P2 profile.

The inversion quality in the comparison of different solutions was directly estimated in the model space by the relative norm of log-resistivity misfit:

$$\delta\rho = \|\ln \rho - \ln \rho_{true}\| / \|\ln \rho_{true}\| \quad (\text{in } \%),$$

with $L2$ norms taken over the area $[0,350] \times [0,400]$ km. This misfit is shown in white over model sections in Fig. 3,4.

The inversion quality was also indirectly estimated by absolute $L2$ data misfits, calculated separately for profile-period arrays (i.e. pseudo sections) of each data component. The absolute misfit for the apparent resistivity data at the log-scale was further recalculated into the relative misfit (in %) for these data at the normal scale. Such “partial” data misfit estimates for a number of presented inversion solutions are summarized in Table 1.

3. Results

Fig. 3 demonstrates the resolution and the convergence achieved in the inversion of truly simulated 8-component data set jointly and separately in its magnetotelluric (MT) and magnetovariational (MV) parts. The separate MT inversion solution used normal weights both for the apparent resistivity and the impedance phase, while in the joint solution the apparent resistivity was strongly (10 times) downweighted just as in the real data inversion for the protection from subsurface galvanic distortions.

The fit of all these solutions to the true model looks really perfect. The model space misfit approaches the level of 5-7%, partial data misfits go down to the level of modeling accuracy (Table 1) and the most of fine details in final inversion models are very like to those in the true model. In spite of relatively longer convergence, MV solution finally gets the quality of two other solutions. Only the resistive layer between sedimentary and crustal conductors seems to be not exactly resolved.

The best fit is achieved in the joint MT/MV inversion. In this solution the surprising recognition of both resistive and conductive structures at depths of 4-30 km takes place below the subsurface conductor with the average conductance spread around 1000 S. Moreover, main upper mantle inhomogeneities are still very well seen below sedimentary and crustal structures of the total conductance exceeding 10000 S.

Finally, in Fig. 4 we study the influence of the random noise intensity on the effectiveness of the considered 8-component inversion scheme applied to noise-contaminated data. At the noise level of 2% and even 5% it is still possible to distinguish the most of fine details of the true model. Even at the noise level of 15% our inversion scheme truly distinguishes the general location of conductive zones and their most intensive parts, but is not accurate in the tracing of their connection and peripheral extension. Notice that partial data misfit estimates are reduced in final solutions to the level of correspondent norms of the simulated noise (Table 1).

It is important to mention a prominent stability of the considered inversion solutions almost for the whole sequence of iterations. This stability comes from the excessive informativeness of the joint MT/MV data ensemble and from the wide variety of stabilization tools adaptively implemented in the applied inversion technique (Varentsov, 2002, 2007b).

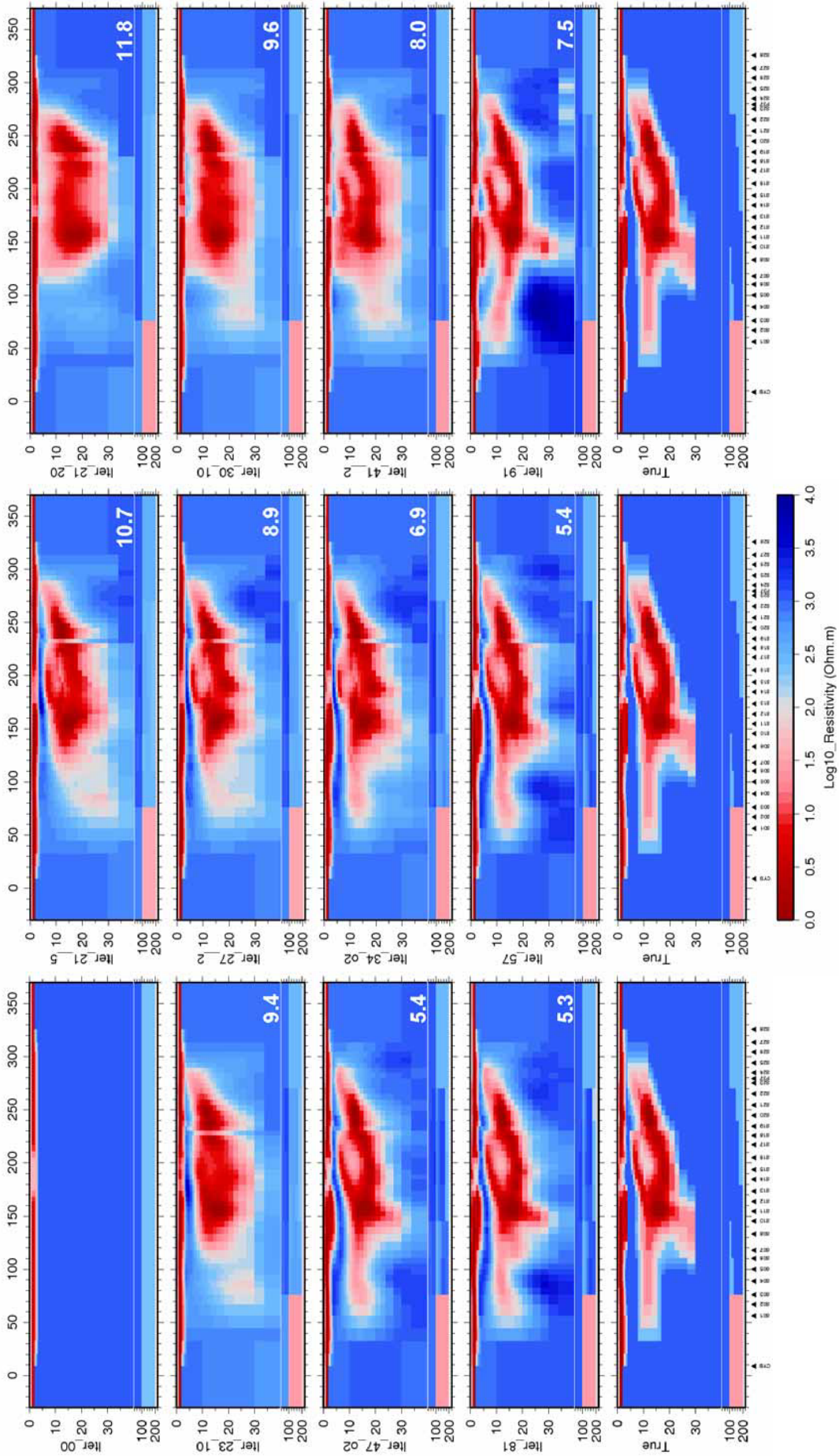


Figure 3. The inversion of different component ensembles from the true data set, from left to right: $Z_{HP}+Z_{EP}+W_{zx}+H_x$, $Z_{HP}+Z_{EP}$, $W_{zx}+H_x$; the starting model is quasi-1D (the left top panel); the true model is shown in bottom panels, white numbers give model space relative error (in %) for the area $[0,350] \times [0,400]$ km.

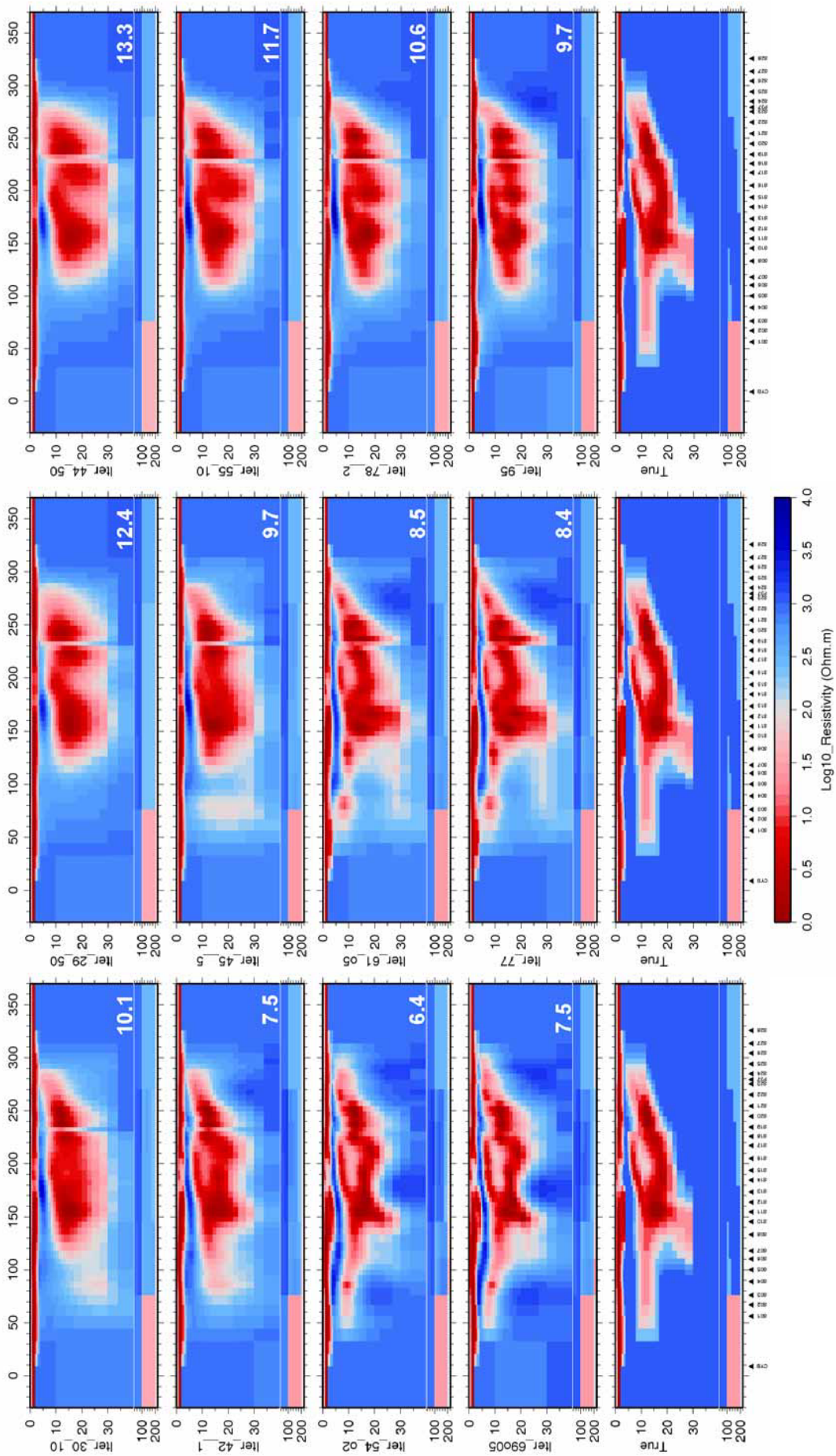


Figure 4. The inversion of 8-component data sets simulated with added random noise (from left to right: 2, 5 and 15%) from quasi-1D starting model; the true model is shown in bottom panels; white numbers give model space relative error (in %) for the area $[0,350] \times [0,400]$ km.

Components	It.	Par. RMS (L2)	Partial MSF (L2) / Noise (L2)								Comments
			Ro HP	PhZ HP	Ro EP	PhZ EP	Mod Hx	Ph Hx	Re Wz	Im Wz	
Ro+PhZ (EP+HP) +Hx+Wzx Ro DW 10x	81	5.33	0.13	.025	0.04	.007	.000	.007	.000	.000	No Noise, Quasi-1D Starting Model
Ro+PhZ (EP+HP) Normal Ro Weight	57	5.41	0.09	.033	0.16	.091					No Noise, Quasi-1D Starting Model
Hx+Wzx	91	7.50					.000	.011	.000	.000	No Noise, Quasi-1D Starting Model
Ro+PhZ (EP+HP) +Hx+Wzx Ro DW 10x	69	7.51	3.50	.245	2.52	.330	.009	.108	.004	.002	2% Noise, Quasi-1D Starting Model
Ro+PhZ (EP+HP) +Hx+Wzx Ro DW 10x	77	8.36	9.06	.646	6.41	.811	.022	.244	.009	.005	5% Noise, Quasi-1D Starting Model
Ro+PhZ (EP+HP) +Hx+Wzx Ro DW 10x	95	9.66	26.9	1.94	18.5	2.44	.066	.745	.030	.014	15% Noise, Quasi-1D Starting Model
			26.5	1.91	17.9	2.36	.064	.667	.027	.014	

Table 1. Summary of misfit estimates (consequently, in the space of model parameters and in the data space) in 2D inversion solutions for different data ensembles simulated with different data noise levels; apparent resistivity misfits are relative (in %), all other partial data misfits are absolute; error norms are given in lower part of table cells in the case of solutions for noise-contaminated data.

Conclusions

We demonstrated quite an optimistic view on the resolution limits of the multi-component 2D inversion in the case of extremely complicated, but truly 2D data even contaminated with random noise at the level of 5-15%. In the most of cases the model resolution successively improves in the course of inversion iterations without serious overfitting effects. The model space misfit estimates formally point at the latest inversion iterations as solutions with the best resolution and stability.

We managed in all presented inversion solutions to reliably separate conducting structures overlapped at several depth levels from the sedimentary cover to the upper mantle and met no prominent peripheral “false” conductors, though noticed the influence of the starting model and the noise level on the resolution of peripheral resistive background.

The MV data inversion competes well with the conventional MT inversion, and no convergence contradictions are met in the most complicated, but also most promising 8-component joint MT+MV inversion.

We got important justification to treat seriously the fact of separation of sedimentary and crustal structures in the real data inversion at Pomeranian profiles and to take into account fine details of conducting anomalies located there at sedimentary, crustal and upper mantle levels.

However, these experiments only partly simplify the understanding of our real data inversion results at Pomeranian profiles. We have to learn better the influence of 3D data distortions on 2D inversion results. The right way to do it is to analyze in the same way 3D model simulations (Kousnetsov et al., 2006), but this is the subject of a separate paper.

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From Precambrian to Variscan basement: Magnetotellurics in the region of NW Poland, NE Germany and South Sweden across the Baltic Sea

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Abstract

The Trans-European Suture Zone runs through the North Sea, South Scandinavia, the Baltic Sea, and Poland into the Black Sea. It divides old Precambrian lithosphere in the Northeast from younger, Caledonian and Variscan one in the Southwest. 2D models of four magnetotelluric profiles crossing this prominent tectonic border are presented and an attempt to integrate them into quasi 3D view on the regional conductivity structure is made in this paper.

1 Introduction

The network of magnetotelluric (MT) stations used in this study consists of two older and two newer profiles plus some single sites in between. The older ones are called P-2 and LT7 (see fig.1 for locations), they have been measured in 2003 and 2004 in the framework of the EMTESZ (*Electromagnetic investigation of the Trans-European Suture Zone*) project. This project has been introduced many times elsewhere (e.g. Brasse et al. [1]), so we will limit ourselves to its most necessary features here.

The aim of EMTESZ is the electromagnetic (i.e. MT) investigation of the Trans-European Suture Zone (TESZ) in NW Poland, a fundamental tectonic boundary dividing the old Precambrian Platform in the NE from the younger Paleozoic, i.e. Caledonian and Variscan, orogenic belts in the SW (see fig.1). The data at the SW end of the LT7 profile which is reaching into Germany suggested further measurements. In an electromagnetic sense, this region is dominated by the roughly EW-running North German Conductivity Anomaly, in contrast to the TESZ itself, where the conducting structures are striking rather NW-SE. In order to comprehend that structure properly, a NS running profile, called MVB, was established along the 14° E-Meridian in Germany close to the Polish border in 2006/07. This profile could benefit from some single sites measured earlier on Bornholm Island and in the Baltic Sea with a Polish offshore instrument.

Offshore technology from IFM Geomar Kiel was also important on the second new profile MVS which runs from the Lake Müritz in the German federal state Mecklenburg-Vorpommern via Rügen Island and across the Baltic Sea into Scania (South Sweden, see fig.1). It has also been measured in 2006/07. The direction of this profile has been chosen to follow a “historic” profile by the BGR from the

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1990ies and to lay perpendicular to the assumed strike of the Sorgenfrei-Tornquist Zone (STZ), a branch of the TESZ (see fig.1).



Figure 1: Overview of measurement area with profiles P-2, LT7, MVB, and MVS. Important tectonic elements are the Trans-European Suture Zone (TESZ) consisting of the Sorgenfrei-Tornquist Zone (STZ) and the Teisseyre-Tornquist Zone (TTZ), the Caledonian Deformation Front (CDF), the Trans-European Fault (TEF), the Variscan Front (VF), the Precambrian Platform or East European Craton (EEC), and the Paleozoic Platform (PP).

2 Data examples of new profiles

Most of the earlier EMTESZ measurements have been carried out from 2003 to 2005. Now there are two new profiles, MVB and MVS, mainly measured in summer and autumn 2006 and some new sites in spring 2007 to improve the data quality. The direction of the new profiles was chosen because of the direction of a former BGR profile and effects seen in the Polish EMTESZ-profile (LT-7).

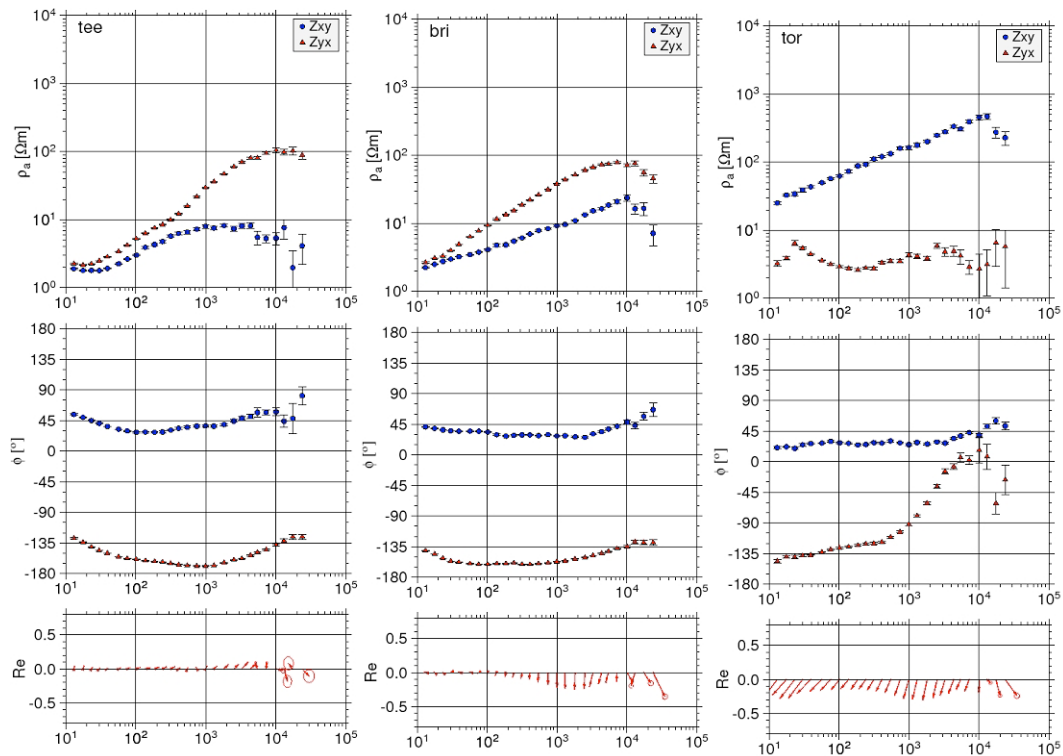


Figure 2: Three sounding examples from the northern, middle and southern part of profile MVB. Station TEE is located on the mainland approximately 20 km from the coast NW of Szczecin. BRI is the next station to the south of the crossing point of two profiles MVB and LT-7 and TOR is the second station from the south. It shows strong 3D-effects in the phase, as they are characteristic for the southern sites. Altogether these three stations have a good data quality and show the characteristics of the profile from the thick, well conducting sedimentary top layer in the north towards the resistive crystalline basement in the south.

Profile MVB (Mecklenburg-Vorpommern – Brandenburg) crosses the Variscan Front (VF) and reaches the Teisseyre-Tornquist Zone (TTZ) at its northern end on Bornholm Island (see fig.1). N-S direction was chosen because of the south-pointing induction arrows at the SW end of the Polish EMTESZ-profiles, cf. fig.4. It has about 260 km length and runs along the 14th degree East meridian. It includes 20 stations with a distance between 6 km and 23 km plus two offshore-stations and one onshore site on Bornholm carried out by the Institute of Geophysics of the Pol-

ish Academy of Sciences, Warsaw. In the northern mainland part there are thick sedimentary covers (up to 10 km) penetrated by saline aquifers and/or other well conducting structures like that one beneath Usedom Island, which was identified as alum shales by the BGR (ρ at low periods around 1-3 Ωm), whereas in the southern part the profile reaches a region where the crystalline basement of the Variscides is almost outcropping (ρ_a has higher values even for low periods), see fig.2.

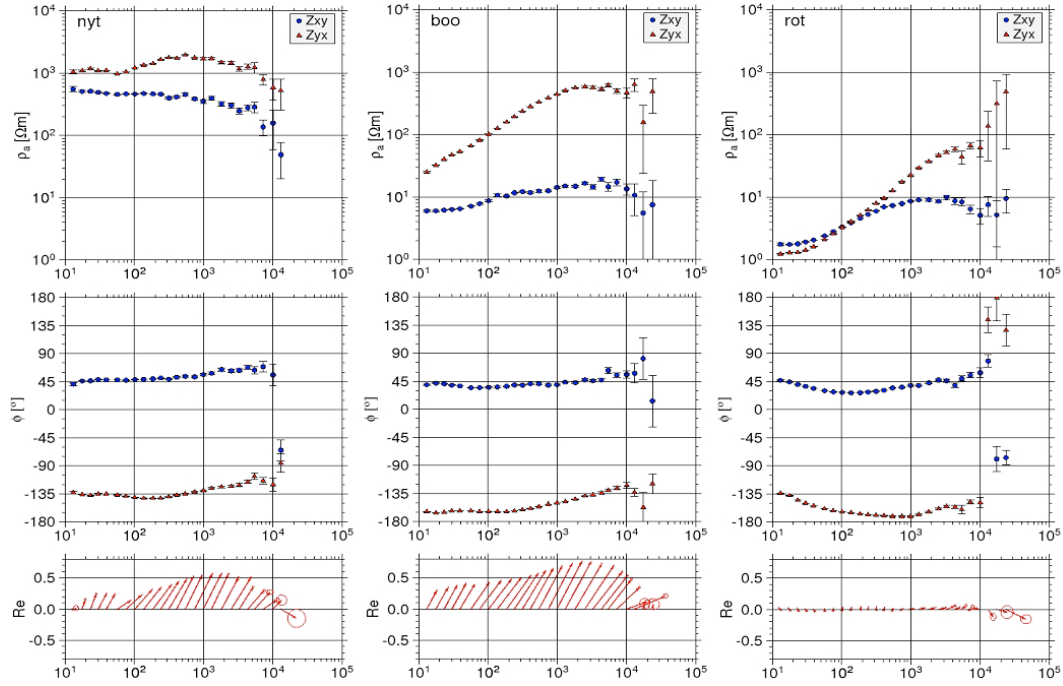


Figure 3: Sounding examples of sites located in profile MVS. From the northern end of the profile there is shown NYT in South Scandinavia, NE of Andrarum (cf. fig.1, a place known for its alum shale outcrops). Site BOO is the southernmost one in Sweden right at the Baltic Sea, and site ROT is at the southern end of the profile in Mecklenburg-Vorpommern.

Profile MVS, meaning Mecklenburg-Vorpommern and Scania, is about 300 km long and runs from north-north-east to south-south-west. It includes 22 stations with a distance between 6 km and 12 km and two offshore-stations from IFM Geomar Kiel that were deployed close to the Rügen and the Swedish coast.

The direction of this profile was chosen because of the induction vectors from former BGR-measurements.

Some transfer functions of MVS are given in fig.3. The ρ_a curves at lower periods show the high apparent resistivity of the Precambrian crystalline basement in the north (station NYT) and at the margin of the sedimentary basin marked by the Baltic Sea (station BOO). Site ROT at the southern part of the profile shows the resistivity of the increasing sedimentary cover (with lower ρ_a at lower periods).

The profile starts in the region of the VF in the south and crosses the TTZ/STZ in South Scandinavia (see fig.1).

Stable transfer functions for all profiles result from magnetotelluric data processing including the remote reference technique according to Egbert et al. [3]. The remote data for MVB and MVS were from the observatories of Belsk and Niemegk. Strike angles were calculated for single and multi sites after Smith [8], which allows to sort out bad data with a weight matrix. The strike angle for profile MVS has a strong variation especially in the northern part; the average for the profile is around 23°. For profile MVB the strike angle varied from 25° in the South to 1° in the North. The average strike angle for profile MVB is 21°.

3 Wide-area consistency in transfer function map views

The map views of induction arrows and perturbation vectors show a very consistent picture over the measurement area. We present them at a period of ca. 1800 s, since there are maximum real arrows and zero imaginary arrows at most stations.

Considering induction arrows (Wiese convention, fig.4), the whole region is divided into three parts.

There are large (0.7-1.0), NNE pointing arrows in the northern and northeastern part that finds its boundary beginning on Central Rügen Island, then following a line slightly north of Usedom Island and the West Polish coast almost till Koszalin. From there it merges in the known course of the TESZ. Hence, the induction arrows are directed more or less perpendicular to the border of the Precambrian Platform and the Baltic Shield, and their large length is partly caused by the strong conductivity contrast at the edge of the basement.

The middle part is characterized by very short arrows with relatively erratic directions. Only on profiles P-2 and LT7 they eventually suggest some small-scale 3D structures. It becomes clear that the good conductors are located in this part, whose southern boundary is located at 53rd degree North latitude.

In the South, the induction arrows have a moderate length of about 0.3 and they point mainly southwards. This behavior is known for many decades as the North German Conductivity Anomaly.

The perturbation vectors

$$\vec{p} = \begin{pmatrix} h_H \vec{u}_x \\ d_H \vec{u}_y \end{pmatrix}$$

and

$$\vec{q} = \begin{pmatrix} h_D \vec{u}_x \\ d_D \vec{u}_y \end{pmatrix}$$

(with $\vec{u}_x, \vec{u}_y$ being unit vectors in the subscripted directions) are a means to visualize the transfer function called perturbation tensor, which correlates the magnetic field components of a local station (B_x, B_y, B_z) with the horizontal magnetic components (B_x^R, B_y^R) measured synchronously at a reference station (Schmucker [7]):

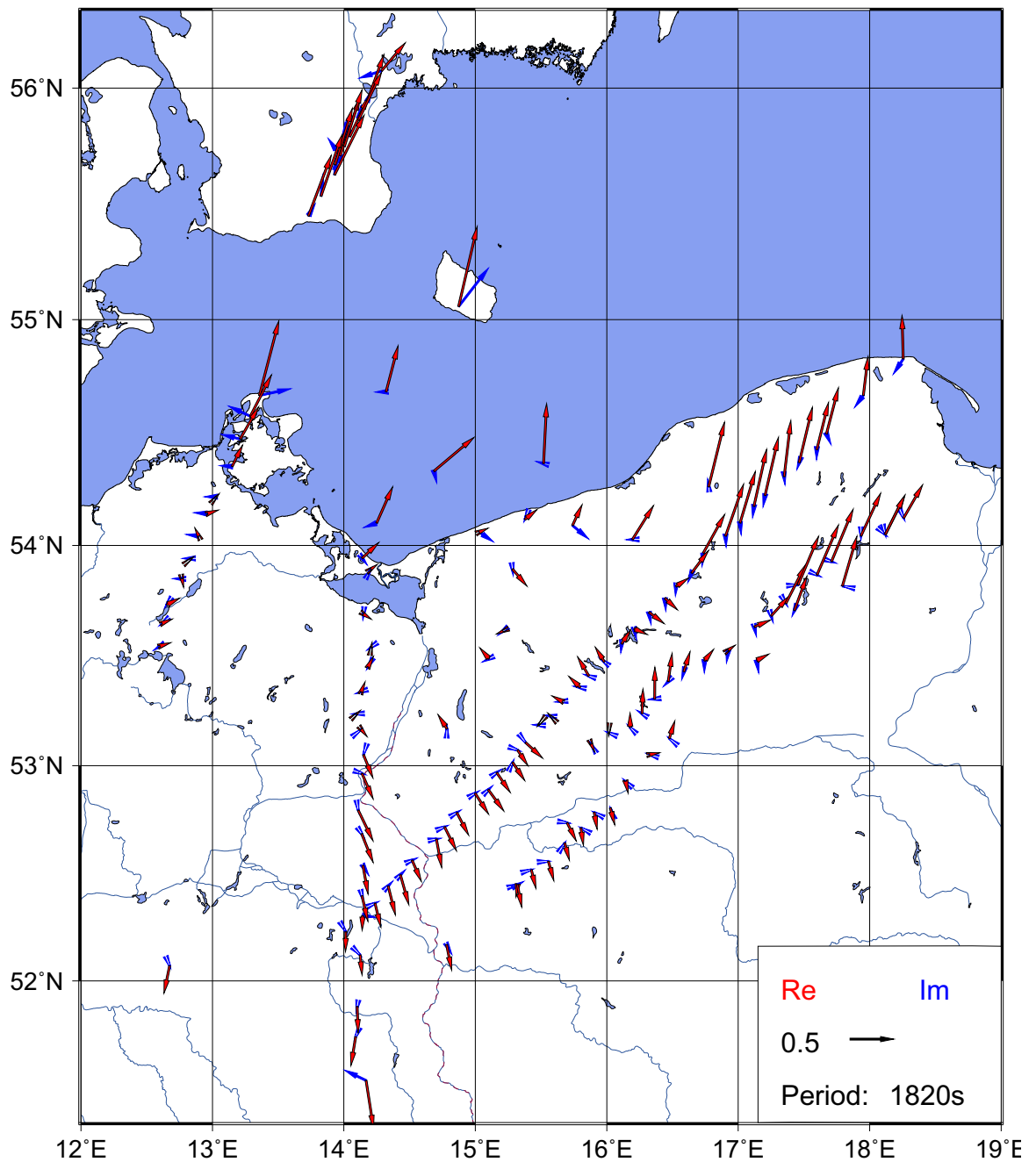


Figure 4: Induction arrows (Wiese convention) at ca. 1800 s indicating a trisection of the measurement area. See text for detailed description.

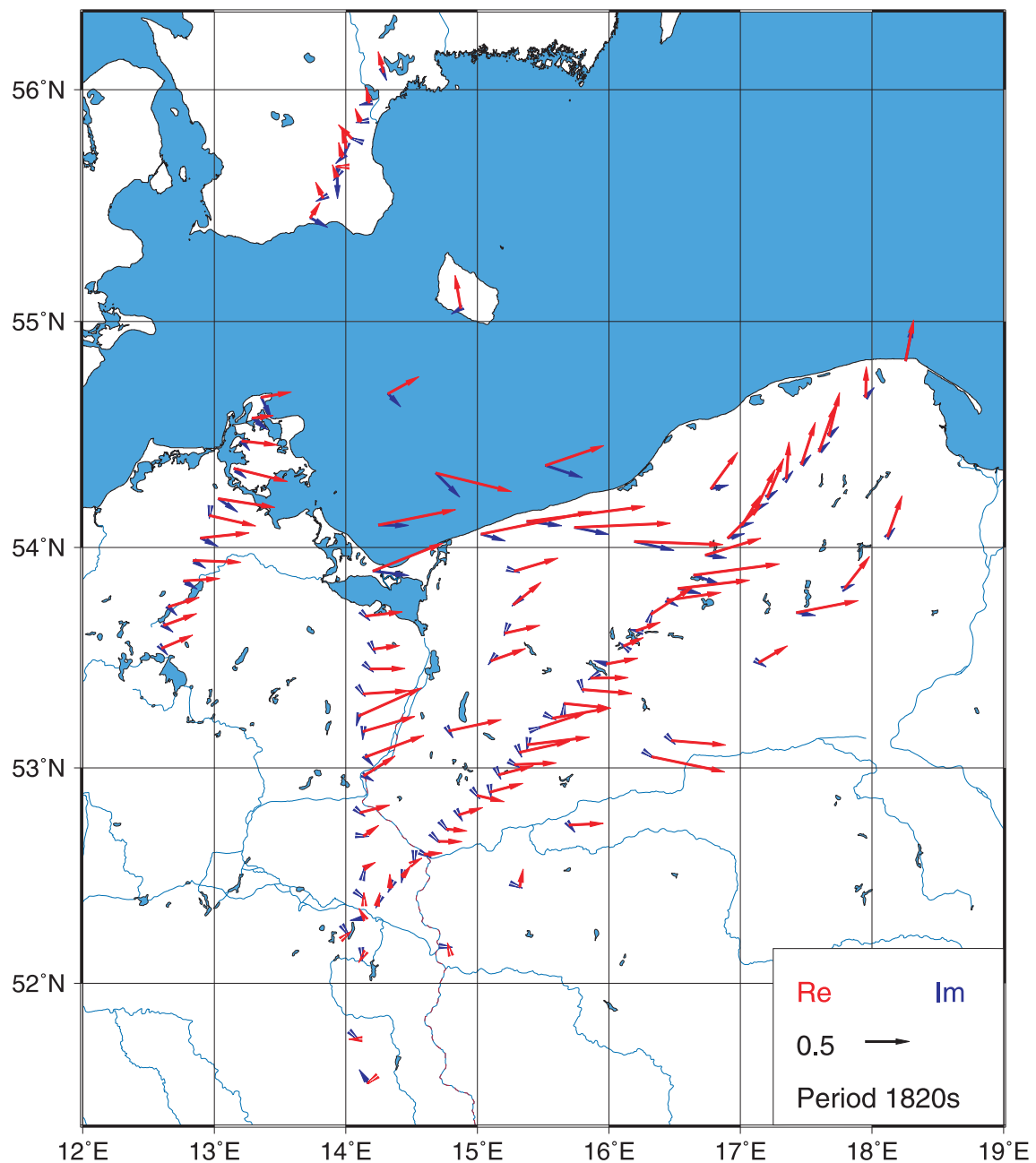


Figure 5: Perturbation vectors (only $\vec{p}$) at ca. 1800 s with respect to Niemegek Geomagnetic Observatory and rotated by 90°. Two prominent conductors emerge in this presentation, one slightly north of 53°N and another one along the Baltic coast between Rügen and Koszalin. See text for more details.

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} h_H + 1 & h_D \\ d_H & d_D + 1 \\ z_H & z_D \end{pmatrix} \begin{pmatrix} B_x^R \\ B_y^R \end{pmatrix}$$

If the reference station is situated on a relatively one-dimensional subsurface, the perturbation vectors reflect nicely the direction and strength of the anomalous (i.e. caused by lateral conductivity contrasts) magnetic field at the local station and thereby eventually the run of well-conducting anomalies.

In our case, Niemegk observatory (NGK) served as reference site. Fig.5 shows the so-called $\vec{p}$ -vectors that are connected to the North component of the reference field variations. Due to the mainly E-W striking geological structures and corresponding induced currents, anomalous magnetic fields are rather observed in North direction and therefore, $\vec{p}$ -vectors are more illustrative than the remaining $\vec{q}$ -vectors.

In fig.5, the vectors are rotated by 90° to visualize the strike of well-conducting anomalies instead of the direction of the anomalous magnetic field itself. Contemplating the region referred to as “middle part” in terms of induction arrows, we can see that perturbation vectors give a more differentiated picture of that generally well conducting region. There can be distinguished two conductivity anomalies. One is obviously identical with the classical North German Conductivity Anomaly, of course extending far into Poland, slightly north of 53°N and striking E-W. The second one is much stronger and runs along the Baltic coast. Its maximum can be described by a line from South Rügen Island over Usedom and the West Polish Baltic coast, then bending to SE and following the Craton edge.

4 2D-modeling

Being aware that some data, particularly tippers, display 3-D characteristics or different strike directions in several areas of Pomerania and NE Germany, we employed 2-D inversion of various transfer functions on all four profiles to obtain a first-order overview on structures of the subsurface. For LT7 and P-2, the data were rotated into the coordinate frame assuming an electrical strike direction of N60°W, which is roughly coinciding with the trend of the TTZ, but problematic in the region of the German-Polish border as mentioned in section 1. The data of MVB and MVS were rotated according to an electric strike direction of N69°W, or N67°W, respectively.

We inverted the classical MT transfer functions (apparent resistivities and phases for both TE and TM modes) and the tipper. Exceptions are the MVS profile and two single offshore sites on MVB, where only the tipper was inverted. The non-linear conjugate gradient algorithm of Rodi & Mackie [6] as an implementation of Tikhonov’s regularization approach allows for a variety of settings which all influence the resulting model space. The most important parameter is the regularization quantity itself; it determines the trade-off between model roughness and data fit. It has been chosen between 5 and 30 leading to RMS fitting values between 1.5 (MVS) and 2.5 (MVB).

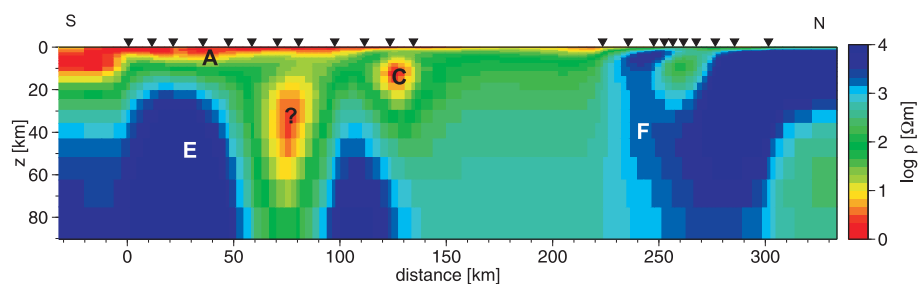


Figure 6: 2D resistivity model of profile MVS. See text for details.

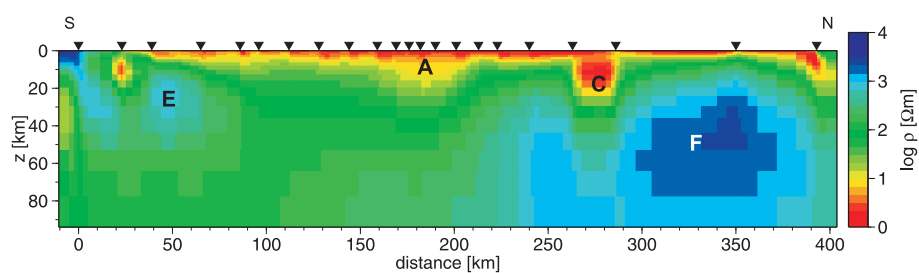


Figure 7: 2D resistivity model of profile MVB. See text for details.

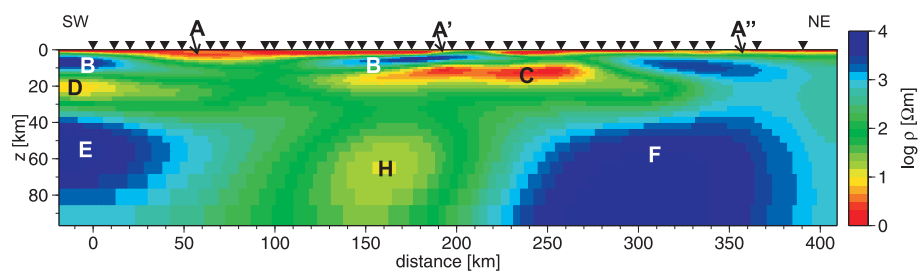


Figure 8: 2D resistivity model of profile LT7. See text for details.

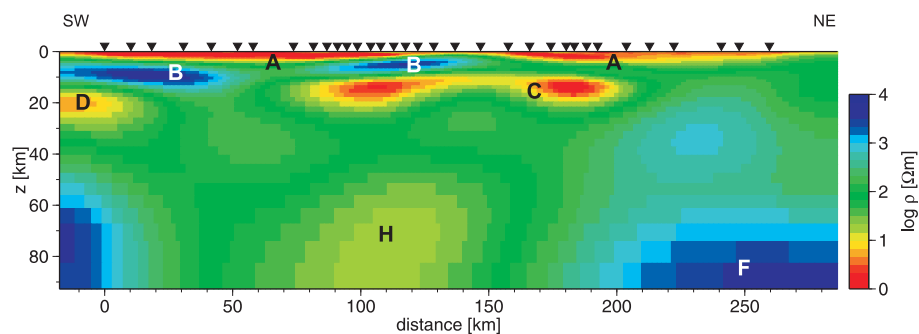


Figure 9: 2D resistivity model of profile P-2. See text for details.

The algorithm further includes the option of penalizing vertical or horizontal structures, inversion for static shift and the assignment of minimum errors (error floors) which may be set in a way that importance of phases is higher than statics-prone apparent resistivities. This large number of settings requires in turn a large number of experiments in order to find a best or best-suited model that explains the data.

The models obtained are shown in figs. 6 - 9. They display several distinct conductive and also resistive features, which are marked by upper-case letters.

- A signifies the very conductive Cenozoic-Mesozoic overburden; its very low resistivity of approx. $1 \Omega\text{m}$ is due to the saline aquifer which is commonly encountered throughout the North German-Polish Basin (e.g., Magri et al. [4]) in a depth of several hundred m. It reaches maximum thicknesses of almost 15 km in the middle of profile MVB, forming a typical basin structure and causing thereby the induction arrow pattern known as North German Conductivity Anomaly (cf. fig.4). This layer vanishes almost completely in the central regions of the Mid-Polish Trough (A' on fig.8, continuing also on fig.9), where older, more resistive sediments are encountered close to the surface due to a partly eroded anticline structure. Note that this layer overlies also the EEC (A'', fig.8) and the Baltic Sea (fig.7). The latter is less clearly visible in fig.6 where a possibly thicker sediment cover remained unresolved due to the lack of stations in the central part of the sea. The upper limit of this layer is not well resolved due to the period range investigated here, but seems to undulate somewhat along LT-7 according to the sealing, Mid-Oligocene clay layer (called "Rupel clay" in the NE German Basin) above.

- B (visible on figs. 8 and 9) is interpreted as the resistive Zechstein layer. It is apparently broken (or less resistive) in westernmost Poland close to the German border.

- The most obvious and pronounced conductor C – with resistivities as low as $2 \Omega\text{m}$ and an integrated conductivity (conductance or conductivity-thickness product) between 1000 and 1500 S – is underlying the whole TTZ at a depth of 10-12 km (figs. 8 and 9). In a more concentrated, less layer-like form (which can be a question of regularization) it is encountered beneath Usedom and Rügen Islands (figs. 7 and 6). It correlates with the Pre-Variscan consolidated crust as deduced for the TTZ region from the analysis of seismic refraction data (Dadlez [2]) with relatively low P-wave velocities of about 5.85 km/s. We may thus infer that the conductor is located in Silurian-Cambrian, pre-Variscan (Caledonian) meta-sediments. From the models alone it is not possible to uniquely deduce the cause of the enhanced conductivities; they may either be due to saline fluids (crustal brines) or electronic conductors like graphite or alum shale. The latter is frequently encountered in boreholes in the northernmost basin areas in NE Germany (e.g., at well G14 close to Rügen island in the Baltic Sea) and crops out in the southernmost Swedish province of Scania. Note that in electromagnetic methods only conductance is resolved within certain limits, not the individual quantities themselves. This layer could thus be thinner than shown in figs. 6 - 9 with an increased conductivity or vice versa.

- D is a mid-deep crustal conductor at approx. 20 km depth (figs. 8 and 9).

Although it lies at the margin of the profiles and may thus not be resolved, data of profile MVB confirm the existence of a conductive lower crust.

- E and F are the very resistive deeper sections corresponding to the Paleozoic Platform in the SW and the East European Craton in the NE. Again, E is at the margin of the profile and poorly resolved.

- H appears to be the most controversial structure in figs. 8 and 9. It may not be well resolved due to its depth and 3-D effects in the central parts of the TTZ, but note that it is modeled in all 3 different inversion schemes on both profiles (*pers. comm.* M. Smirnov, I. Varentsov). However, if the resistivities in this "anomaly" are set equal to the surrounding (i.e., 50 Ωm), the resulting rms is 1.7585 (instead of 1.7450 for the original LT7 model) or 1.2389 instead of 1.2293 at central site POM just above. This is an insignificant change and we may safely assume more normal resistivities here.

The most suspicious and immediately obvious overall appearance of the models in figs. 7 - 9 is that they resemble the picture of a subduction zone. This is not as clearly expressed in the images of P-2 and MVB as beneath LT7, but all three models indicate a rise of the moderately conductive zone associated with the upper mantle towards the NE and on top of the roots of the EEC. Although being aware that resolution deteriorates at large depths, this image may not be completely arbitrary. The idea of the TTZ being the location of an ancient subduction zone was introduced by Zielhuis & Nolet [9], based on anomalously low S-wave velocities at great depth beneath the TTZ. The ancient subduction setting of East Avalonia and Baltica together with the asthenospheric high in the domain of the Polish Trough is accordingly visualized by Mazur & Jarosinski [5]. The "anomaly" H may thus be regarded as a relict of Caledonian subduction.

5 A regional synopsis of resistivity structure

Fig.10 shows an attempt to combine the four models in a map in order to get an overview of the regional resistivity structure. The models extend to a depth of 90 km. The view is directed on the area from Northeast.

The resistive, massive block of the Precambrian Platform can be seen in all models in the foreground. On the German-Polish mainland as well as partly in the Baltic Sea, very well conducting sediments cover the surface. They seem to deepen on the MVB profile where the North German Conductivity Anomaly is expected according to the perturbation vector map view (section 3), a fact suggesting that this anomaly is just caused by the geometry of the sediment-filled North-German-Polish Basin. The other big anomaly indicated by perturbation vectors is represented on every profile by a prominent, rather compact mid-crustal conductor of only a few Ωm (letter C in figs. 6 - 9). Since their locations coincide with the edge of the Precambrian Platform, it is plausible that the development of well-conducting material, e.g. black shales, took place in the frame of the continental plate collision of Baltica and Avalonia. The southern ends of the profiles show rather heterogeneous structures,

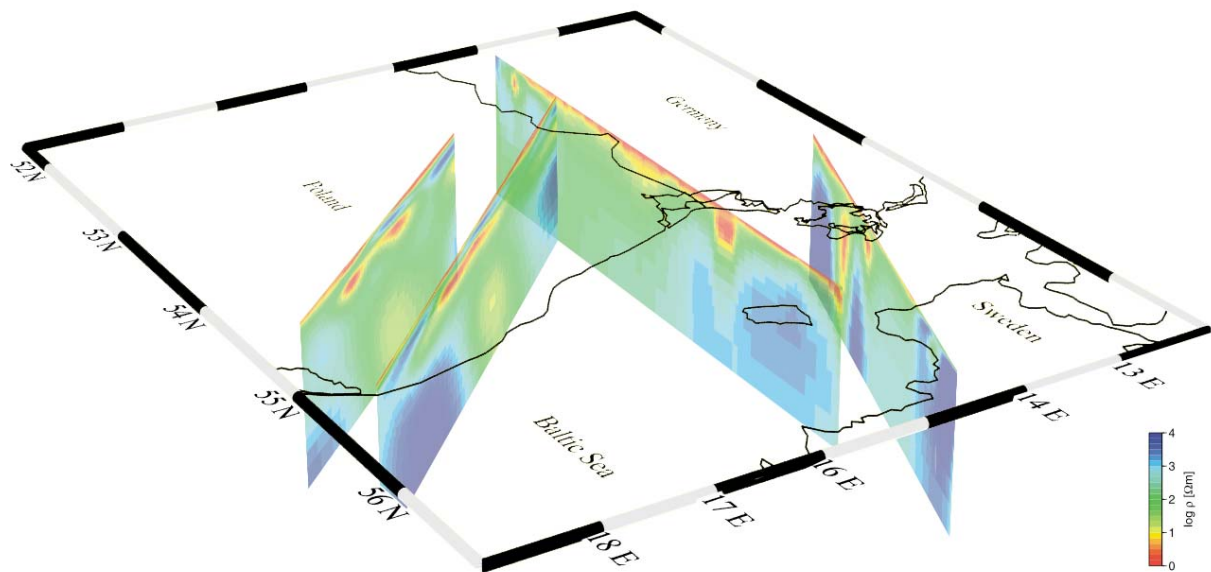


Figure 10: View from NE on four 2D resistivity models introduced in section 4 and arranged according to their geographic position. Depth extend of models 90km. The continuity of structures throughout profiles and the accord with information given by induction arrows (conf. fig.4) and perturbation vectors (fig.5) is significant.

although a rising resistivity towards the Variscan basement can be generally stated. The big differences in ρ values can have their reason in large distances between the profile ends (in case of P-2 and MVS) as well as in different assumed strike angles of the profiles although they have a common cross-over point (in case of LT7 and MVB).

Comparing figs. 1 and 10 one has to state the following: The main anomaly (C in figs. 6 - 9) coincides with the Teisseyre-Tornquist Zone on the Polish mainland, but obviously does not continue to the NW with the Sorgenfrei-Tornquist Zone (but note that good quality sites are missing in the Baltic Sea close to the Swedish coast). It bends to W at the Baltic coast, instead, thereby coinciding with the Trans-European Fault. This fact is certainly worthwhile considering in the discussion about the tectonic development of this region.

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Contact impedance of grounded and capacitive electrodes

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Abstract

The contact impedance of electrodes determines how much current can be injected into the ground for a given voltage. If the ground is very resistive, capacitive coupling may be superior to galvanic coupling. The standard equations for the impedance of capacitive electrodes assume that the halfspace is an ideal conductor. Over resistive ground at high frequencies, however, the contact impedance will depend on the electrical properties, i.e. electrical conductivity and permittivity, of the subsurface. Here, I review existing equations for the resistance of a galvanically coupled, spherical electrode in a fullspace. I extend the theory to the general case of a sphere in a spherically layered fullspace which may display both galvanic and capacitive coupling.

For a capacitively coupled electrode, the common assumption of an ideally conducting fullspace (or halfspace) breaks down if the displacement currents in the fullspace become as large as the conduction currents. For a moderately resistive medium with $1000 \Omega\text{m}$ this is the case for frequencies larger than 100 kHz. For very high resistivities around $1 \text{ M}\Omega$, the transition frequency reduces to 100 Hz. Thus, in principle, one may determine electrical resistivity and permittivity by measuring magnitude and phase of the electrode contact impedance.

Introduction

DC resistivity measurements are usually carried out with four electrodes. This way, the ratio between measured voltage and injected current is independent of the grounding resistance of the electrodes. However, calculation or estimation of the electrode resistance may be important in some situations. If the ground is very resistive, technical issues may limit the current that can be injected into the ground. When trying to decrease contact resistance, for example by watering electrodes, the exact dependence on ground resistivity or geometry is important to find an optimum strategy. Finally, the contact resistance itself might be used to obtain information on the ground resistivity (Dashevsky et al., 2005).

For galvanically coupled electrodes, equations describing the injected current as function of voltage have been derived for different electrode geometries by Krajew (1957). Capacitive electrodes normally consist of sheets close to the ground with no direct contact. They are used with an alternating current of sufficiently high frequency such that the impedance is sufficiently low. They may be particularly useful if the ground is very resistive and galvanic coupling is not feasible, or if fast measurements with a moving system are to be carried out. Kuras et al. (2006) describe the theory behind 4-point resistivity measurements with capacitive electrodes and discuss the conditions under which inductive currents may be ignored. To estimate the contact resistance of capacitive electrodes, the halfspace is normally assumed to be an ideal conductor. Over very resistive ground, however, the assumption of an ideal conductor is no longer valid, and the contact resistance of capacitive electrodes will depend on electrical conductivity and dielectric permittivity of the halfspace.

Here, I review the equations for galvanically coupled electrodes and extend the theory to capacitively coupled spheres in a fullspace. I investigate under which conditions the assumption of an ideally conducting halfspace breaks down and 2-point measurements might be feasible to determine conductivity and dielectric permittivity of the ground.

The basic setup is sketched in figure 1. A DC voltage is applied to galvanically coupled electrodes (top panel) or an AC voltage to capacitively coupled electrodes (bottom panel). The aim is to derive equations for the resistance R , required to calculate the current I from the applied voltage U via:

$$R = U / I \quad (1)$$

where R depends on resistivity for galvanic coupling, and on resistivity and electric permittivity for capacitive coupling.

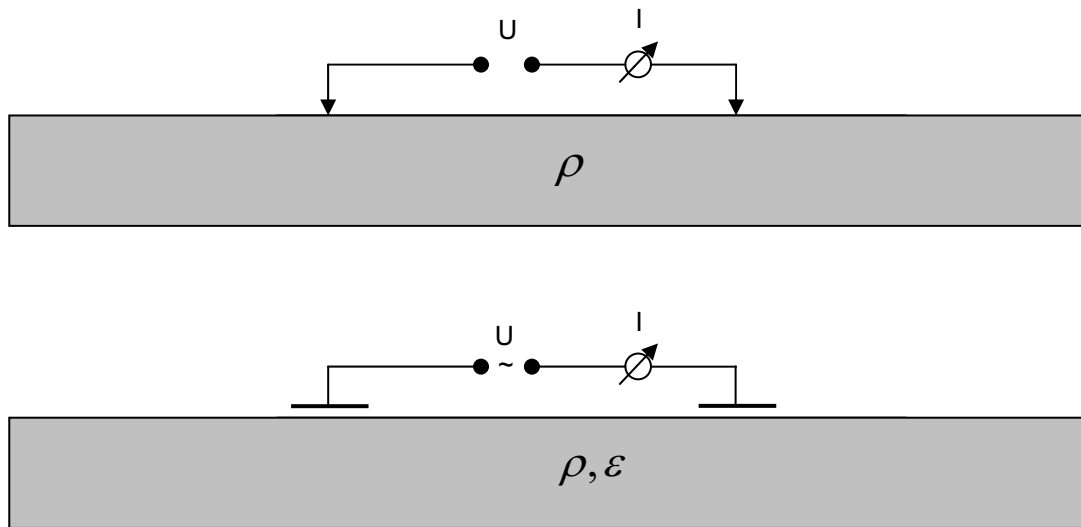


Figure 1: Sketch of the basic setup. Top panel: DC voltage applied to galvanically coupled electrodes. Bottom: AC voltage applied to capacitively coupled electrodes.

Galvanically coupled spherical electrode in fullspace

The calculation of the resistance of arbitrary electrodes over a halfspace depends on the shape of the electrodes and requires numerical solution. Therefore, I simplify the problem by considering spherical electrodes in a fullspace. This strongly deviates from the situation sketched in figure 1, but in order to obtain physical insight, simple analytic equations are desired. The equation for the contact resistance of a single galvanically coupled spherical electrode in a fullspace was given by Krajew (1957):

$$R = \frac{\rho}{4\pi r_0} \quad (2)$$

where ρ is the resistivity of the fullspace and r_0 is the radius of the sphere. One important implication is that the resistance is inversely proportional to the radius, and not to the surface of the electrode. This will apply to other types of electrodes as well, in a sense that the spatial dimension of the electrode enters linearly into the resistance. The linear dependence might be counterintuitive, because one could expect the resistance to decrease with the surface area of the sphere. The important point is that the electric field at the surface of the sphere decreases

with $1/r_0$, which compensates one spatial dimension, as can be seen from the derivation in appendix 1.

Another useful assumption is that the distance between the two electrodes is large compared to the size of the electrodes. In that case, each electrode may be treated independently. The distance between the electrodes drops out of the equations and the total resistance will simply be the sum of the two single electrode resistances (Krajew, 1957).

Equation (2) can easily be extended to the situation where the electrode is surrounded by spherical shells. The parameters for the case of two spheres, which will be sufficient to describe most of the practical situations, are defined in figure 2:

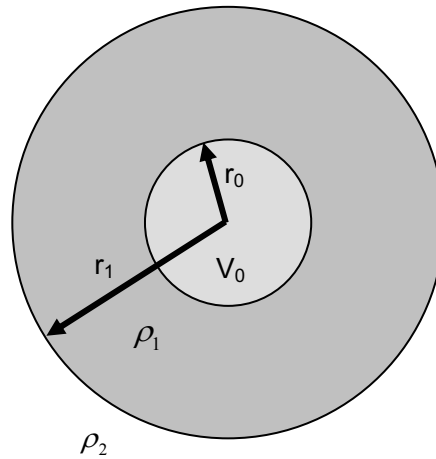


Figure 2: Geometry of a spherical electrode, radius r_0 with potential V_0 , surrounded by a spherical shell with radius r_1 and resistivity ρ_1 , in the fullspace with resistivity ρ_2 .

The resistance of the spherical electrode is given by:

$$R = \frac{\rho_1}{4\pi r_0} \left(\frac{1 - \frac{\rho_1}{\rho_2} + \frac{\rho_1}{\rho_2} \frac{r_1}{r_0}}{\frac{\rho_1}{\rho_2} \frac{r_1}{r_0}} \right) = \frac{\rho_2}{4\pi r_0} \left(\frac{1 - \frac{\rho_1}{\rho_2} + \frac{\rho_1}{\rho_2} \frac{r_1}{r_0}}{\frac{r_1}{r_0}} \right) \quad (3)$$

A derivation slightly deviating from that of Krajew (1957) is given in Appendix 1.

Equation (3) may be used in different forms to study the dependence of resistance on the resistivity distribution of the volume surrounding the electrodes. It is common practice to decrease contact resistance by pouring water into the ground near the electrode, and we may estimate the amounts of water and the resistivity contrast which is required to achieve a certain reduction in resistance. We assume that the water fills a spherical shell of radius r_1 and reduces the resistivity to ρ_1 compared to ρ_2 of the undisturbed formation. The decrease of contact resistance is then expressed as

$$\frac{R}{R_0} = \left(\frac{1 - \frac{\rho_1}{\rho_2} + \frac{\rho_1}{\rho_2} \frac{r_1}{r_0}}{\frac{r_1}{r_0}} \right) \quad (4)$$

where $R_0 = \frac{\rho_2}{4\pi r_0}$ (5)

denotes the resistance in a fullspace with resistivity ρ_2 , which would exist if no watering was applied.

Figure 3 illustrates the reduction of electrode resistance by a conductive spherical shell surrounding the electrode. The resistance quickly decreases with the size of the conductive shell, but for radii larger than 10 times the electrode size, a further increase is not efficient any more. The behavior with respect to resistivity contrast is similar: Once a reasonable resistivity contrast of 1:10 is reached, a further increase does not lead to a significant decrease of resistance.

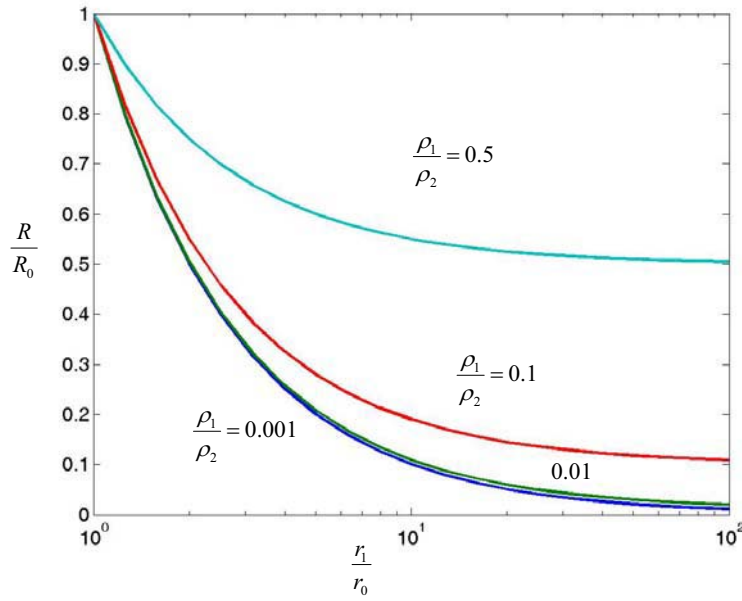


Figure 3: Reduction of electrode resistance as function of radius of the conductive shell for different resistivity ratios between outer fullspace and conductive shell. Note the logarithmic radius axis.

Capacitively coupled sphere

For the capacitively coupled sphere, it is useful to use electrical conductivity instead of resistivity. We may use the same equations derived for the static case if we replace the electrical conductivity σ by a complex conductivity defined by:

$$\sigma^* = \sigma + i\omega\varepsilon \quad (6)$$

where ε is the dielectric permittivity. This substitution is justified in detail in Appendix 2. One assumption which is not expanded on here is that induction effects may be ignored. This aspect was discussed in some detail by Kuras et al. (2006). The complex electrode impedance is obtained by rewriting eq(3) with the substitution defined in eq. (6):

$$Z = \frac{1}{4\pi r_0 \sigma_1^*} \left(\frac{1 - \frac{\sigma_2^*}{\sigma_1^*} + \frac{\sigma_2^* r_1}{\sigma_1^* r_0}}{\frac{\sigma_2^* r_1}{\sigma_1^* r_0}} \right) \quad (7)$$

A capacitively coupled electrode may be studied by setting conductivity and relative dielectric permittivity in the inner shell to the values of air ($\sigma_1=0$, $\varepsilon_{r1}=1$). If the fullspace surrounding the electrode is sufficiently conductive, the common ideal conductor assumption will hold, and the resistance will not depend on the electrical parameters of the fullspace. This can be seen by writing eq. (7) in the limit $\sigma \rightarrow \infty$:

$$Z = \frac{r_1 - r_0}{i\omega\varepsilon_0 4\pi r_0 r_1} \quad (8)$$

which may be compared with the impedance of a plate over an ideally conductive halfspace:

$$Z = \frac{d}{i\omega\varepsilon_0 A} \quad (9)$$

where d is the distance between the halfspace and the plate, and A is the area of the plate. Obviously, the thickness of the inner shell ($r_1 - r_0$) corresponds to d , and $4\pi r_0 r_1$ corresponds to the area A .

However, for a resistive fullspace, this approximation will not be valid any more. The transition is illustrated in figure 4, which shows the resistance for a spherical capacitive electrode with 1mm separation between electrode and fullspace, calculated from eq. (7). The curve for $\sigma_2=1$ S/m represents the ideally conducting fullspace. The resistance follows a $1/\omega$ frequency dependence over the entire frequency range, and does not depend on conductivity or permittivity of the fullspace. The upper limit is set by the curve for very low conductivities ($\sigma_2=10^{-12}$ S/m) which represents a spherical electrode in the air.

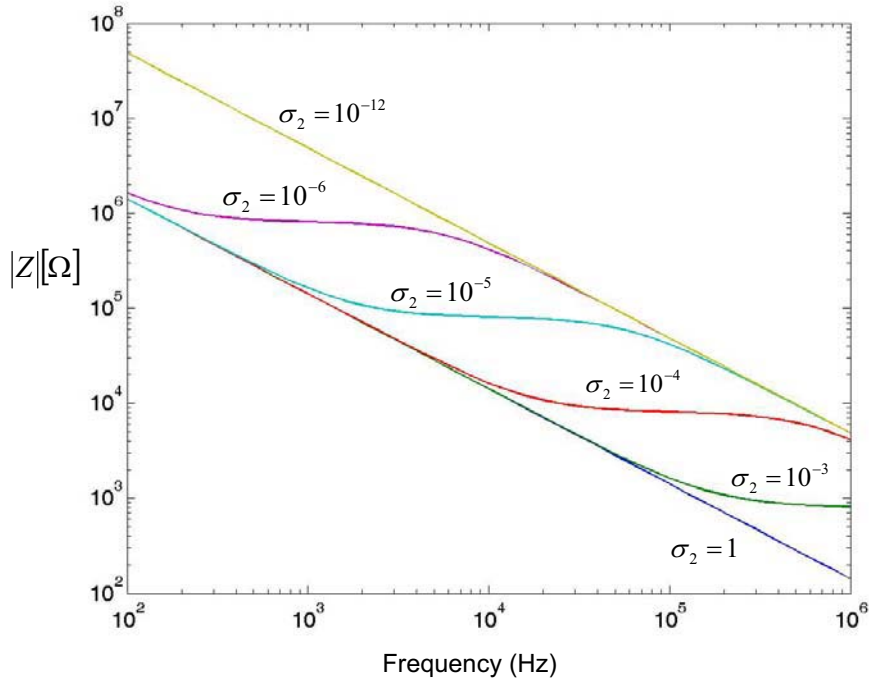


Figure 4: Amplitude of the complex impedance as function of frequency for different electrical conductivities (in S/m) of the fullspace. The radius of the spherical electrode is $r_0=0.1m$, the shell between the fullspace and the electrode is 1 mm thick ($r_1-r_0=0.001m$), and the relative permittivity of the fullspace is $\varepsilon_r=3$.

If the fullspace is moderately resistive (i.e. $\sigma_2=10^{-3} \text{ S/m}$), the electrode resistance starts to deviate from the ideal conductor limit at approx. 100 kHz . If the fullspace is very resistive (i.e. $\sigma_2=10^{-6} \text{ S/m}$), the transition starts at relatively low frequencies around 100 Hz . Of course, the transition frequency corresponds to the point where displacement currents start to become as large as conduction currents. Thus, if a capacitive electrode system is used over permafrost areas, over very dry rock, or on space missions landing on asteroids or comets, the ideal conductor equations will break down.

Figure 5 illustrates the behavior of the phase of the impedance. In the limit of an infinitely conductive or resistive fullspace ($\sigma_2=1$ or 10^{-12} S/m), the impedance behaves like that of an ideal capacitor, and the phase is -90 degrees. For finite fullspace conductivities, the phase will be sensitive to variations in conductivity (and permittivity, not illustrated), which may in principle be used to determine those parameters. Measuring amplitude and phase of the injected current related to the source voltage gives two equations which are required to solve for the two unknowns σ and ϵ_r . In practice, however, the additional dependence on the distance between electrode and fullspace or halfspace, and capacitive coupling between cables and the measuring device may create difficulties. Dashevsky et al., (2005) suggested to measure the difference of the impedance for two different heights in order to remove coupling effects, and used this approach to evaluate pavement quality.

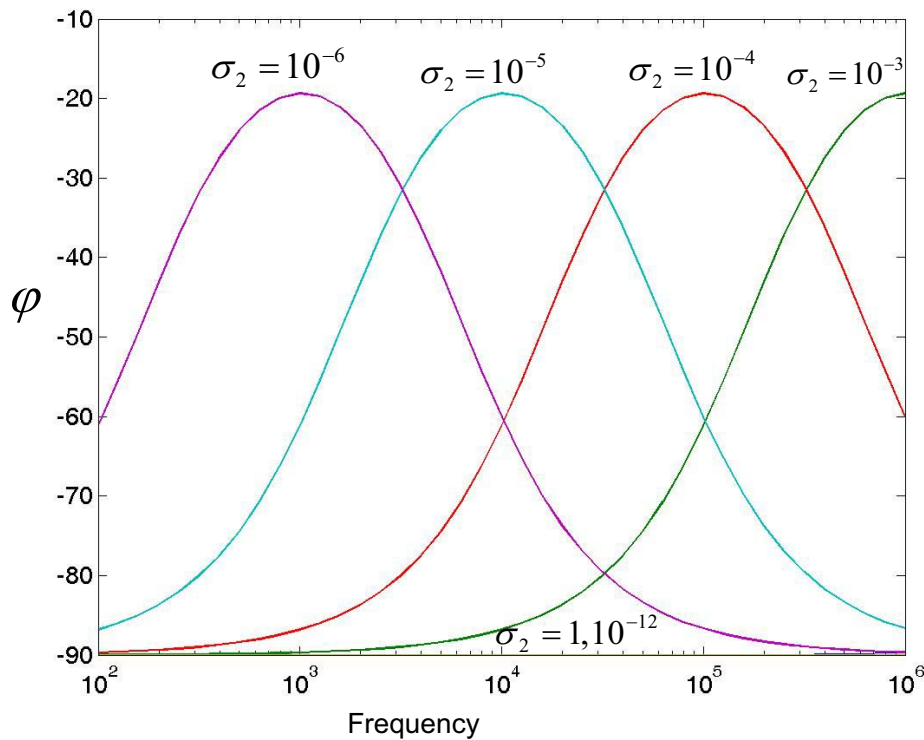


Figure 5: Phase in (degrees) of the contact impedance of a capacitive electrodes for different conductivities in S/m of the spherical fullspace. Parameters are the same as in figure 3.

Conclusions

For a single, galvanically coupled sphere, the resistance decreases with the radius of the sphere, and not, as one might expect, with the area of the sphere. Thus, if in practice the contact area is increased by using many metal sticks in parallel, the decrease of resistance will be proportional only to the square root of the number of sticks. When reducing contact

resistance by watering, there is a saturation effect with respect to both resistivity contrast and volume. Below a resistivity contrasts of 0.1 between water and undisturbed ground, the resistance does not further decrease. Thus, there is no point using excessive amounts of salt to create extremely conductive water.

For capacitively coupled electrodes, the common assumption of an ideal conductor breaks down for resistive ground and high frequencies. Depending on electrode size and geometry, the electrode impedance may be underestimated by two orders of magnitude if the finite conductivity is neglected. In principle, two-point measurements to determine electrical parameters of the subsurface with capacitive electrodes are feasible. However, the penetration depth of such measurements is only in the order of the size of the electrodes. Moreover, distortion effects by capacitive coupling between cables and the measuring device have to be carefully controlled.

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Appendix 1: Derivation of the electrode resistance for the spherical shell model.

A1.1 Spherical electrode in a homogeneous fullspace

We use the geometry sketched in figure 2. We assume a constant potential V_0 on the spherical electrode with radius r_0 . At any distance r from the center of the electrode, the potential for $r > r_0$ must follow:

$$V(r) = V_0 \frac{r_0}{r} \quad (\text{A1})$$

because from potential theory it will decay with $1/r$, and $V(r_0) = V_0$ has to be fulfilled. Therefore, the electric field at r is:

$$E(r) = -\frac{\partial V}{\partial r} = V_0 \frac{r_0}{r^2} \quad (\text{A2})$$

and in particular:

$$E(r_0) = \frac{V_0}{r_0} \quad (\text{A3})$$

This allows us to calculate the current density at the surface of the electrode and the total current by integrating over the area of the sphere:

$$j = \frac{E}{\rho} = \frac{V_0}{\rho r_0} \quad (\text{A4})$$

and

$$I = 4\pi r_0^2 j = 4\pi \frac{V_0 r_0}{\rho} \quad (\text{A5})$$

Finally, we obtain the electrode resistance from the ratio between potential and current:

$$R = \frac{V_0}{I} = \frac{\rho}{4\pi r_0} \quad (\text{A6})$$

which is equal to eq. (2).

A1.2 Spherical electrode within a spherical shell in a fullspace

In order to fulfil Laplace's equation for the potential, in the outer fullspace ($r > r_I$) it must follow:

$$V(r) = \frac{b}{r} \quad (\text{A7})$$

where b is a yet unknown constant to be determined from the boundary conditions. Within the inner shell ($r_0 < r < r_I$) we use the form:

$$V(r) = V_0 \frac{r_0}{r} + a \frac{r - r_0}{r} \quad (\text{A8})$$

Obviously, this form of $V(r)$ fulfils Laplace's equation, and the constant a must be determined from the boundary conditions. At the edge outer of the inner shell ($r = r_I$), the two potentials must be equal:

$$V(r_I) = V_0 \frac{r_0}{r_I} + a \frac{r_I - r_0}{r_I} = \frac{b}{r_I} \quad (\text{A9})$$

and thus:

$$b = V_0 r_0 + (r_I - r_0)a \quad (\text{A10})$$

The second condition results from the continuity of current density at the boundary. Inside the boundary ($r < r_I$), the electric field is:

$$E_1 = -\frac{\partial V}{\partial r} = V_0 \frac{r_0}{r^2} - \frac{r_0 a}{r^2} \quad (\text{A11})$$

and outside ($r > r_I$) it is :

$$E_2 = -\frac{\partial V}{\partial r} = \frac{b}{r^2}$$

Continuity of current density at $r=r_l$ requires that

$$\frac{E_1}{\rho_1} = \frac{E_2}{\rho_2} \quad (\text{A13})$$

and thus

$$\frac{1}{\rho_1} \left(V_0 \frac{r_0}{r_1^2} - \frac{r_0 a}{r_1^2} \right) = \frac{1}{\rho_2} \frac{b}{r_1^2} \quad (\text{A14})$$

We now have two equations (A10 and A14) for the two unknowns a and b , and we obtain

$$a = V_0 \frac{1 - \frac{\rho_1}{\rho_2}}{1 - \frac{\rho_1}{\rho_2} + \frac{\rho_1 r_1}{\rho_2 r_0}} \quad (\text{A15})$$

The solution allows us to calculate the current density, which may be expressed as:

$$j = j_0 \frac{\frac{\rho_1}{\rho_2} \frac{r_1}{r_0}}{1 - \frac{\rho_1}{\rho_2} + \frac{\rho_1}{\rho_2} \frac{r_1}{r_0}} \quad (\text{A16})$$

where

$$j_0 = \frac{V_0}{\rho_1 r_0} \quad (\text{A17})$$

is the current density of the sphere in a fullspace without a spherical shell. We finally obtain the resistance in the form given in equation (3) through

$$R = \frac{V_0}{I} = \frac{V_0}{4\pi r_0^2 j} \quad (\text{A18})$$

Appendix 2: Derivation of the potential equation in the complex case

Ampere's law states that:

$$\text{rot} \underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t} \quad (\text{A19})$$

where $\underline{H}$ is the magnetic field, $\underline{j}$ is current density and $\underline{D}$ is the electric displacement. By taking the divergence, we obtain:

$$\text{div} \left(\underline{j} + \frac{\partial \underline{D}}{\partial t} \right) = 0 \quad (\text{A20})$$

With

$$\underline{j} = \sigma \underline{E} \quad (\text{A21})$$

and transformation to the frequency domain, such that the time derivative becomes a multiplication with $i\omega$, we get:

$$\text{div}(\sigma \underline{E} + i\omega \varepsilon \underline{E}) = 0 \quad (\text{A22})$$

If we introduce the complex conductivity

$$\sigma^* = \sigma + i\omega \varepsilon \quad (\text{A23})$$

(A22) writes:

$$\text{div}(\sigma^* \underline{E}) = 0 \quad (\text{A24})$$

Finally, Faraday's law states that

$$\text{rot} \underline{E} = -\frac{\partial \underline{B}}{\partial t} \quad (\text{A25})$$

If induction effects can be ignored, then

$$\text{rot} \underline{E} = 0 \quad (\text{A26})$$

and the electric field may be obtained from a scalar potential V :

$$\underline{E} = -\text{grad} V \quad (\text{A27})$$

We thus obtain the basic equation for V

$$\text{div}(\sigma^* \text{grad} V) = 0 \quad (\text{A28})$$

which is the basis for the derivation of the electrode resistances. It is identical to the equation used in the static case, the only difference being that it is complex and the DC conductivity was replaced as defined in eq. (A23). Thus, all arguments apply for the complex case as well, and eq. (3) may directly be transferred into eq. (7).

TEM on Lake Holzmaar, Eifel

A feasibility study

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Abstract

We developed a floating TEM setup with a transmitter size of $18 \times 18 \text{ m}^2$. Due to its modular design it can be handled by two operators and is easy to transport. As lacustrine sediments in maar lakes provide paleoclimatic proxy data, an extensive TEM survey at Lake Holzmaar (Eifel) was carried out to investigate the sediment thickness. The data collected can be explained well by means of three dimensional Finite Element Modelling.

Introduction

Lacustrine sediments provide excellent paleoclimatic proxy data. Depending on the sediment rates, which vary between 0.1 and 6 mm a^{-1} , information about the last 200,000 years can be obtained with a yearly resolution. Lake sediments are mainly archives on paleoclimatic fluctuations, geomagnetic field variations and volcanic activities. Even human impact on soil erosion or heavy metal accumulation is preserved (Zolitschka [1998]).

Maar lakes act as superior sediment archives as they possess deep and undisturbed bodies of water with reducing conditions at great depth. Steep slopes and plane lake bottoms help to build up laminated sediments by steady deposition in a

non-turbulent environment. Drilling cores of several european dry maars (silted up former lakes) and maar lakes have been recovered up to a depth of 200 m. They can be correlated and absolutely dated based on warve counting and radiocarbon dating. To find the most promising sites and drilling locations, precoring seismic and geological surveys have to be carried out (Negendank and Zolitschka [1993a]).

It will be shown that sediment thickness can be estimated by applying electromagnetic methods on the lake's surface. Therefore, we designed a floating TEM setup which is combined with the existing units of Zonge Engineering and Research Organisation Inc. An extensive survey at Lake Holzmaar, Eifel, was carried out. Along two sections data was obtained at 16 sites across the water surface area.

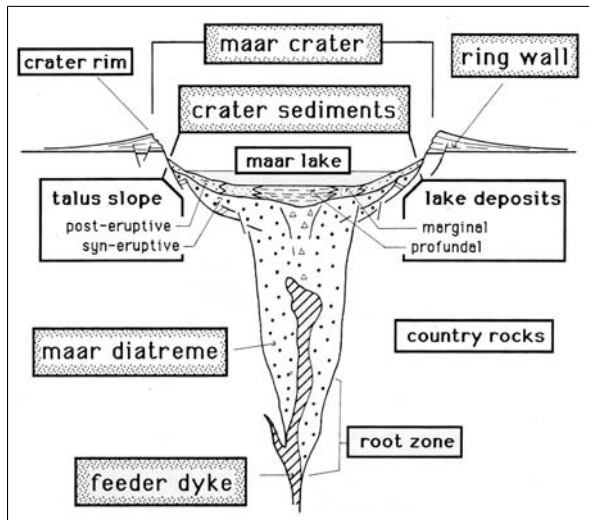


Figure 1: Schematic plot of a maar showing crater, ring wall, crater sediments and diatreme. [Büchel [1993]]

Geological background

Maars are volcanic craters with up to 2 km in diameter. According to Noll [1967] maars are formed by strong hydrothermal eruptions. The uprising melt reacts with confined groundwater and fractures the host rock. As the overburden collapses and pressure is released, the superheated water evaporates and blows out the fractured material. The eruption process, which lasts several hours or days, results in a bowl-like depression cut into the surrounding host rock. Most of the maar tephra is transported up to several hundreds of kilometres. Only a small part builds up a confining ring wall as shown in figure 1. During the post-eruptive processes a maar lake will form for the groundwater level is undercut by the crater. Mass movements start to fill up the crater and decrease the inclination of its walls (Büchel [1993]). At its last stage a

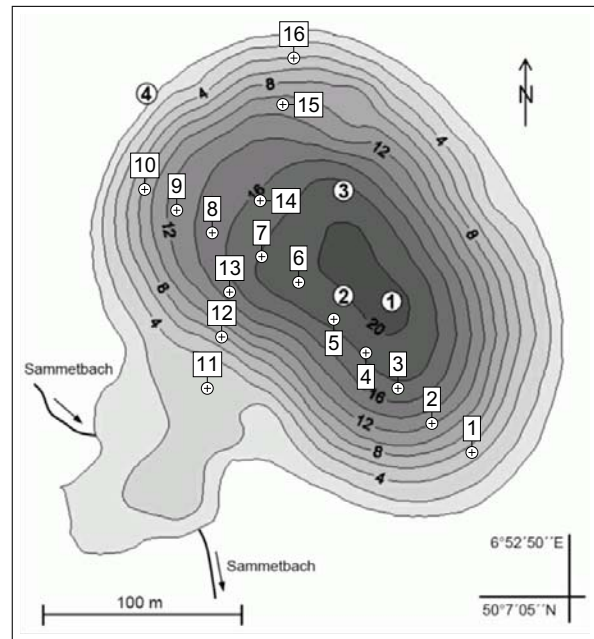


Figure 2: Depth contours [m] and receiver sites [1] to [16]. [modified according to Moschen [2004]]

maar will only show the resistant diatreme filling as a carved out mountain if erosion and denudation are effective enough.

Lake Holzmaar

Lake Holzmaar, a 40.000 to 70.000 year old maar lake, is the smallest water filled maar of the Quaternary Westeifel Volcanic Field, which is situated 100 km south of Cologne. Its body of water is approximately 290 x 210 m² wide with a depth of up to 20 m, steep slopes and a plane bottom. By building a weir in the late middle ages the natural water level rose about 3 m, causing the southwestern bank to become a shallow bay. The bathymetry is shown in figure 2. The resistivity of the water is known

to be $\rho_w \approx 30 \Omega m$ derived from in-situ measurements. From the late 1980's to the mid 1990's a total of 18 cores have been recovered from Lake Holzmaar, reaching to a sediment depth of 32 m below lake bottom. They can be dated back 13.000 years establishing a varve year calender of central Europe (*Negendank and Zolitschka* [1993b]).

Floating TEM deployment

The central loop TEM method is well known and widely used in geophysical prospecting. Common transmitter sizes vary from $10 \times 10 \text{ m}^2$ in NanoTEM mode up to $400 \times 400 \text{ m}^2$ for greater exploration depth. In the recent past water-borne TEM applications have been presented (*Goldman et al.* [2004], *Barrett et al.* [2005]).

Our aim was to develop a floating TEM deployment to work with the existing devices of Zonge Engineering and Research Organisation Inc. According to *Spies* [1989] the depth of investigation z_{max} (at a given resistivity σ) depends on the transmitter moment

$$z_{max} \approx 0.55 \left(\frac{I A}{\sigma \eta_\nu} \right)^{\frac{1}{5}}$$

where I is transmitted current, A the transmitter area and η_ν the noise level. As the transmitted current is limited to 18 Ampère by the Zonge unit, one has to maximize the transmitter size. This can be easily done land-borne, but floating device has to be mechanical rigid stable to avoid errors caused by changing transmitter-receiver geometry.

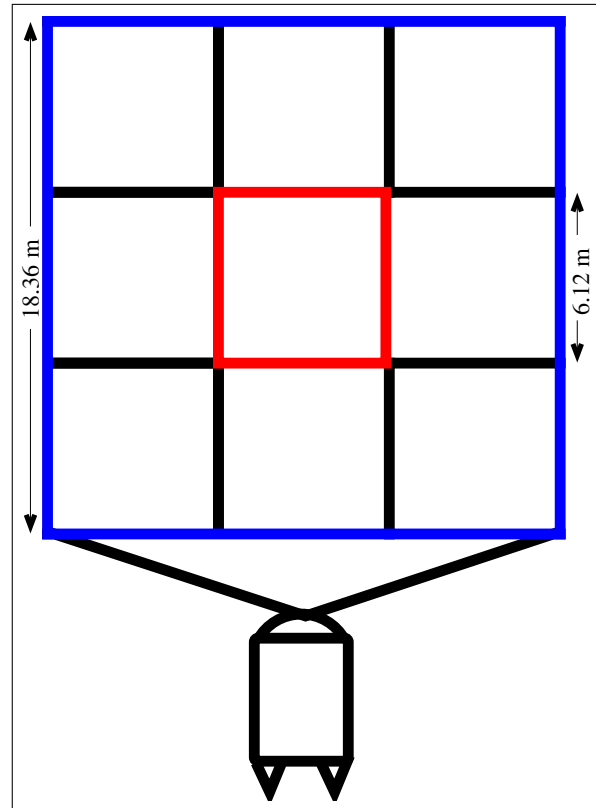


Figure 3: Floating TEM deployment.

The design presented consist of about 100 standard PVC tubes which can be combined in a modular way. Since the PVC tubes are only 2 m long, they can be put together and detached easily. The entire setup is reusable and can be transported in a van.

The deployment consists of 9 squares ($6 \times 6 \text{ m}^2$ each), which add up to a chessboard-like framework of $18 \times 18 \text{ m}^2$ as shown in figure 3. The transmitter loop consists of an insulated wire inside the outer edges of the framework. The receiver antenna is placed inside the edges of the innermost $6 \times 6 \text{ m}^2$ square. With 4 turns a total receiver moment of 144 m^2 is achieved.

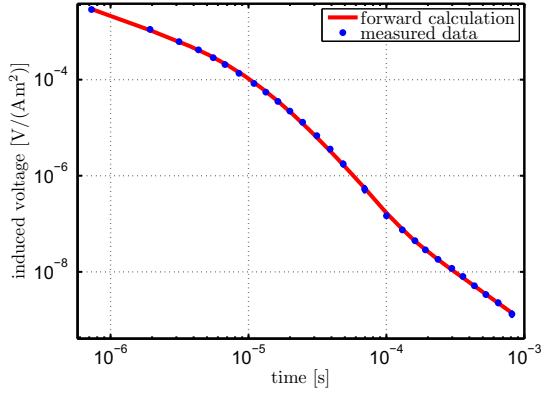


Figure 4: Data from reference site.

Transmitter and receiver devices, batteries and equipment like GPS receivers are stored in the towing boat. The entire deployment can be set up within a few hours and is operated by two people.

Survey at Lake Holzmaar

As Lake Holzmaar is a nature reserve, any kind of combustion engine is strictly forbidden. We used an electric outboard motor to maneuver on Lake Holzmaar. At prior measurements it was figured out that the electromagnetic noise produced by the motor will impair the data quality significantly. Due to strong winds, ropes had to be drawn across the lake to avoid drifting while collecting data.

Two profiles with a whole of 16 sites have been investigated as shown in figure 2. In addition, data at a reference site 250 m outside Lake Holzmaar was collected.

As shown in figure 4 the data recorded at the reference site can be explained well by 1-D Marquardt-Levenberg inversion resulting

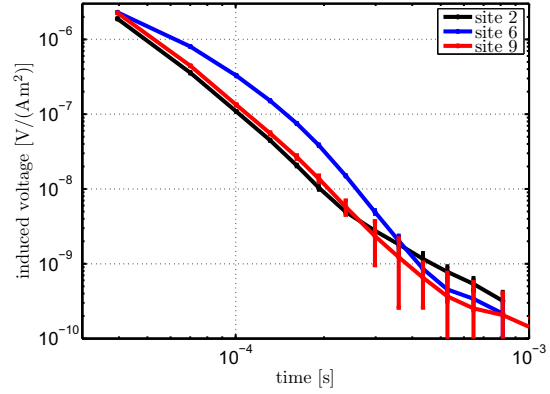


Figure 5: Data of profile 1.

#	resistivity [Ωm]	thickness [m]
1	1016	12
2	46	18
3	1877	112
4	64	∞

Table 1: Model at reference site.

in a model with four layers as shown in table 1. The resistivity distribution agrees with the geology assumed. Pyroclastic eruptive rocks form a highly resistive overlaying strata. The second layer might be a ground-water aquifer since the first boundary in a depth of 12 m corresponds to the maar lake's water level. Deeper layers consist of greywacke and shales (*Meyer [1988]*).

Figure 5 shows data from three sites of profile 1, which extends from the southeastern to the northwestern bank as shown in figure 2. Obviously the data is widely influenced by three dimensional effects. All of the transients recorded on the surface of the lake show a huge dynamic and some kind of rotational symmetry.

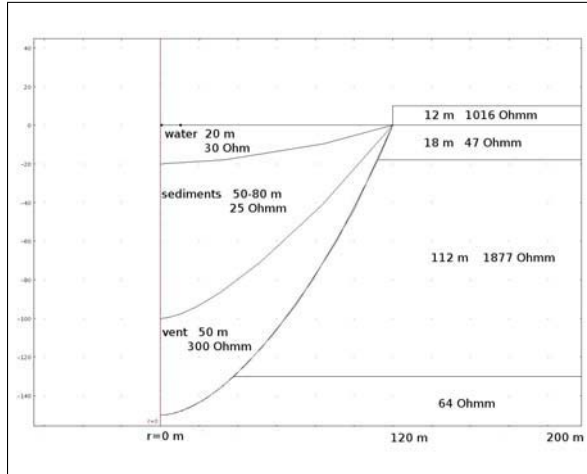


Figure 6: Simple model of a maar lake with symmetry of rotation integrated into model as shown in table 1.

To interpret the data multi-dimensional modeling is necessary. As site 6 is situated approximately in the middle of the allmost circular lake, it can be analyzed by a model using symmetry of rotation.

A simple model of a maar lake was set up with COMSOL Multiphysics as shown in figure 6 and 7. The maar consists of a vent, sediments and water. It is integrated into the layered model obtained at the reference site as shown in table 1.

The symmetry of rotation implies several conditions for the fields along the axis of rotation (z-axis):

$$\begin{aligned} B_r &= 0 \\ \frac{\partial B_z}{\partial r} &= 0 \end{aligned}$$

The fields are continuous at all interior boundaries. The model space is 3000m x 3000m wide to avoid effects of its outer boundaries where the following con-

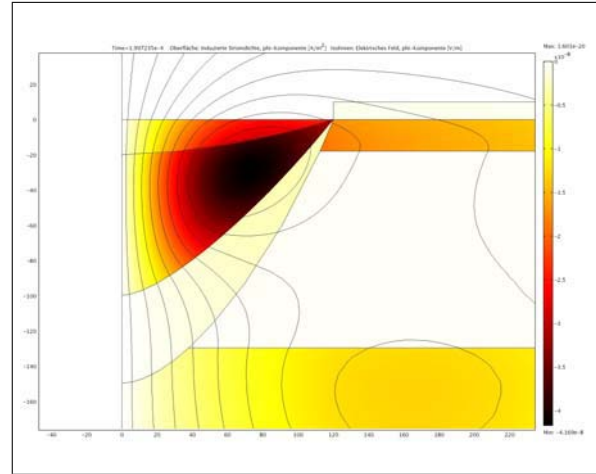


Figure 7: Induced current density (coloured, [A/m²]) and E_ϕ -component (isolines, [V/m]) at $t = 2e^{-4}s$ after switch off.

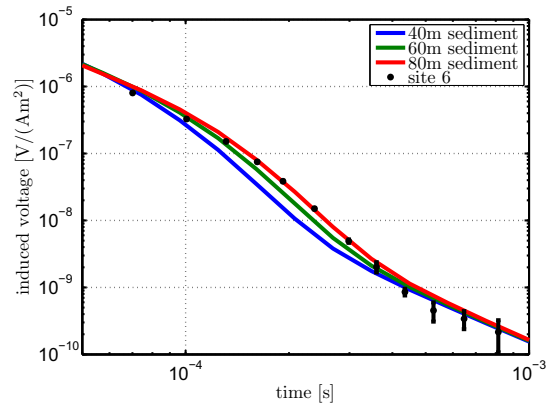


Figure 8: Effect of sediment thickness.

ditions for $\mathbf{E}$ and $\mathbf{H}$ have to be met:

$$\begin{aligned} \mathbf{n} \times \mathbf{E} &= 0 \\ \mathbf{n} \cdot \mathbf{H} &= 0 \end{aligned}$$

Forward calculations for several resistivities and sediment thickness were carried out by means of Finite Element modeling.

As shown in figure 8 the data of site 6 can be reproduced well by sediments with

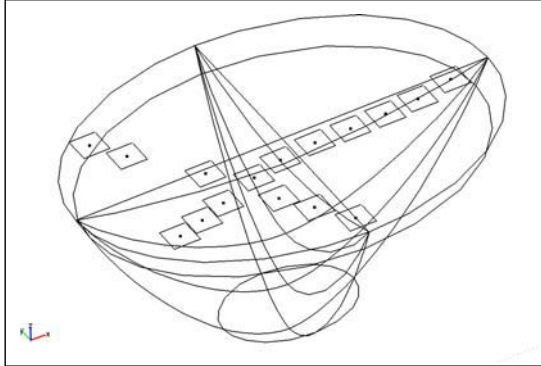


Figure 9: *Elliptic model with exact receiver sites.*

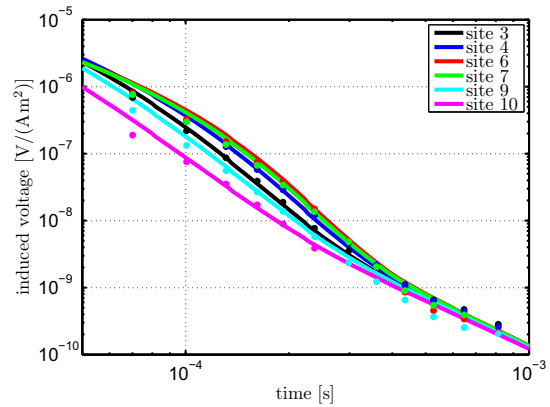


Figure 10: *Synthetic (lines) and measured data along profile 1.*

a resistivity of $25 \Omega\text{m}$ and a thickness of 80 m. These values should be considered as a guess since a lot of simplifications have been made.

The intense decay of the induced voltage between $t = 2e^{-4}\text{s}$ and $t = 6e^{-4}\text{s}$ after transmitter switch off can be explained by the induced current system being 'trapped' in the highly conductive sediments. Instead of moving down- and outwards the electric field is deformed due to the shape of the maar as shown in figure 7. The maximum of E_ϕ is caught at shallow depth until leaping into the surrounding geology causing a huge decrease of the induced voltages as seen in figure 8.

3-D modeling

By taking advantage of the rotational symmetry short computation time can be achieved. To explain the data of both profiles and illustrate the effect of the shores as shown in figure 5, 3-D model-

ing is indispensable. An elliptical model of $290 \text{ m} \times 210 \text{ m}$, which is also integrated into the layered model obtained at the reference site, allows to calculate synthetic data at the exact receiver locations as on Lake Holzmaar (see figure 9). It revealed that the systematic change in the induced voltages measured along the profiles is consistent with the simulation. The best fit for all sites was achieved with a model showing 55 m of sediments with a resistivity of $20 \Omega\text{m}$. Synthetic and measured data for several sites of profile 1 is shown in figure 10.

Conclusion

A water-borne central loop TEM system has been developed. It worked well at several test sites and allowed to carry out a survey at Lake Holzmaar. Due to the small diameter of the lake the data is strongly influenced by non 1-D effects and can only be explained with 3 dimensional modeling. The bottom of the sediments seems to be

below 70 m. As the lake's radius is just about 130 m the value is vague. Effects caused by the banks and the surrounding geology conceal the sediment thickness.

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Application of Transient Electromagnetics for the Investigation of a Geothermal Site in Tanzania

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Introduction

In 2006 and 2007 geothermal exploration field surveys were carried out in Tanzania within the framework of the GEOTHERM programme. The project partners were the Tanzanian Ministry of Energy and Minerals (MEM), the Geological Survey of Tanzania (GST), the Tanzanian Electric Supply Company Ltd. (TANESCO) and BGR on behalf of the Federal Ministry for Economic Cooperation and Development (BMZ). GEOTHERM is a programme of technical cooperation initiated by the German government. It started in 2003 and promotes the use of geothermal energy in partner countries by kicking off development at promising sites (Fig. 1). East Africa is in the major regional focus of the programme. The project activities there are supported by the African Rift Geothermal Facility (ARGeo). Geophysical methods (Transient Electromagnetics (TEM), Magnetotellurics (MT) and Geoelectrics) have been applied in order to determine the electrical conductivity of the subsurface, therewith providing information on lithology and structure of the underground. Additionally, the area was investigated geologically and geochemically and counterpart colleagues were trained “on the job” in the exploration methods applied. First results of the TEM survey are presented here.

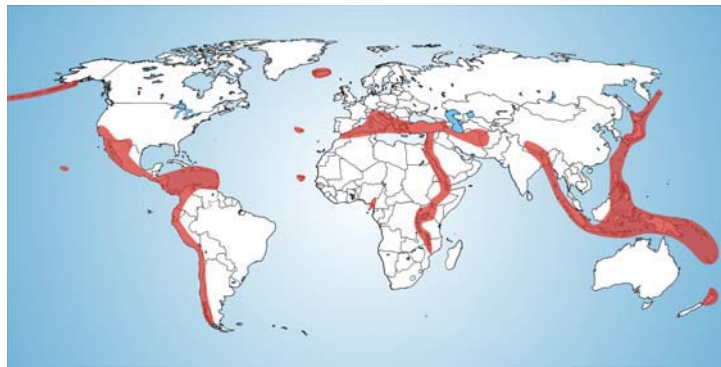


Fig. 1: The map shows the world wide distribution of areas with high geothermal potential. The GEOTHERM programme has a regional focus on East African countries which are close to the East African Rift Valley. The project countries here are Eritrea, Ethiopia, Uganda, Kenya, Tanzania and Rwanda.

1. Motivation and survey area

Electromagnetic and geoelectrical measurements provide information on the distribution of the electrical resistivity in the subsurface. In geothermal systems, electrical resistivity variations are predominantly caused by hydrothermal alteration zones. The hot fluids of a geothermal system lead to the formation of a sequence of hydrothermal alteration products depending on the temperature. At the top of a high-

enthalpy geothermal system a clay cap with expandable clay minerals uses to occur. Its resistivity is generally lower than in overlying rocks exposed to lower subsurface temperatures. Below the clay cap a higher resistive core is to be found representing the geothermal reservoir. Thus, the succession of a low resistive region (the clay cap) and a high resistive surrounding (the core below and low temperature alterations above) are somehow indicative for a geothermal reservoir (Fig. 2 and 3). Electrical resistivity methods are well suitable to detect this pattern. Fig. 4 illustrates the TEM method applied in this survey.

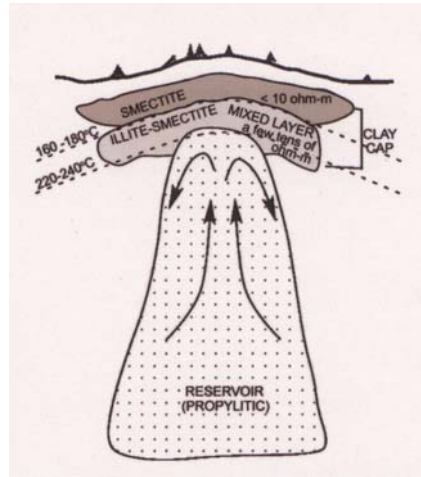


Fig. 2: Geothermal resistivity model, modified after Pellerin et al., 1996. Oskooi et al. 2005.

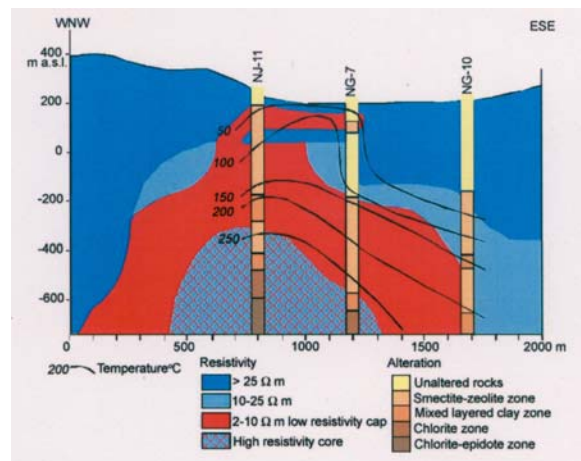


Fig. 3: Resistivity model results over a geothermal system in Iceland, after Arnason et al. 1986. Oskooi et al. 2005.

Principle of Transient ElectroMagnetic measurements

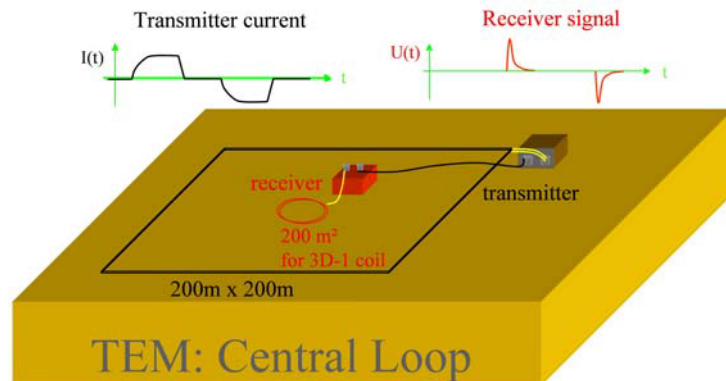


Fig. 4: Transient electromagnetic (TEM) measurements provide information on the distribution of the electrical resistivity of the subsurface. The TEM method uses transmitter and receiver coils with different sizes and configurations. A changing primary field caused by a short current impulse in the transmitter coil induces eddy currents in the ground, which are decaying with time causing electromagnetic induction in the receiver coil. The resulting decaying voltage is recorded over a certain time window; it contains information on the resistivities of the subsurface. The exploration depth depends on the local noise level, and therefore is influenced by the transmitter power. By varying coil size and time windows the TEM system is able to provide information on the resistivities at depths in the range from some metres to several hundreds of metres below surface.

The geophysical ground surveys were performed in the area of the Songwe valley and the Ngozi volcano (Fig. 5 and 6). Due to geochemical indications the area is supposed to contain a geothermal reservoir. The zone is expected to extend in north westerly direction from Ngozi volcano toward the Songwe valley. In this report only some main results of the TEM survey are shown.

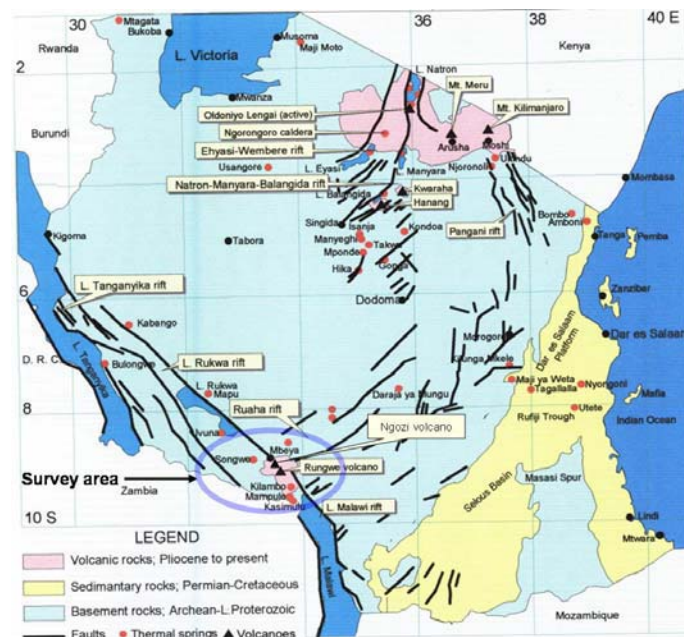


Fig. 5: Simplified geological map of Tanzania showing the major faults of the East African Rift System (EARS), hot spring locations and volcanic fields.

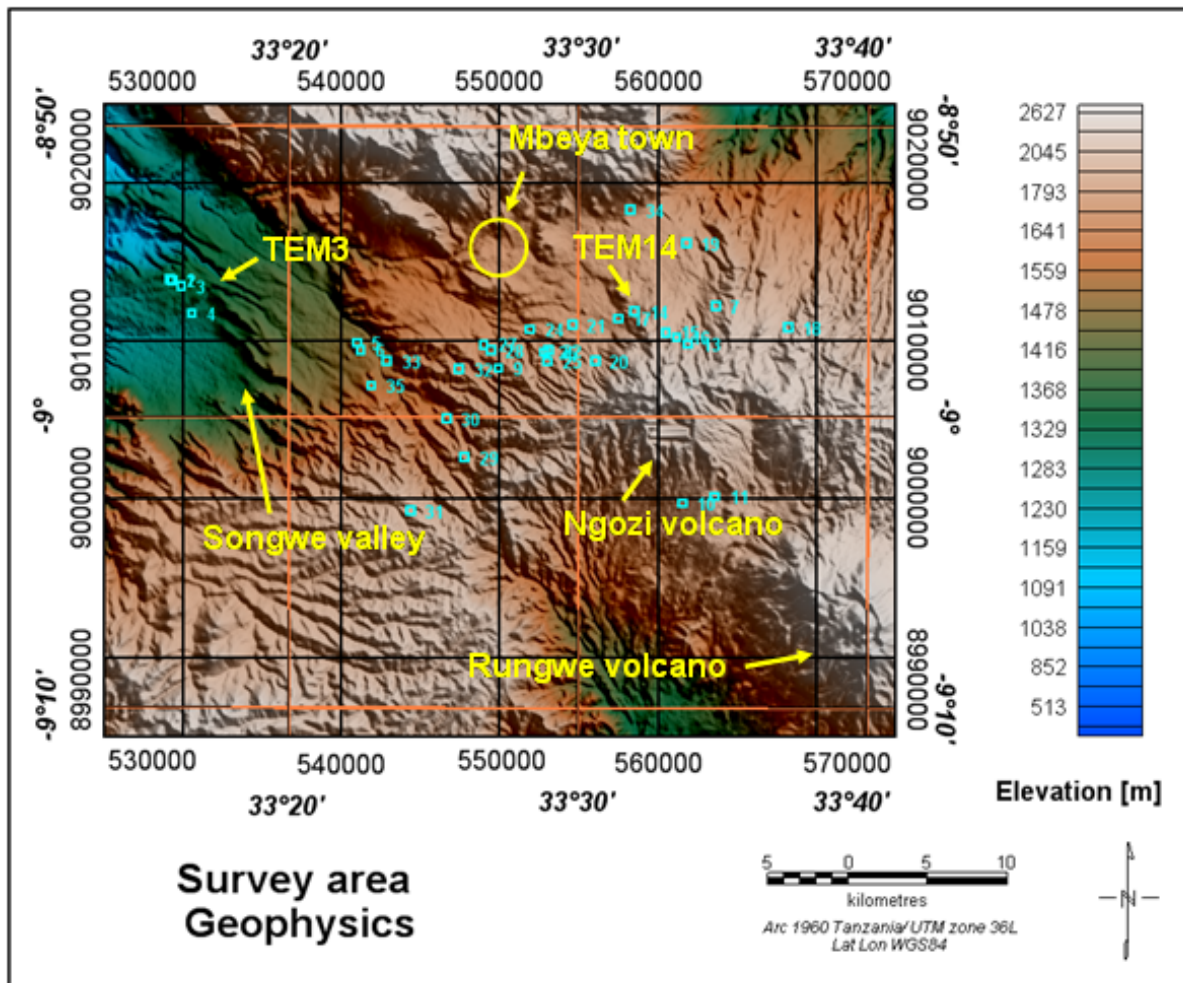


Fig. 6: Distribution of the TEM sounding sites (light blue boxes) in the Songwe valley and around Ngozi volcano.

The measurements yield apparent resistivities as function of the time after switch off of the primary current pulse. After data processing, true resistivities as a function of the depth of 1-dimensional models, consisting of horizontally infinitely extended layers are calculated. The resistivity provides information on the rock types and it also depends on their water content. It is expected, as already mentioned, that the alteration process created a low resistive clay cap which overlays a higher resistive core, the latter indicating the geothermal reservoir.

2. First results of the TEM measurements

Examples of measured (crosses) and model (continuous curves) apparent resistivities after inversion, and the respective 1D models are shown in Fig. 7, 8 and 9.

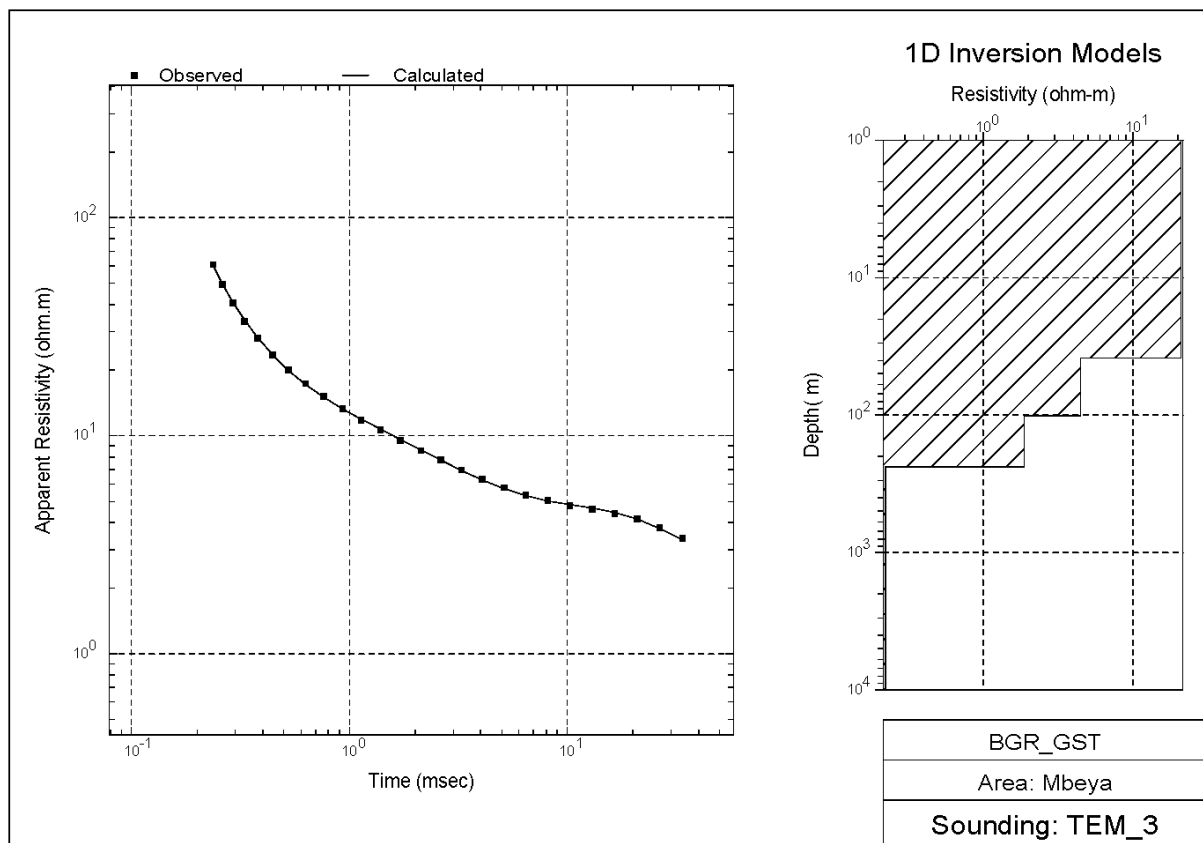


Fig. 7: Apparent resistivity curve and 1-dimensional model result at TEM site 3 in the Songwe valley.

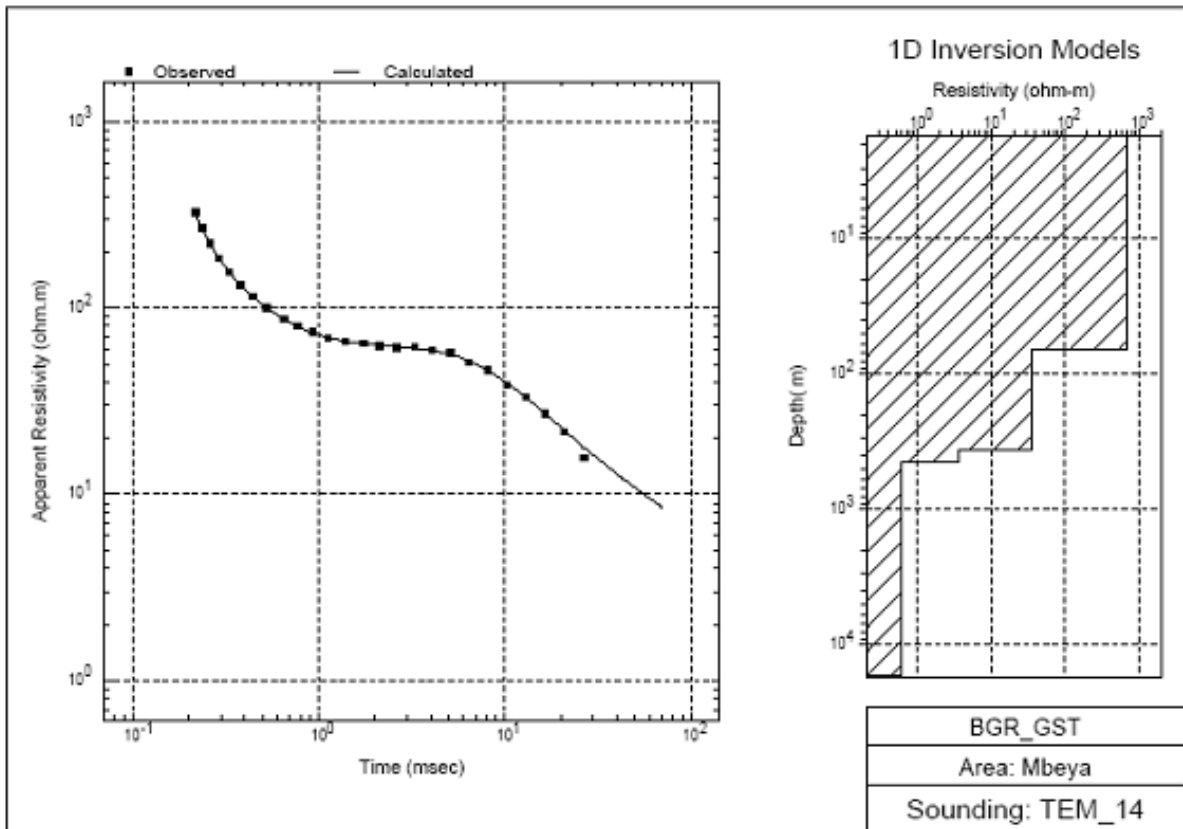
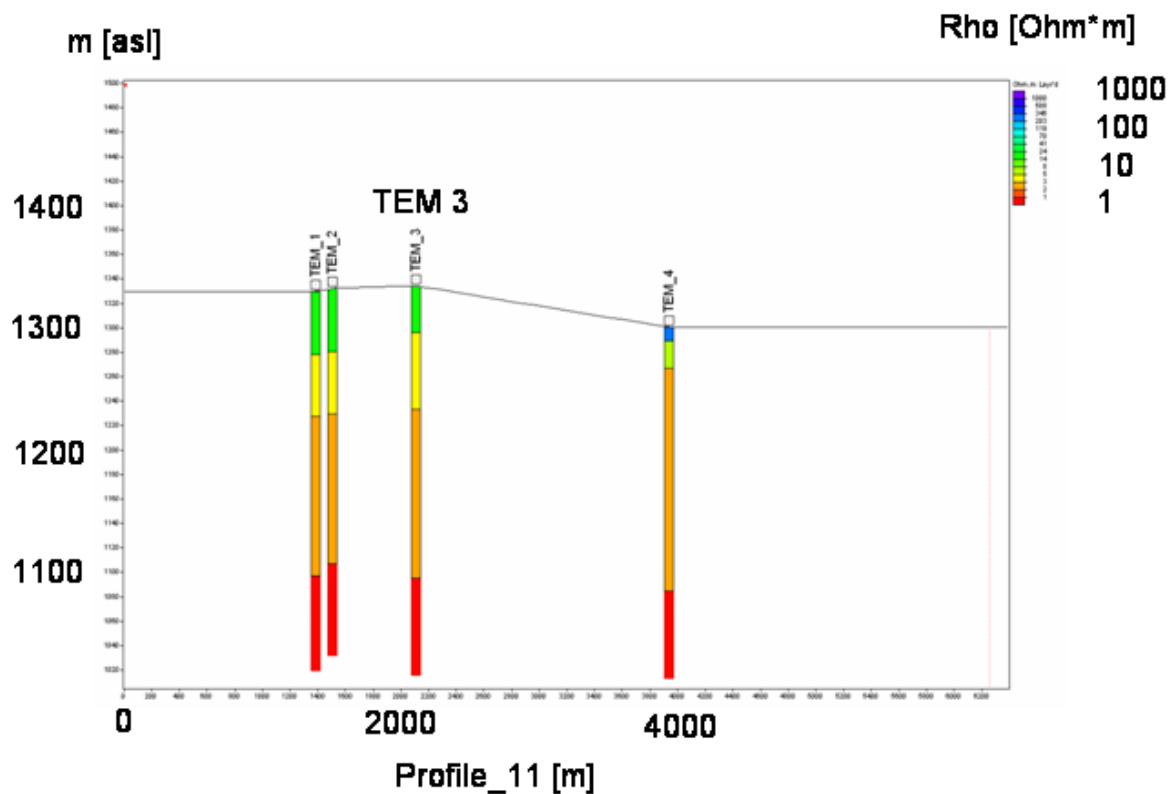
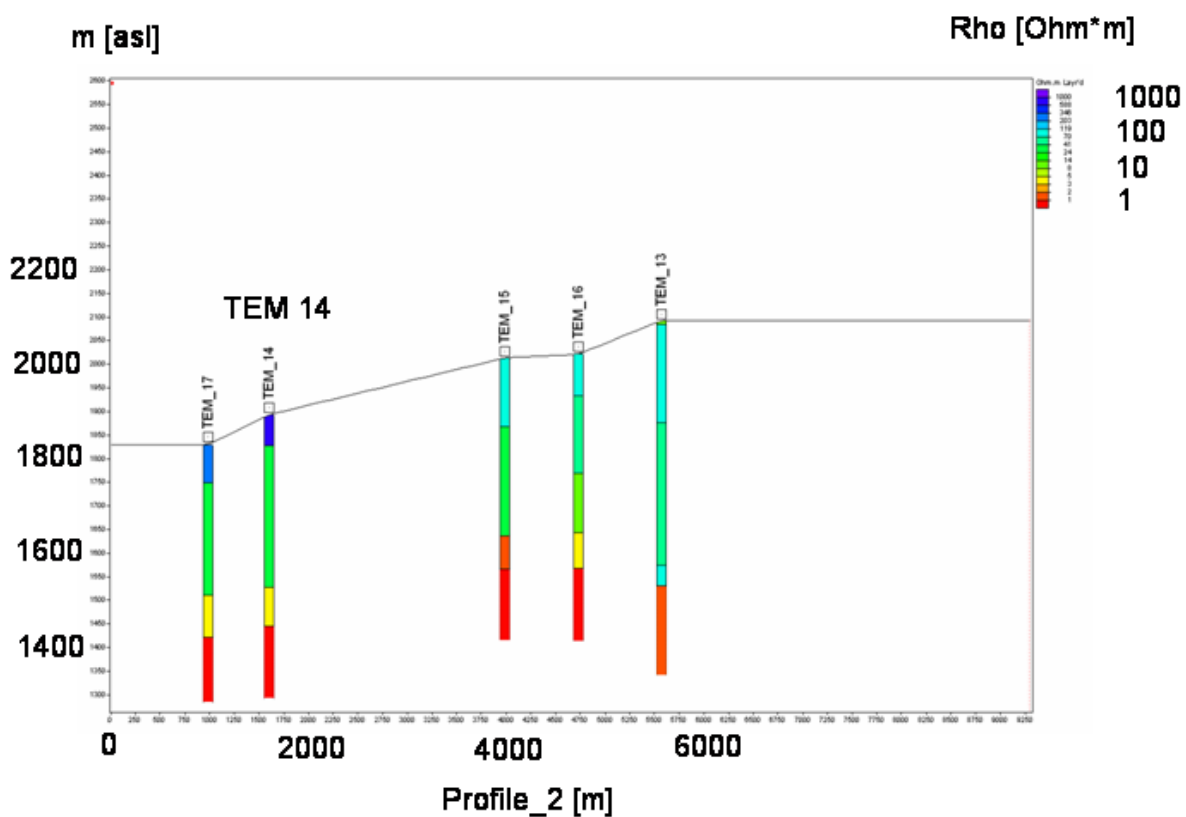


Fig. 8: Apparent resistivity curve and 1-dimensional model result at TEM site 14 north of Ngozi volcano.

Most of the apparent resistivity curves are decreasing with time. At most of the sites a very low resistive layer can be determined, e.g. at about 200 m in the Songwe valley (supposed to be alluvial sediments) at TEM site 3 (Fig. 7) and at about 500 m at TEM site 14 north of Ngozi volcano (Fig. 8). For some sites it was not possible to find a model which fits the apparent resistivities at late times. Therefore some curves were cut at the end (Fig. 8). Fig. 9 (a, b) shows vertical resistivity sections for two profiles (no. 11 and no. 2) in these areas. Both show the mentioned low resistive layer.



a)



b)

Fig. 9: 1-dimensional vertical resistivity sections from TEM data for the Songwe valley along profile 11 (a) and north resp. northwest of Ngozi volcano along profile 2 (b).

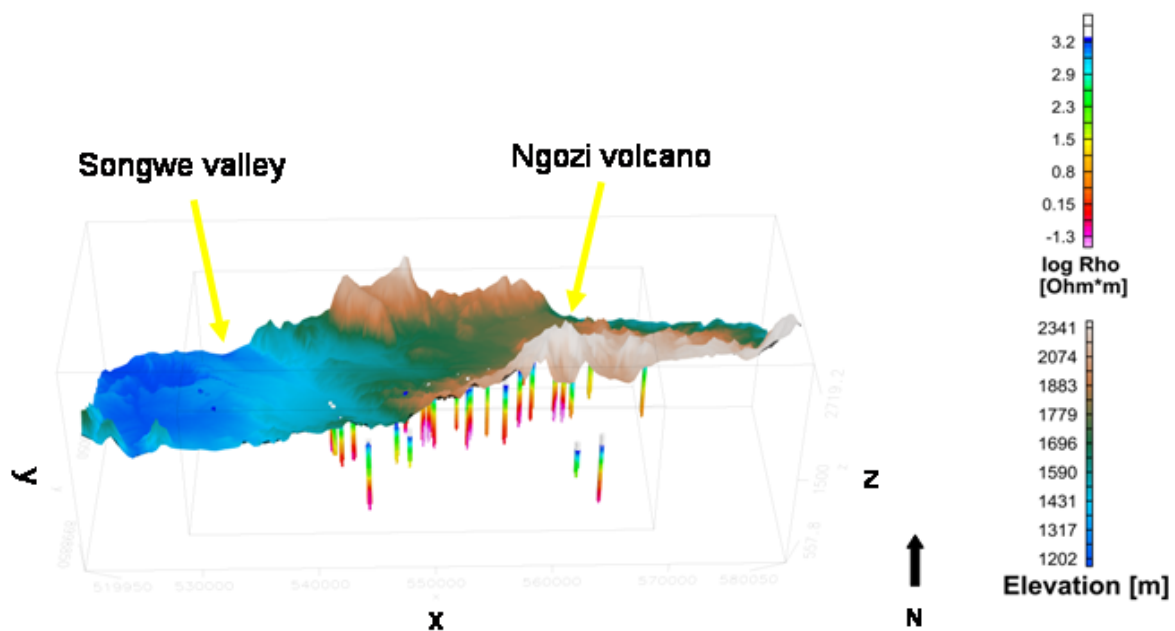


Fig. 10: 3-dimensional display of the 1D inversion results for the TEM sites in the whole survey area.

Fig. 10 shows a 3-dimensional visualization of the 1-dimensional inversion results. Volume pixels (Voxel) with certain cell sizes, depending on the desired resolution were used for it. Too big cell sizes cause interpolation into parts of the area, where no survey sites exist. Therefore a smaller cell size of 100mx100mx25m was chosen. The SRTM topography is drawn over and cut along a west easterly line crossing the Ngozi volcano. Below the volcano and at most parts of the survey area as well a low resistive region is indicated at depths of around several hundred metres. Its lateral extension cannot be determined very well because of the limited number of measurements. The limitation is mainly caused by the rugged terrain.

3. Conclusions

Most of the resistivity curves within the entire survey area show a decrease to later times, indicating lower resistivities at greater depths. This result may be indicative for the existence of a clay cap at depth, but has to be proven by the results of MT measurements, which simultaneously have been carried out in the same area. The horizontal expansion of the low resistive structure, found by the TEM, is not very clearly determined; the 3-dimensional display has also to be treated with caution because of possible interpolation artefacts, which may fill many parts of the survey area, where no data exist.

Many of the MT sites also coincide with the TEM sites. For extending the 1D model to greater depth a combination of the results of the two geophysical surveys is intended. The interpretation of the MT survey, however, is still unfinished at the moment. The geochemical and geological results too have to be considered for the interpretation of the low resistive layers. These results also are still in preparation.

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A TEM survey for exploring a hot water aquifer in South Chile

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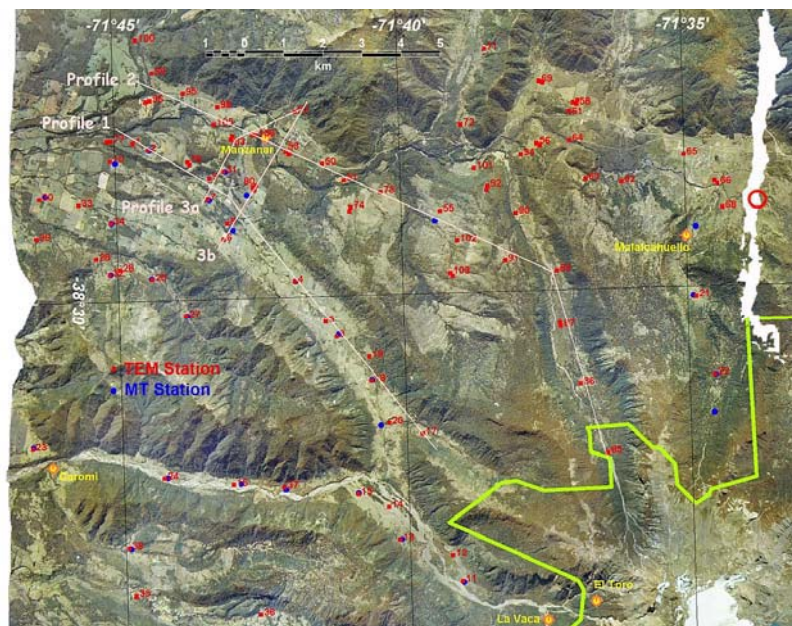
Introduction



In the frame of the BGR project **GEOTHERM**, a project for promoting geothermal exploration in developing countries, the area NW of the Sierra Nevada, 9th region of Chile, was surveyed geophysically during a field campaign in 2006.

Transient-Electromagnetic (TEM) measurements were performed in order to support the interpretation of magnetotelluric (MT) data, which had been collected simultaneously (c.f. contribution of U. KALBERKAMP, this volume) and possibly to discover anomalous zones within the first hundreds of meters of the subsurface which might be caused by hot water reservoirs. One result of this survey was the clear indication of well conducting layers at depths ≥ 100 m at a few points close to the very northern and western borders of the area covered. We interpreted this observation that we are just seeing the edge of an aquifer with

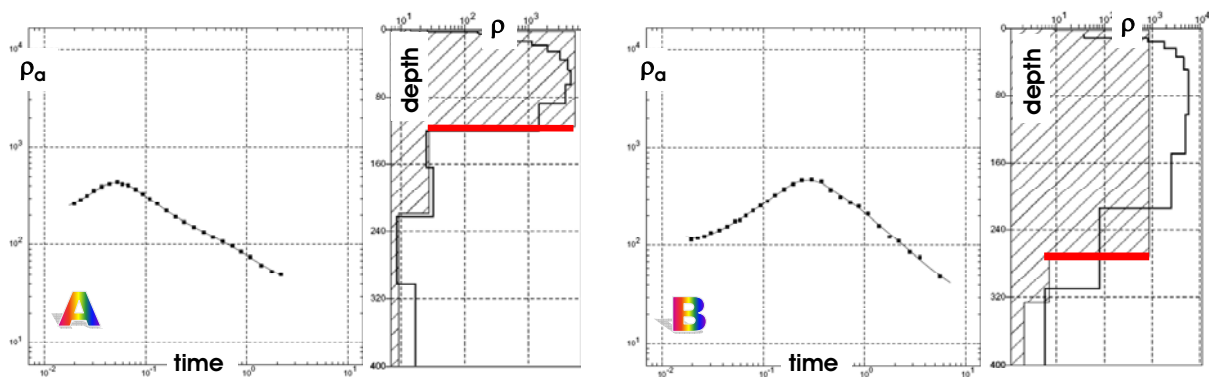
hot water which is used at the hotel complex of Manzanar, a few km farther to the north. As this aquifer is little known, in particular its extension, and as it might be of considerable economic interest to the local community we decided to realize a second field campaign in order to gather more information on this anomalous zone. The 38 points from 2006, just touching the margin of the Manzanar aquifer, could eventually be complimented with an additional 57 new points in 2007.



Aerial photo of the survey area. Thick blue dots are MT stations, small red squares TEM stations. Hot water wells are marked with their names in yellow and the light green polygon in the right low corner is the border of the national park

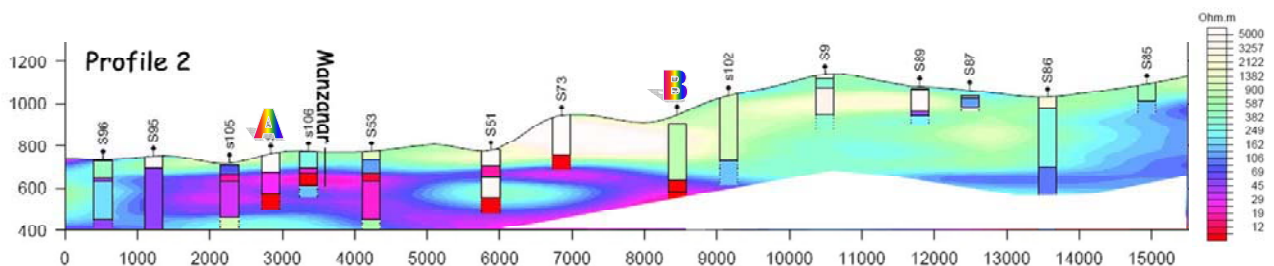
Field examples

The survey has clearly proven the existence of a low resistivity ($<10 - 20 \Omega \cdot \text{m}$) structure in the vicinity of the Manzanar well. Two sounding examples of points over the anomalous zone are shown in the figures below. The strong decrease of the apparent resistivity with increasing decay times indicates well conduct-

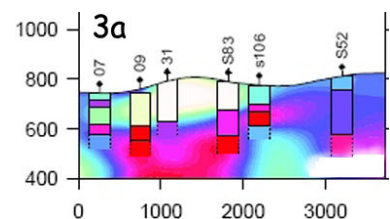


ing layers at depth. At sounding **A** (left) the decrease starts early at around 50 μs , i.e. the depth of the good conductor is relatively close to the surface at ~ 120 m; sounding **B** (to the right) shows the decrease much later at about 300 μs : the depth of the well conducting layer is ~ 250 m. The reason for this is the higher altitude of the second point of ~ 140 m. In the left part of the graphs the apparent resistivity is represented as function of the decay time double-logarithmically: discrete points are the measured values and the continuous curves are the responses for the models represented to the right beside it. The layer resistivities in $\Omega\cdot\text{m}$ are thereby logarithmically plotted along the x axis and the depth of the layers in meters linearly downward.

Vertical sections

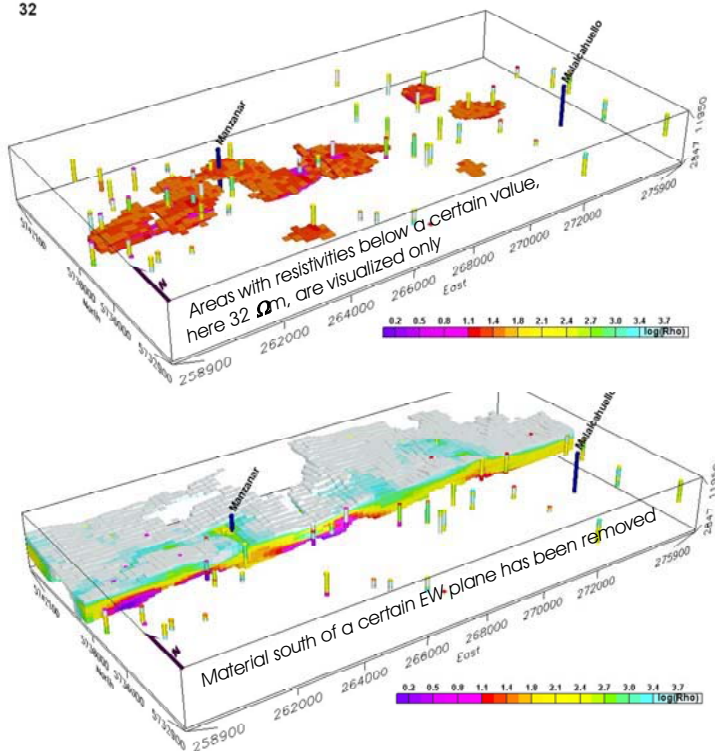


The two soundings shown are lying on the NW-SE profile 2 (cf. the aerial photo above). The 1-D models together with the results of all points along this profile were used to produce a vertical resistivity section. The selected colour scale emphasizes ranges of low resistances by red or violet colouring. The Manzanar anomaly in the left (NW) half of the panel immediately attracts attention. It starts west of Manzanar and extends some km farther ESE. Its depth is ~ 80 to ~ 250 m below surface, depending on the altitude of the point, and it has a thickness of at least 100 m. Profile 3a (to the right) runs perpendicularly to the longitudinal profile 2. From this a width of the anomaly of ~ 1 -2 km can be inferred.



3-dimensional visualization of the resistivity models found

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In order to better visualize the spatial distribution of the low resistivity structures a 3-dimensional presentation was attempted. The subsurface is portioned into so-called voxels, homogeneous prism shaped volume units, each which an electrical resistivity, calculated by interpolation from the interpreted TEM models. The voxels size used for the presentations shown to the left is 200 x 200 x 10 m. The view is from above and from the SW; the vertical exaggeration is five fold. The one-dimensional models found at each point measured are represented by small circular columns and the hot springs of Manzanar and Malalcahuello are labelled for a better orientation.

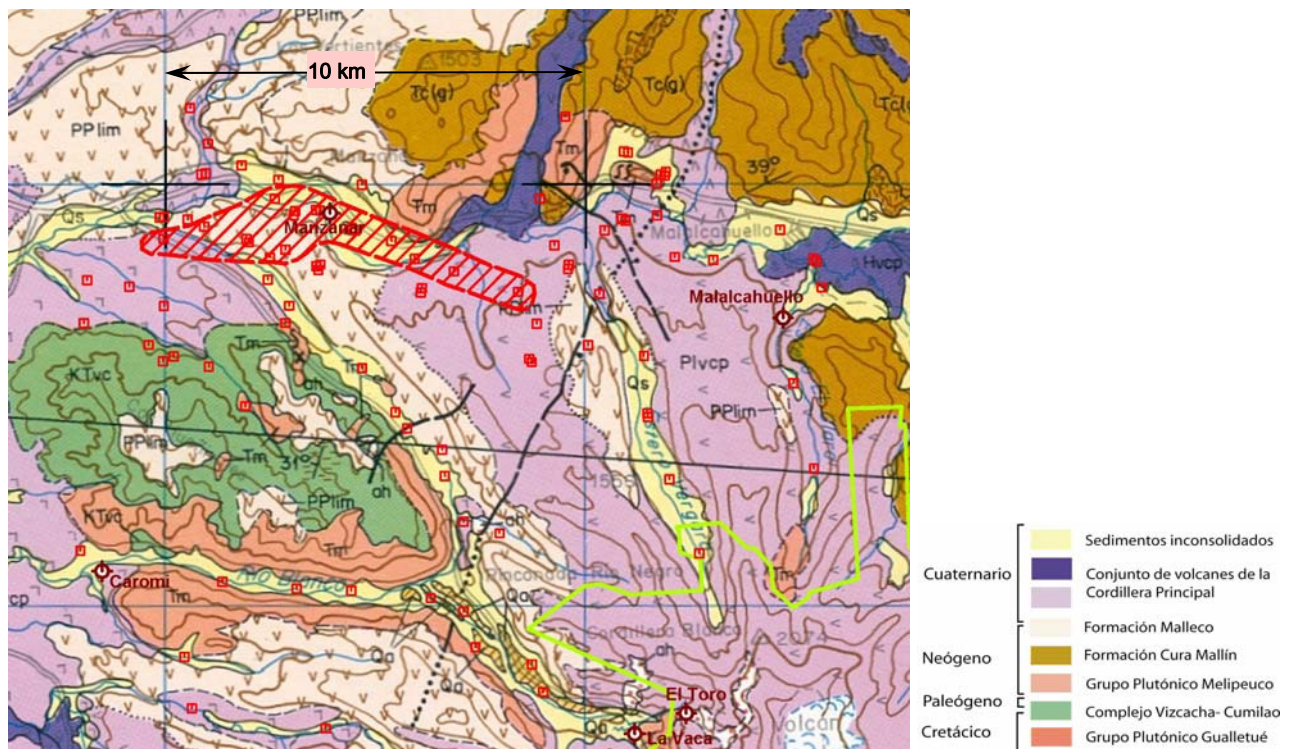
Results

The survey has clearly proven the existence of a low resistivity ($< 10\text{-}20 \Omega\cdot\text{m}$) structure in the vicinity of the Manzanar well. It lies at a depth of ~ 80 to 120 m below surface and has a thickness of at least 100 m. It starts ~ 2 km west of Manzanar and extends an additional ~ 5 km farther to the ESE. Its width is ~ 1.5 km. The first part of the structure seems to extend along the valley of the Cautín River. Farther on it apparently continues below the mountains south of the valley where it is logically at greater depth below surface. Because of the narrowing of the valley towards the East measurements were not possible for 2 to 3 km there. So the information is not sufficient to confirm the continuation of the low resistivity structure described along the valley. In any case it does not seem to be very simply shaped. The 3-dimensional presentations give the impression of two separate zones where Manzanar lies in between the both. There might be, however, a problem of interpolation due to the too sparse point density, as already indicated. Additional uncertainties in the interpretation arise of course from the intrinsic ambiguity of electromagnetic modeling due to the principle of equivalence.

The other important hot water occurrence in the area, the thermal springs of Malalcahuello in the very Eastern part of the area interestingly does not show any anomaly at all. There were only a few soundings accomplished in this area, admittedly, and in particular none in the immediate proximity of the hotel com-

plex (due to artificial interferences suspected). A clear result is, however, that Malalcahuello and Manzanar are different hot water occurrences and continuity between both does obviously not exist.

There is little doubt that the low resistivity zone of Manzanar is due to mineralized hot waters. Resistivities below $10 \Omega \cdot \text{m}$ at depth can only be explained with elevated contents of ions. All the soundings confirm very high resistivities above the good conductor mentioned. Consulting the geological map (below) gives us immediately the explanation: the hot water aquifer is covered by volcanic layers of quaternary and Pliocene/Pleistocene ages mainly consisting of basaltic and andesitic lavas. The whole structure of the Manzanar aquifer can of course be verified by drilling holes only: locations one km west of Manzanar and one to two km east of it should be good selections. For evaluating the potential hot water resource pump tests have to be performed subsequently.



Geological map of the survey area. Small red squares are TEM stations. Hot water wells are marked by little brownish circles and their names. The Manzanar anomalous zone, hatched in red, lies below tertiary and quaternary lavas which came from the Sierra Nevada volcano some 20 km to the SE.

Exploration of Geothermal High Enthalpy Resources using Magnetotellurics – an Example from Chile

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Introduction

Geothermal energy sources are formed by heat stored in rock at depth. In regions with high heat flow, like at volcanically active plate margins, high total thermodynamic energy is accumulated in the so called high enthalpy resources. To explore for these resources up to a pre feasibility stage a range of geoscientific methods is used in a defined sequence starting with regional reviews and remote sensing followed by geologic-, hydrologic-, geochemical and geophysical surveys.

The applied geophysical methods usually comprise temperature measurements (gradient boreholes), seismology, magnetics and resistivity methods, including Magnetotellurics (MT). Geothermal surface manifestations like hot springs, fumaroles, geysers and the associated geological and geochemical settings are indicating the presence of a geothermal reservoir. Particularly resistivity methods may be used for delineating the lateral and depth extensions of such potential reservoirs. The Magnetotelluric method is frequently used for this purpose since it easily covers the necessary exploration depth down to 5 km.

The role of electrical resistivity

Unaltered volcanic rocks generally have high resistivities which can be changed by hydrothermal activity. Hydrothermal fluids tend to reduce the resistivity of rocks

- by altering the rocks,
- by increase in salinity or
- due to high temperature.

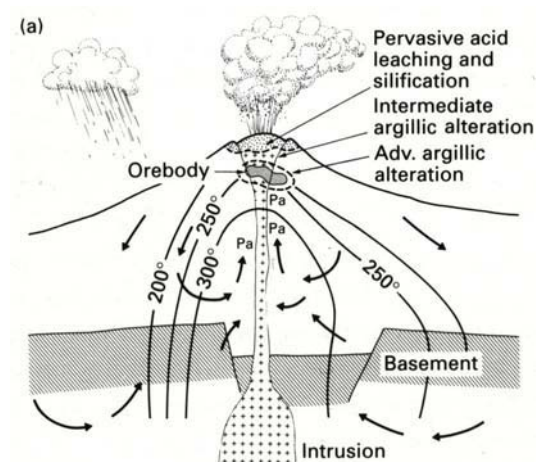


Figure 1: Sketch of a geothermal resource in volcanic terrain of the acid sulphate type and associated alteration. Arrows indicate the circulation of meteoric water (after Evans 1997).

In high enthalpy reservoirs, i.e. fluid temperatures above 200 °C, hydrothermal alteration plays the predominant role. In a volcanic terrain (fig. 1) the acid-sulphate waters lead to different alteration products depending on the temperature and thus on the distance from the heat source. With basalts as country rock smectite becomes the dominant alteration product in the temperature range from 100 °C to 180 °C. At higher temperatures mixed layer clays and chlorite become dominant (fig. 2).

Laboratory measurements indicate that smectite (expandable) clay minerals show very low resistivities even with resistive fluids as saturant (fig. 3, Emerson and Yang 1997). While smectite clay exhibit resistivities below 5 Ohm*m other (non expandable) clays have higher resistivities. Since the abundance of smectite is restricted to a temperature range from 100 °C to 180°C a smectite layer (or cap) is formed around a hot reservoir at the corresponding distance. Layers both above and below this smectite layer have higher resistivities, thus a succession of high-low-high resistivities with depth is indicative for a geothermal reservoir of this type (fig. 4) where the higher resistivities below the clay cap point to the core of the reservoir and represent a possible drilling target. The MT method may detect this pattern which is expected at depth of several hundreds of meters down to 1500 m (restricted due to feasibility reasons).

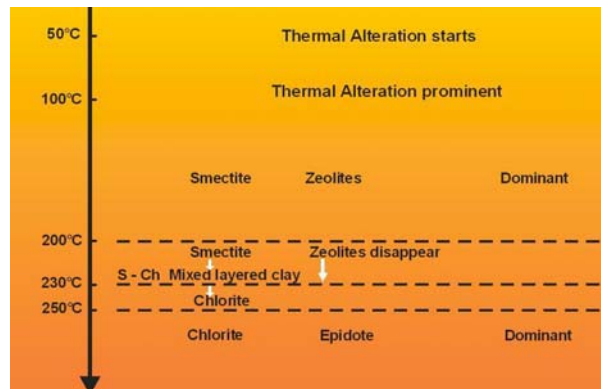


Figure 2: Alteration mineralogy with increasing temperature in basaltic country rock. In the temperature range 100°C to 180°C smectite becomes the dominant alteration product and generally forms a smectite/bentonite clay cap (source: Geological Survey of Iceland ISOR).

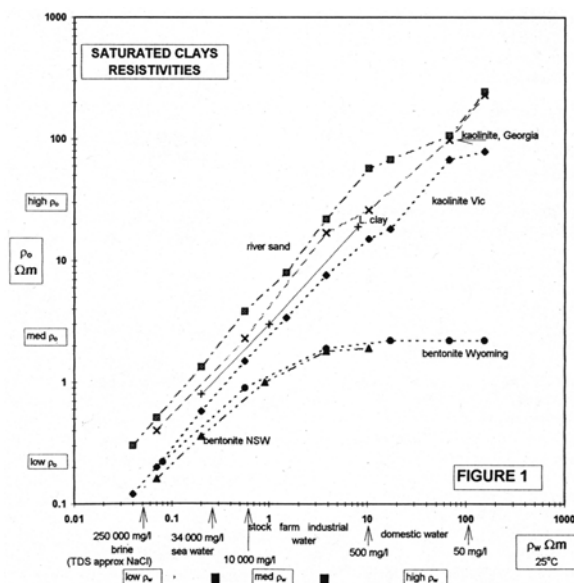


Figure 3: Bentonite clay (mineralogy: smectite) exhibits resistivities around 2 Ohm*m even if the saturant fluid has high resistivity (after Emerson & Yang 1997).

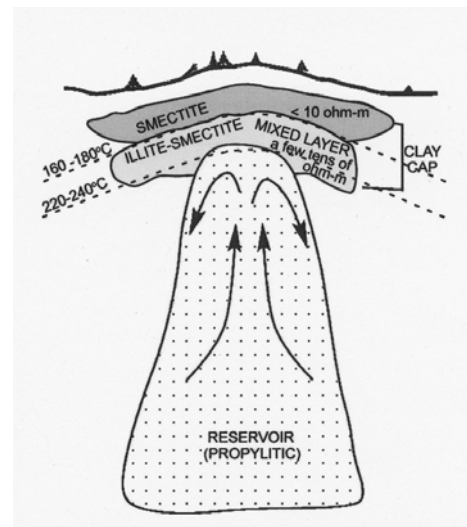


Figure 4: Schema of a generalised geothermal system. The smectite cap formed exhibits resistivities in the range of 2 Ohm*m, the mixed layer around 10 Ohm*m (modified after Johnston et al. 1992).

An Example from Chile

Within a geothermal exploration program electromagnetic surveys have been conducted in the 9th region of Chile, northwest of the Sierra Nevada volcano (fig. 5). Along MT profile P1 (fig. 6) in a NW bearing from the Sierra Nevada volcano 10 MT soundings (up to 100 s period) have been recorded.



Figure 5: Chile location map. Right circle: Sierra Nevada prospect area. Source: <http://www.turistel.cl>

Time series processing included visual inspection of the recorded data and excluding disturbances and heavily noise affected parts of the time series. Although very time consuming this approach proved to be the best means to extract maximum information from the noise contaminated time series. These preconditioned time series were then transformed into the frequency domain by a FFT using adapted window lengths. With a coherency based algorithm the Fourier spectra were then

averaged and the impedance tensor estimated. The impedance tensor has been rotated mathematically by a constant angle derived from the swift angle at low frequencies. This results in a data set with consistent orientation with one axis approximately along the valleys which is taken as direction for the E-field and assigned the TE-mode. Static shifts of the resistivity curves have been removed by means of the MT response derived from TEM measurements at the same locations.

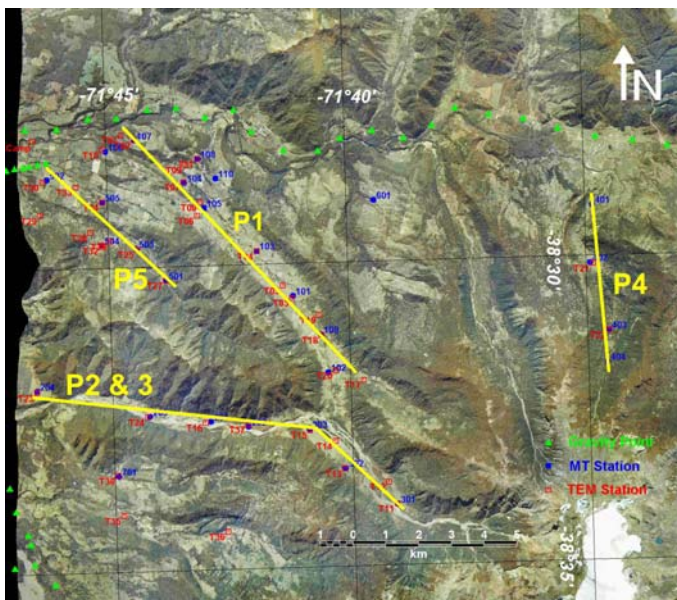


Figure 6: MT profile locations superimposed on an aerial photo of the survey area. The Sierra Nevada volcano is located in the SE corner. In the North you see the Rio Cautin (along the green Gravity stations). Only MT profile P1 shows resistivity lows of interest to geothermal exploration.

The subsequent 2D modelling has been performed using an algorithm by Mackie implemented in the software WinGLink. Both, TE- and TM- modes were taken into account in the 2D modelling.

A shallow conductor in the range of soundings 107 and 108 (fig. 7) is due to a hot aquifer (Manzanar aquifer) and has been investigated in detail with TEM (see

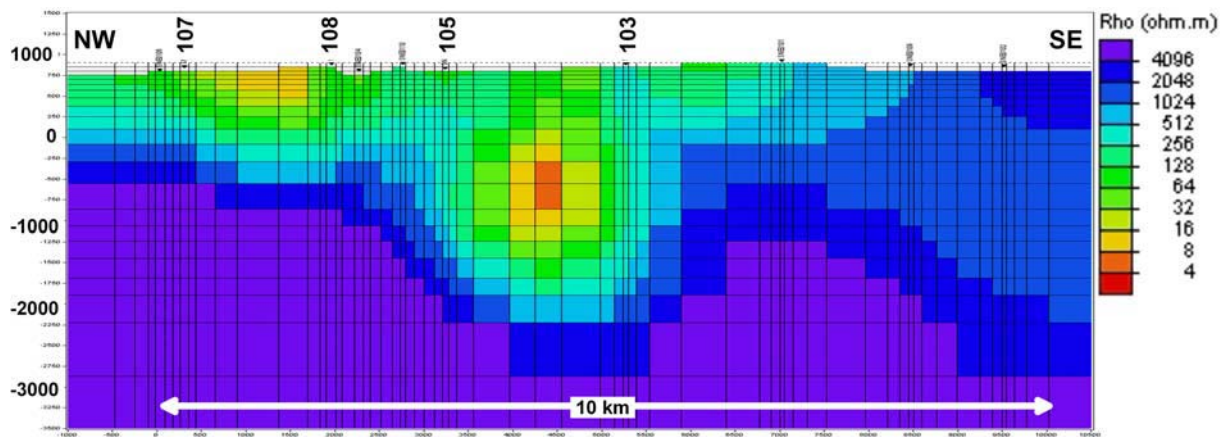


Figure 7: MT profile P1 trending from NW to SE. The interpretable depth reaches about 6 km. Resistivity lows are found at 1.3 km depth (550 mbsl) between stations 105 and 103 as well as a shallow anomaly between stations 107 and 108.

contribution by G. Reitmayr, this volume). In addition to the TEM interpretation given there we can conclude from the MT model a depth extension of this aquifer to a maximum of 600 masl, i.e. an aquifer thickness of max. 400 m.

At a depth of approximately 600 mbsl a good conductor shows up between soundings 105 and 103. This conductor could be associated with alterations or hydrothermal fluid flows. To the SE end of the profile resistivities increase above 1000 Ohm*m and do not indicate any alteration zone or pathways for highly mineralised geothermal waters, flowing from the Sierra Nevada (EL Toro fumarole and hot well) to the Cautin river at Manzanar or

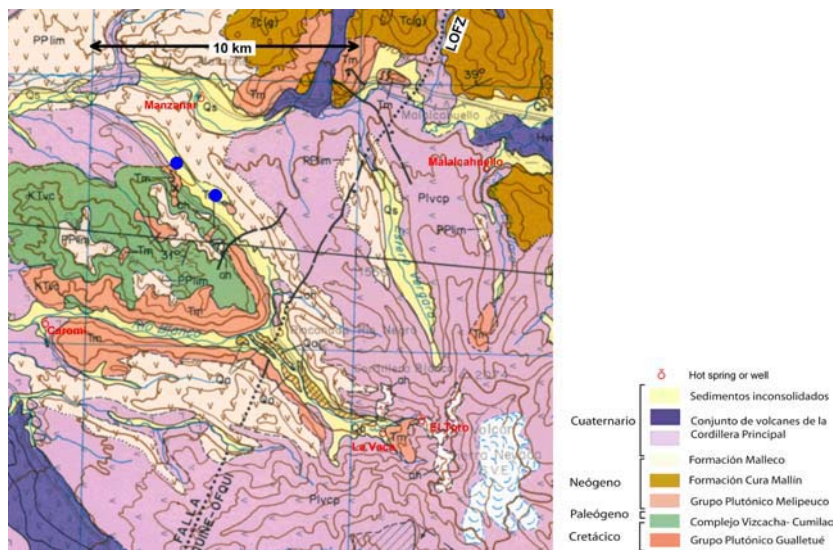


Figure 8: Part of the geological map covering the survey area. Main elements are quaternary volcanic rocks (violet, blue), volcanic-sedimentary unit (green) and quaternary valley infills. LOFZ= Lique-Ofqui Fault Zone, major dextral strike slip system (transpressional regime), NNE trending. Blue dots = MT sounding points 105 and 103.

Malalcahuello (fig. 8). Fluid flows, as proposed by conceptual models based on geochemical analyses, will probably be restricted to faults linked to the dextral strike slip system of the Lique-Ofqui Fault Zone (LOFZ) but are not detected by the MT data gathered.

Possible causes for the resistivity low

The central resistivity low could be caused by argillic alteration of a recent geothermal system. Since the extension of the anomaly is rather small and its resistivity in the range from 4 to 8 Ohm*m, it seems more likely due to mineralised fluid flow in faults. The aerial photo (fig. 6) shows a SW-NE trending tectonic lineament in that area supporting this assumption. The shallow resistive zone is most probably linked to the Manzanar aquifer. In the given environment also fossil alterations, altered pyroclastics or glacial clays could at least contribute to low resistive anomalies.

Acknowledgements

The MT survey in Chile has been carried out as part of a cooperation project between Fundacion Chile and BGR as part of the GEOTHERM Programme. We gratefully acknowledge the cooperation with Carolina Munoz and Oscar Coustasse (Fundacion Chile). GEOTHERM is a technical cooperation programme financed by the Federal Ministry for Economic Cooperation and Development (BMZ) to promote the use of geothermal energy in partner countries (www.bgr.de/geotherm/).

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1D-Laterally Constraint Inversion (1D-LCI) of radiomagnetotelluric data from a test site in Denmark

T. Schmalz¹, B. Tezkan²

Abstract

Within an EU-project a radiomagnetotelluric system (RMT-F) was developed allowing the simultaneous recording of time series of four components of the electromagnetic field. The device operates within a frequency range from 10 kHz to 1 MHz. With the aim to characterize an aquifer for groundwater exploration the RMT-F has been applied on a test site close to the city of Århus (Denmark). The measured time series were processed and interpreted by 1D-Laterally Constraint Inversion (1D-LCI) and 2D inversion. The results were then compared with other geophysical methods (PACES, SkyTEM, GroundTEM).

Introduction: the radiomagnetotelluric method

Radiomagnetotellurics (RMT) is an electromagnetical method that uses civilian and military radiostations within a frequency range from 10 kHz to 1 MHz as transmitters. The electromagnetic waves radiated from these stations diffuse into the conductive earth and induce current systems which are connected with secondary electric and magnetic fields.

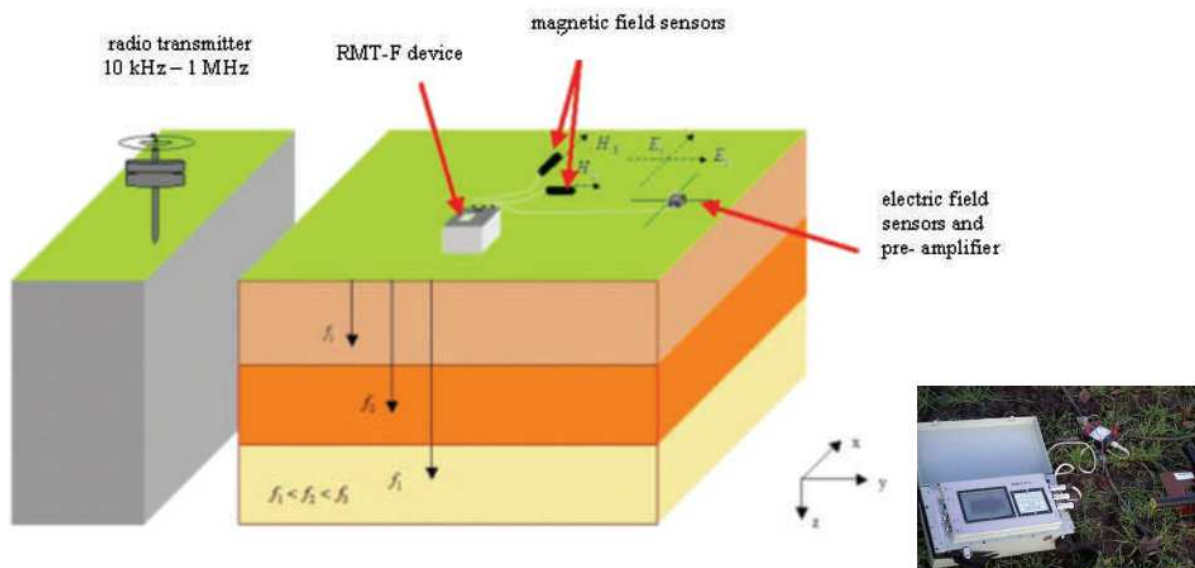


Fig. 1: The radiomagnetotelluric method (Schmalz, 2007).

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² Institute of Geophysics and Meteorology, University of Cologne, Albertus-Magnus-Platz, D-50923 Cologne, Germany

Due to the fact that the penetration depth increases for lower frequencies and decreases for higher frequencies, these fields are measured with electric and magnetic sensors for several frequencies (fig. 1).

The RMT-F system

The RMT-F system (fig. 1) was developed within an EU-project and allows the recording of time series for both electric and magnetic fields. The system consists of a receiver unit, an electric antenna to observe the electric field and two magnetic sensors for the registration of the magnetic field in perpendicular directions. Two different frequency bands are used: one consists of frequencies from 10 kHz to 100 kHz with a sampling frequency of 312.5 kHz and another one which consists of frequencies from 100 kHz to 1 MHz with a sampling frequency of 2.5 MHz. Time series are saved in the receiver and can later be transferred to a computer via an Ethernet connection.

Processing software SM 25

The processing software SM 25 was developed together with the RMT-F system. The program can be used to display the time series in order to check the data quality. Auto and cross spectra of the electric and magnetic time series are calculated and can also be viewed with the software. The user can define an azimuth range to select the available transmitters. In addition, the coherency between the observed electric and magnetic fields oriented perpendicular to each other can be selected by the user. The transfer functions are only calculated for frequencies with coherencies greater than the selected coherency level (fig. 2) and for radio transmitters within the azimuth range defined by the user (fig. 3).

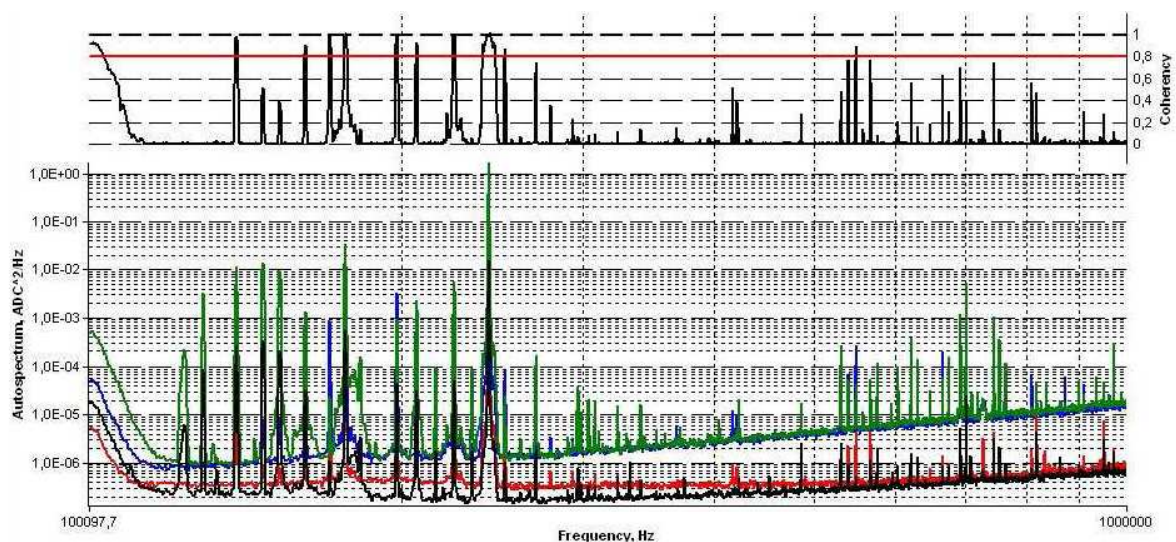


Fig. 2: Auto spectra and coherency for the recorded time series (H_x : red, E_y : blue, E_x : green, H_y : black) in a frequency range from 100 kHz to 1 MHz.

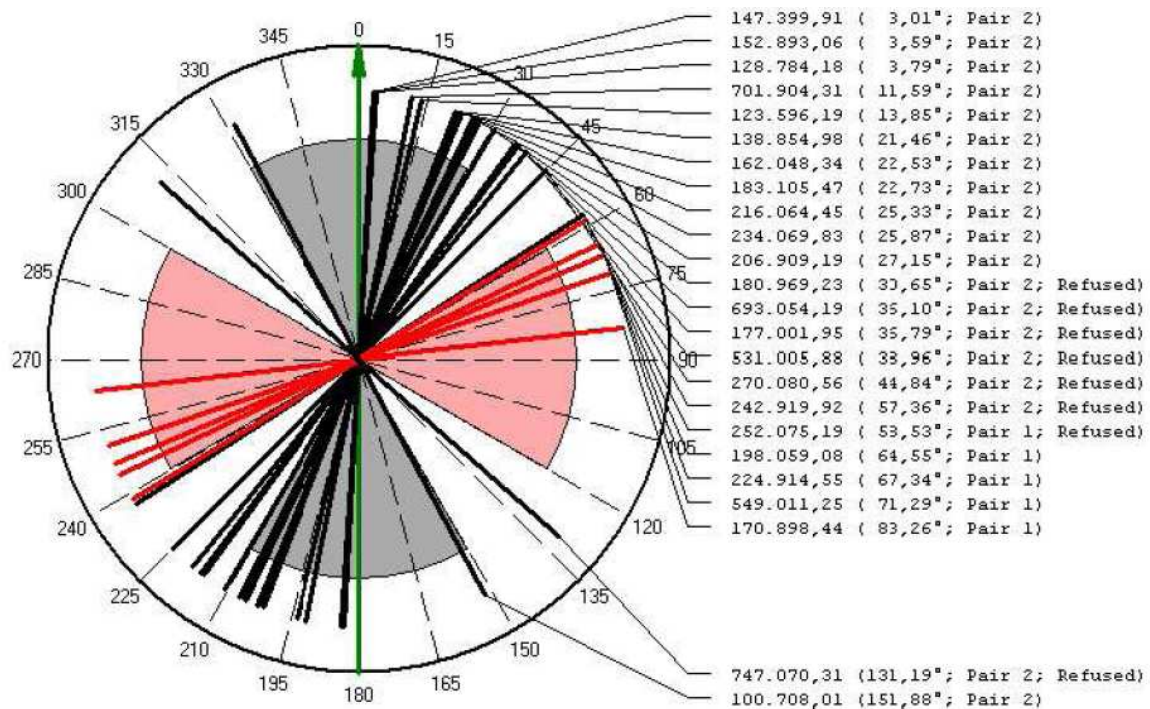


Fig. 3: Visualisation of the azimuth range and the transmitter directions.

1D-Laterally Constraint Inversion (1D-LCI)

A “workbench” program was developed during the EU-project to interpret the data by model calculations. The heart of this program is the 1D-Laterally Constraint Inversion algorithm (*Auken and Christiansen, 2004; Auken et al., 2005*). Here, the underground is divided into separate models. During the inversion these models are tied together by lateral constraints (fig. 4). The constraints contain information about the geological variability of the underground. A priori information, for example from well logs, can also be added and all information migrates through the lateral constraints to the adjacent nodes. The inversion results can be visualized with the program and can be considered as 2D-pseudo-sections due to the employment of lateral constraints.

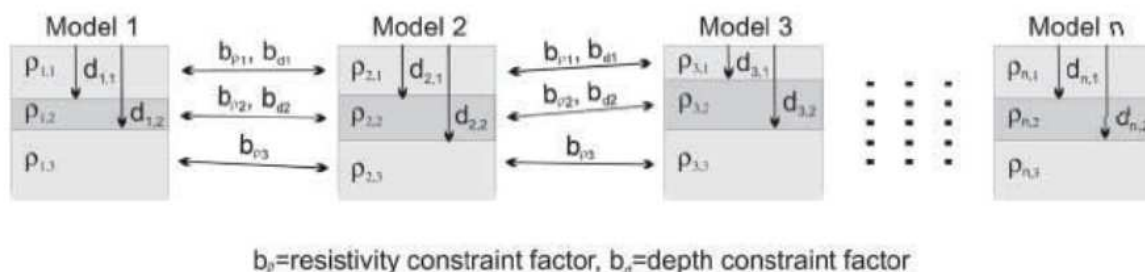


Fig. 4: The Laterally Constraint Inversion (LCI) schematic (taken from *Auken et al., 2005*).

Field example from Denmark

Extensive field measurements were carried out with the RMT-F system on a test site close to the danish city of Århus (*Schmalz, 2007*). The aim of these measurements was the

characterization of an aquifer for groundwater exploration. Overall 89 soundings were carried out on seven profiles. The data was later processed and then inverted using the “Workbench” program (1D-LCI) and a 2D-Inversion program by *Rodi and Mackie* (2001) which uses the nonlinear conjugate gradient method (NLCG). These inversion results were then compared with other geophysical methods (PACES, SkyTEM, GroundTEM). Fig. 5 shows the inversion results of the 1D-LCI and the 2D-Inversion (NLCG) for one selected profile together with a 2D-inversion result of a PACES profile. Comparison of the inversion results (1D-LCI and NLCG) shows a good agreement with respect to conductivities and layer thicknesses. Only for the upper two layers the RMT inversion results can be compared with the PACES inversion result due to the deeper penetration depth of the RMT method.

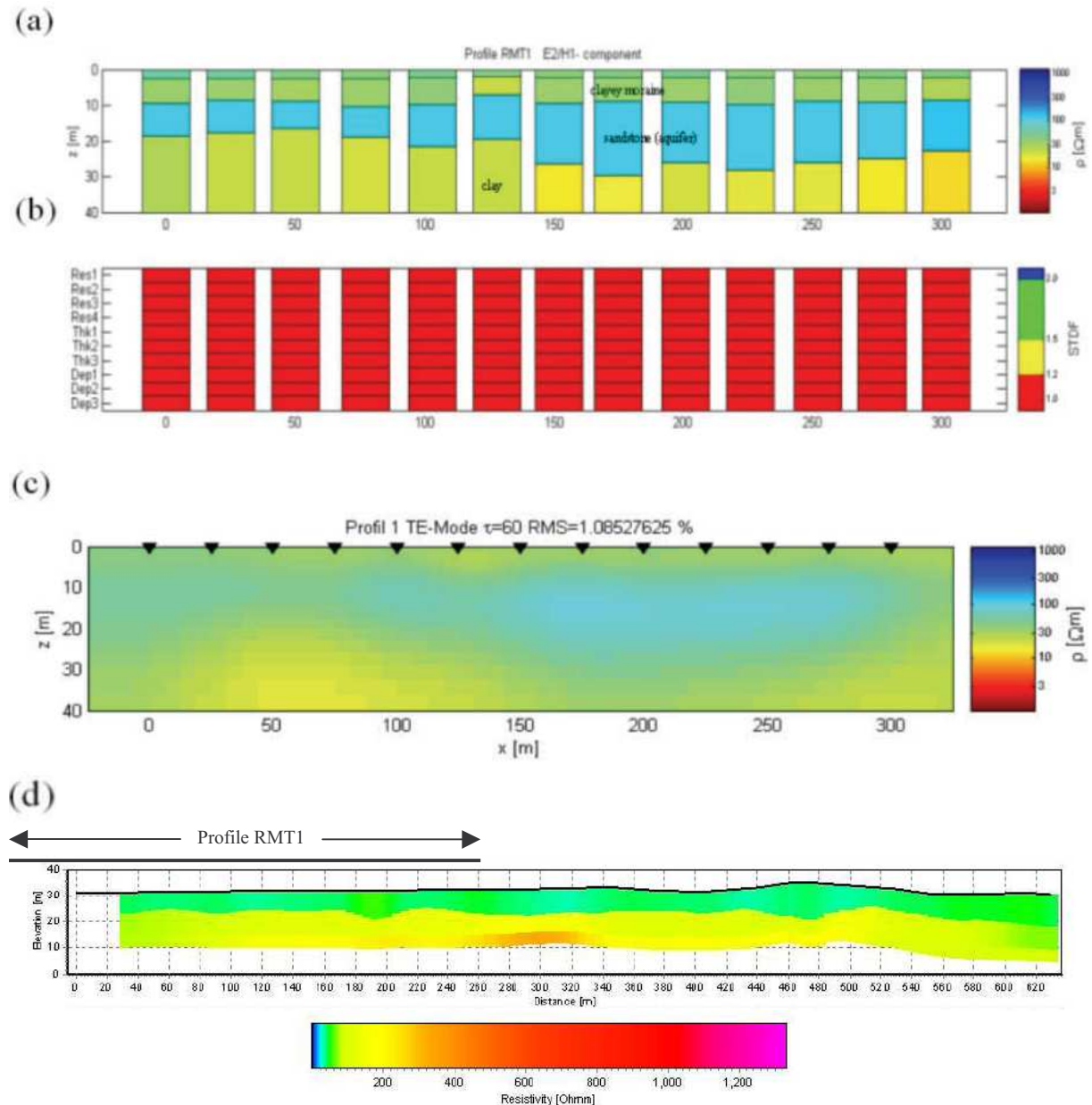


Fig. 5: (a) 1D-LCI result of a RMT profile. (b) Analysis of the model parameters (resistivities: Res1-Res3; thicknesses: Thk1, Thk2; depths: Dep1, Dep2). (c) 2D-Inversion result of the same RMT profile (d) 2D-Inversion result of a PACES profile.

The first and third layer with resistivities between 30 to 50 Ωm can be identified as clay. The second layer with resistivities around 100 Ωm (sandstone) represents the aquifer.

The analysis of the model parameters from the 1D-Laterally Constraint Inversion can be used to assess the resolution of the inverted model. Standard deviations on model parameters are calculated as the square root of the diagonal elements in the covariance matrix C_{est} . Because the model parameters are presented as logarithms, the analysis gives a standard deviation factor (STDF) for the parameter m_s :

$$STDF(m_s) = \exp\left(\sqrt{C_{est,ss}}\right).$$

The theoretical case of perfect resolution has $STDF=1$. A factor of $STDF=1.1$ is approximately equivalent to an error of 10 %. For well resolved parameters $STDF < 1.2$, for moderate resolved parameters $1.2 < STDF < 1.5$, for poorly resolved parameters $1.5 < STDF < 2.0$ and for unresolved parameters $STDF > 2.0$ (Auken and Christiansen, 2004). The colored scale in Fig. 5 (b) indicates these representations. It shows the good resolution of all parameters.

Fig. 6 shows the good model data fit of one selected station taken from the 1D-LCI.

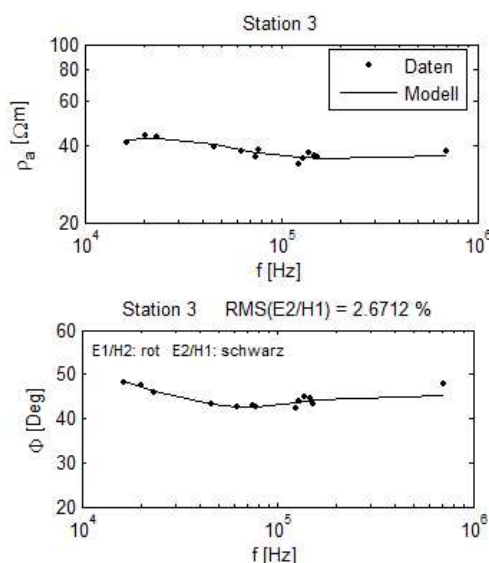


Fig. 6: Model data fit for one selected station of the 1D-LCI result (fig.3, station 3).

Conclusion

Finally, we can say that the measurements with the RMT-F and the application of the processing software SM 25 and the “Workbench” inversion program were successful. The first and third low resistive layers (fig. 5 (a)) can be identified with clay while the second high resistive layer (sandstone) clearly implicates the aquifer. This could also be approved with the 2D-inversion program by Rodi and Mackie (2001) and agree with the inversion result of the PACES profile for the upper two layers. Here, differences occur due to the deeper penetration depth of the RMT method.

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Voruntersuchung zur Entwicklung und Anwendung eines geoelektrischen Bodenexperimentes zur Untersuchung von Körpern im Sonnensystem

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Zusammenfassung

Gegenstand dieser Arbeit sind Voruntersuchungen zur Entwicklung eines elektrischen Bodenexperimentes, welches auf einem Körper des Sonnensystems den Untergrund erforschen kann.

Die Rahmenbedingungen der Untersuchungen orientieren sich an der russischen Phobos-Grunt-Mission. Bei diesem Unternehmen wird ein Lander-System auf dem Mars-Mond Phobos landen und dort vielfältige Untersuchungen, vor allem des Untergrundes, durchführen.

An die Füße dieses Landers wird vorgeschlagen, Elektroden anzubringen, welche kapazitiv einen Strom in den Phobos-Untergrund einspeisen können. In Abhängigkeit von der verwendeten Frequenz und dem Widerstand des Untergrundes kann so eine Bestimmung des Widerstandes und der dielektrischen Permittivität durchgeführt werden.

Im Rahmen dieser Weltraummission herrschen besondere technische Bedingungen, welche in dieser Arbeit untersucht wurden. So besitzt der Phobos-Grunt-Lander lediglich drei Beine. Um einen Strom in den hochohmigen Phobos-Untergrund einzuspeisen und stabile Ergebnisse zu erhalten, müssen die vier benötigten Elektroden jedoch möglichst nah und unter definierten Verhältnissen an den Boden gebracht werden. Hierfür wurden zwei als realistisch betrachtete technische Elektroden-Konstruktionen vorgeschlagen und untersucht.

Weitere Aspekte dieser Arbeit sind der Strom, der über die Elektroden in den Phobos-Boden eingespeist werden kann, und der zu minimierende Stromfluss im Lander selbst auf Grund kapazitiver Wechselwirkungen der Elektroden und der Kabel mit dem Spacecraft.

1 Einleitung

Geoelektrische Verfahren liefern seit einem Jahrhundert verlässliche und glaubwürdige Ergebnisse. Für diese geophysikalische Methode existieren eine Vielzahl an Auswerttechniken und Erfahrungen. Ziel aktueller Forschungen ist es nun, ein elektrisches Bodenexperiment nicht nur auf der Erde, sondern auch mit Hilfe eines Lander-Systems auf anderen Planeten, Asteroiden oder Monden einzusetzen.

Auf solchen Körpern werden jedoch Untergründe erwartet, welche einen hohen Widerstand besitzen und auf welchen es schwierig wird, galvanisch gekoppelte Elektroden einzubringen. Unter diesen Bedingungen ist es sinnvoll, die Stromeinkopplung und Spannungsmessung über kapazitive Elektroden zu realisieren.

Die Anwendung solcher kapazitiver Elektroden wurde erstmalig in den 60er-Jahren vorgeschlagen [Cook, 1956]. Anfang der neunziger Jahre wurde die Idee, Messungen mit Hilfe kapazitiv-gekoppelter Elektroden auf Körpern unseres Sonnensystems durchzuführen, durch Grard wieder aufgegriffen ([Grard, 1990a], [Grard, 1990b]). Diese beiden Arbeiten können durchaus als Geburtsstunde jüngerer Bemühungen angesehen werden, ein kapazitives Messsystem zur Erkundung von Untergründen im Verlauf einer Weltraummissionen zu verwenden.

In allen diesen Arbeiten wird lediglich eine Einführung in die Technik der kapazitiven Widerstandsbestimmung gegeben. Eine vollständigere und erweiterte Theorie wird hier jedoch nicht geschaffen. Dies geschieht in der Arbeit von Kuras [2002]. Hier wird ein Formalismus zur Interpretation kapazitiver Messungen entwickelt, welcher auch die Phasenmessung berücksich-

tigt. Des Weiteren wird eine Rückführung der Kapazitiven-Widerstands-Technik auf die Gleichstromgeoelektrik erreicht.

Mit der Permittivity Probe der Rosetta-Mission ist ein geoelektrisches Experiment zum Kometen Tschurjumow-Gerasimenko unterwegs [Seidensticker et al., 2007].

2 Grundlagen

Das Prinzip einer kapazitiven Elektrode ähnelt dem eines Kondensators. Die eine Platte des Kondensators ist die leitfähige Elektrode, die andere Platte des Kondensators wird durch den Untergrund gebildet. Die Luft oder bei einem Weltraumexperiment der freie Raum dazwischen stellt das Dielektrikum dar. Eine zeitlich veränderliche Ladung Q auf der Elektrode hat nun eine Ladungstrennung durch Influenz im Untergrund zur Folge. Über das elektrische Feld koppeln Elektrode und Boden und ein Wechselstrom wird im Untergrund generiert.

Bei dem hier geschilderten geoelektrischen Experiment werden vier solcher Elektroden benötigt. Zwei Elektroden (A und B) speisen einen Strom I in den Boden ein und erzeugen so an den zwei Potentialsensoren (M und N) eine Spannungsdifferenz ΔU . Dies ist schematisch in Abbildung 1 dargestellt.

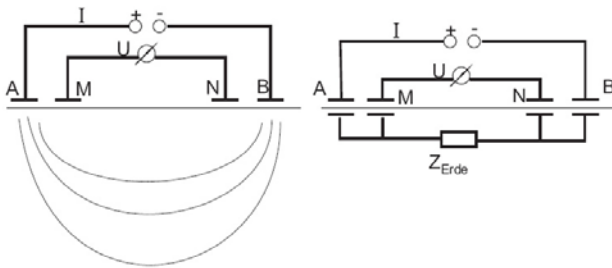


Abbildung 1: Schematische Darstellung des Messprinzips kapazitiver Elektroden.

Die Übergangswiderstände werden als Kapazitäten in Reihe mit der Impedanz des Bodens behandelt. Wegen ihres kapazitiven Blindwiderstandes gelangt mit zunehmender Frequenz mehr Strom in den Untergrund.

Das Signal des zu vermessenden Systems (hier Z_{Erde}) ergibt sich in Form einer komplexen Transfer-Impedanz, also einem Output-Signal $X_{out}(i\omega)$ des untersuchten Systems geteilt durch

eine Input-Anregung $X_{in}(i\omega)$. Im Fall des ERIC-Experimentes ist das Output-Signal eine Spannung und die Anregung ein Strom:

$$G(i\omega) = \frac{X_{out}(i\omega)}{X_{in}(i\omega)} = \frac{U(i\omega)}{I(i\omega)} = Z(i\omega). \quad (1)$$

Diese Transfer-Impedanz ist vom Strom unabhängig und ergibt sich für den Fall von vier kapazitiven Elektroden auf einem Untergrund zu:

$$Z = \frac{1}{i\omega C_0}(1 - K\alpha) = Z_0(1 - K\alpha). \quad (2)$$

Eine Herleitung dieser Beziehung ist in Kuras [2002] zu finden.

Die Größe $Z_0 = \frac{1}{i\omega C_0}$ ist die Vakuum-Impedanz, also die Impedanz eines Vierpols im freien Raum¹. In dieser Vakuum-Impedanz bezeichnet C_0 die Vakuumkapazität der Elektrodenanordnung. Diese Kapazität einer Vierpunktanordnung ist geometrieabhängig:

$$C_0 = \frac{4\pi\epsilon_0}{\frac{1}{r_{AM}} + \frac{1}{r_{BN}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}}}. \quad (3)$$

Der Parameter K stellt den Geometriefaktor für eine spezielle Konfiguration der Elektroden dar:

$$K = \frac{\frac{1}{r'_{AM}} + \frac{1}{r'_{BN}} - \frac{1}{r'_{BM}} - \frac{1}{r'_{AN}}}{\frac{1}{r_{AM}} + \frac{1}{r_{BN}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}}}. \quad (4)$$

Er ist von der Geometrie abhängig, was bedeutet, dass auch die Höhe h der Elektroden über dem Untergrund in diesen Faktor eingeht. Jene Einbeziehung der Höhe macht einen ganz entscheidenden Unterschied zur Gleichstromgeoelektrik aus, bei welcher die (Spieß)Elektroden als auf dem Boden liegend bzw. in dem Boden steckend angenommen werden. Der Einfluss der Höhe der Elektroden macht sich besonders in der Phasenmessung bemerkbar. Befinden sich die kapazitiven Elektroden sehr nah am Boden, so ist $K \approx 1$. Bei großen Abständen der Elektroden über dem Untergrund gilt $K \rightarrow 0$.

¹Bei dem Phobos-Grunt-Lander wird zu dieser Vakuum-Impedanz noch der Einfluss des Landers selbst hinzu kommen.

α ist in Gleichung (2) der Faktor, welcher die elektrischen Eigenschaften des Untergrundes² enthält. Für einen Wechselstrom ist diese Größe komplex:

$$\alpha = \frac{\rho \omega \varepsilon_0 (\varepsilon_r - 1) - i}{\rho \omega \varepsilon_0 (\varepsilon_r + 1) - i} \quad (5)$$

ω stellt die Kreisfrequenz, ρ den spezifischen Widerstand, ε_0 die Permittivität des Vakuums und ε_r die relative Permittivität dar.

Bei einer bekannten Geometrie kann also auf Grund der gemessenen Transfer-Impedanz (Betrag und Phase) der spezifische Widerstand ρ und die relative Permittivität ε_r eines Untergrundes ermittelt werden.

3 Die Phobos-Grunt-Mission

Das russische Phobos-Grunt -Unternehmen ist eine sample-return-Mission, bei welcher ein Lander (Abbildung 2) auf dem Marsmond Phobos landen und mittels einer Rückkehrkapsel Bodenproben zur Erde zurückbringen soll.



Abbildung 2: Modell des Phobos-Grunt-Landers.

Im Verlauf dieser Mission ist nun geplant mit einem Boden-Experiment, basierend auf kapazitiver Stromeinkopplung den Untergrund des Marsmondes zu untersuchen. Dieses Experiment wird den Namen ERIC (**E**lectrical **R**esistivity **I**maging through **C**apacitive **E**lectrodes) tragen.

²und im Fall einer Messung im Weltraum auch die Eigenschaften des umgebenden Vakuums

4 Anbringung der vierten Elektrode

Wie in Abbildung 2 zu erkennen, besitzt der Phobos-Grunt-Lander lediglich drei Beine. Nach der Theorie werden für ein geoelektrisches Bodenexperiment jedoch vier Elektroden benötigt, zwei Stromelektroden und zwei Potentialsensoren. Diese müssen für die Stromeinspeisung bzw. Spannungsmessung, und für die Eindeutigkeit der Interpretation der Ergebnisse, so nah wie möglich an den Untergrund des Phobos-Mondes gebracht werden. Drei Elektroden können unter den Lander-Beinen befestigt werden, für die Anbringung der vierten Elektrode werden hier zwei Konstruktionsmöglichkeiten vorgestellt und diskutiert.

4.1 Anbringung der vierten Elektrode unter dem Lander-Körper

Bei dieser Lösung werden drei Elektroden unter den drei Lander-Füßen angebracht, eine vierte Elektrode wird mittels einer Konstruktion unter dem Lander-Körper befestigt. Dies ist in Abbildung 3 abgebildet.

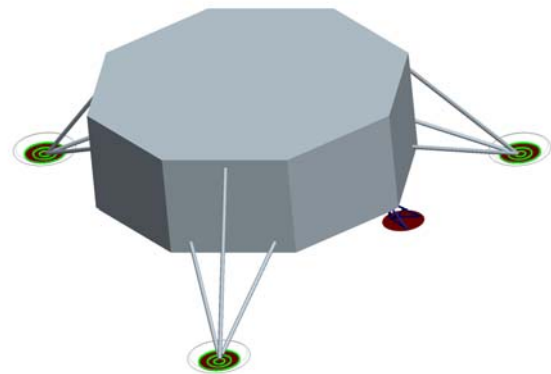
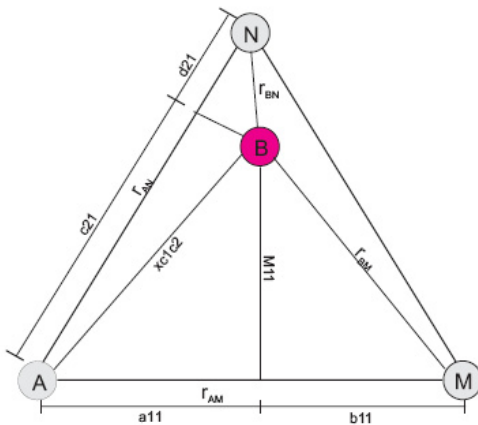


Abbildung 3: Die vier ERIC-Elektroden am Phobos-Lander. Die vierte Elektrode ist am Lander-Körper befestigt. Die Elektroden werden durch PEEK-Ringe (in grün eingezeichnet) von den Lander-Füßen fern gehalten (vergleiche Abschnitt 5).

Die vierte Elektrode befindet sich in einiger Erhöhung über dem Phobos-Untergrund, was sowohl eine Stromeinspeisung als auch eine Potentialmessung erschwert.

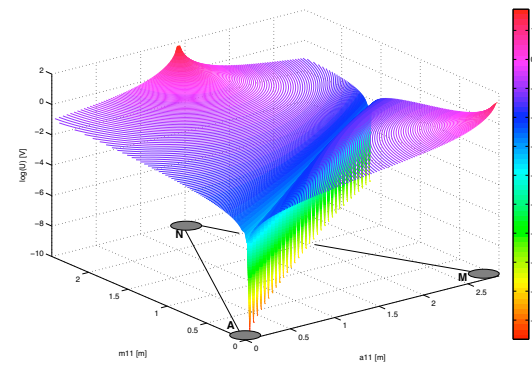
Die Fragestellung, welche sich für diese Elektrodenanbringung stellt ist: Wie wirkt sich die Höhe der vierten Elektrode auf die Stromeinspeisung/Spannungsmessung aus, und wo ist

Um diese beiden Aspekte zu beantworten, ist zunächst in Abbildung 4 die Elektrodengeometrie für eine vierte Elektrode unter dem Lander-Körper dargestellt. Die vierte Elektrode B wandert unter dem Lander, die eingezeichneten Hilfsgrößen variieren.



In Abbildung 5 ist die Spannung an den Potentialsensoren M und N in Abhängigkeit von der Position der Stromelektrode B abgebildet. Die Elektroden A , M und N sind fest unter dem Lander angebracht, B kann variieren. Diese Elektrode befindet sich hier 30 cm über dem Boden. Wie der Strom bestimmt wurde, wird in Abschnitt 5 behandelt.

Um heraus zu finden, welche Position der Elektrode unter dem Lander am sinnvollsten ist, wird die Transfer-Impedanz (Gleichung 2) in Abhängigkeit von der Frequenz und vom Geometriefaktor K (Gleichung 4) in Abbildung 6 dargestellt. Dieser K -Wert variiert hier lediglich von $K = 0,9 - 1$.



Unter einer Frequenz von $f \approx 100\text{ Hz}$ (im Betrag) bzw. $f \approx 1000\text{ Hz}$ (in der Phase) hat der Geometriefaktor einen großen Einfluss auf die Impedanz, besonders auf die Phase. Um so die im Verlauf einer Mission gemessene Impedanz eindeutig interpretieren zu können, sollte $K \approx 1$ gelten. Ansonsten kann nicht unterschieden werden, wo die Messung von der Geometrie oder den Bodenparametern bestimmt werden.

Je weiter sich die Stromelektrode B von A entfernt, desto geringer wird auch der Geometriefak-

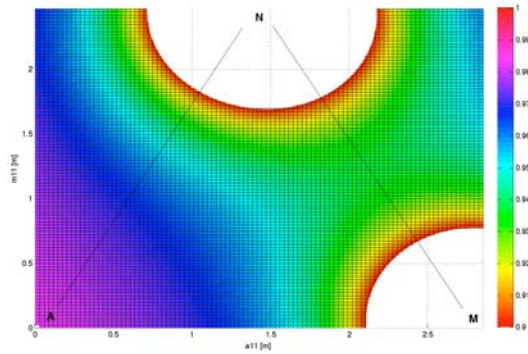


Abbildung 7: Geometriefaktor in Abhängigkeit von der Position der Stromelektrode B (vergleiche Abbildung 4). Die Positionen der drei anderen Elektroden sind eingezeichnet. Der K-Wert zur jeweiligen Position von B ist im Farbbalken kodiert. Im weißen Bereich sinkt der K-Wert unter 0,9 und ist nicht mehr dargestellt.

tor. Um die vierte Elektrode so an den Lander anzubringen, dass die Ergebnisse eindeutig interpretiert werden können, sollte die Elektrode B möglichst auf der Verbindungslinie $\overline{AN}$, jedoch nah an A positioniert werden. Auch das Spannungssignal ist an dieser Stelle nicht kritisch (vergleiche Abbildung 5).

4.2 Die Doppel-Elektroden-Konfiguration

Ein anderer Vorschlag ist, jeweils zwei ERIC-Elektroden unter die mit ca. 46 cm Durchmesser relativ großen Lander-Füße zu befestigen. Dies ist schematisch in Abbildung 8 einzusehen.

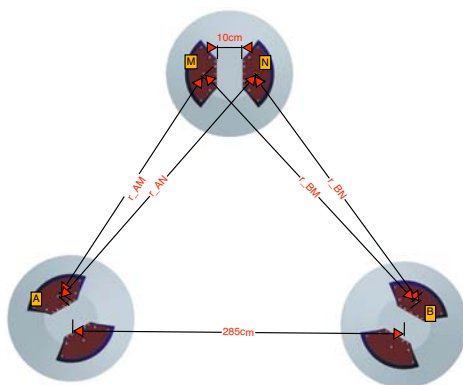


Abbildung 8: Schematische Darstellung der Elektroden-Geometrie. Jeweils zwei Elektroden befinden sich unter einem Lander-Fuß (nicht maßstabsgetreu).

Für diese Konfiguration wurde die Spannung an den Potentialelektroden auf Grund des frequenzabhängigen Stromes errechnet, was in Abbildung

9 abgebildet ist.

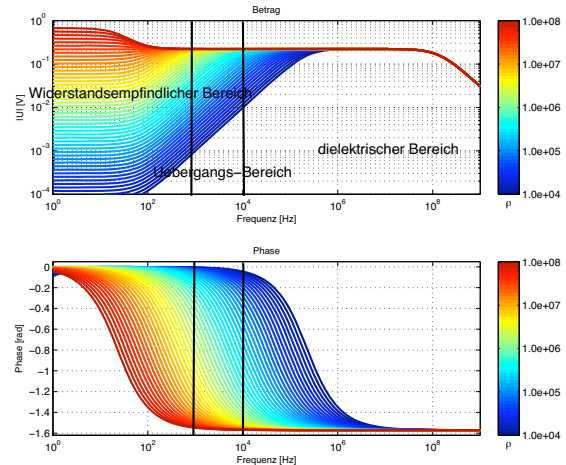


Abbildung 9: Betrag und Phase der an den Potentialelektroden gemessenen Spannung über der Frequenz. Zusätzlicher Parameter: spezifischer Widerstand des Bodens. Annahme: $\varepsilon_r = 7$.

Für das Verhalten des spezifischen Widerstandes bzw. der Permittivität in Abhängigkeit von der Frequenz gilt folgende Beziehung:

$$\varepsilon = \varepsilon_0 \varepsilon_r - i \frac{1}{\rho \omega}. \quad (6)$$

Im unteren Frequenzbereich von Abbildung 9 (für einen spezifischen Widerstand von $\rho = 10^6 \Omega m$ erstreckt sich dieser Bereich bis ca. 5 kHz) zeigt der Betrag der Spannung einen konstanten Verlauf, welcher für verschiedene spezifische Widerstände auch verschiedene (konstante) Spannungsbeträge annimmt. Dies ist der Bereich, bei welchem die Messergebnisse durch den Widerstand des Untergrundes bestimmt werden. Hier dominiert der Widerstands- bzw. Leitfähigkeitsterm $1/(\rho \omega)$ den dielektrischen Term. In diesem Regime ist die Phase Null und somit kann, wenn die Elektroden nah am Untergrund sind ($K \approx 1$), die Theorie der DC-Geoelektrik angewandt werden und der Gleichstromgeometriefaktor Verwendung finden.

In einem mittleren Übergangs-Frequenzbereich (bei $\rho = 10^6 \Omega m$ $f \approx 10 kHz$) ist die Spannung sehr empfindlich. Eine Unterscheidung auf Grund des Widerstandes ist hier nicht mehr eindeutig gegeben. Besonders empfindlich ist hier die Phase, welche in dieser Region beginnt, gegen $-\pi/2$ abzufallen, sich aber noch bei $-\pi/4$ befindet. Dieser Punkt, an welchem die Phase

ihren maximalen Gradienten erreicht, stimmt mit dem Bereich überein, an welchem im Betrag ein asymptotischen Anstieg gegen einen dritten Bereich beginnt (der Anstieg ist eine Folge des mit der Frequenz größer werdenden Stromflusses im Boden). Dieser hier beschriebene Übergangsbereich ist von dem Widerstand des Untergrundes und der Frequenz abhängig. Je höher der Widerstand des Untergrundes ist, desto weiter verschiebt sich das Übergangsregime zu geringen Frequenzen. Physikalisch entspricht dieser Bereich der Situation, in welcher der dielektrische Term (zweiter Ausdruck auf der rechten Seite von Gleichung 6) und der Widerstands-Term (erster Ausdruck auf der rechten Seite von Gleichung 6) sich in der gleichen Größenordnung befinden.

Bei hohen Frequenzen ist der Betrag der Spannung auf Grund des spezifischen Widerstandes im Boden nicht mehr unterscheidbar (bei $\rho = 10^6 \Omega m$ ab einer Frequenz von $f \approx 10 kHz$). Die Phase beträgt $-\pi/2$. Hier dominiert der dielektrische Term ($\epsilon_0 \epsilon_r$) gegenüber der Leitfähigkeit. Die Spannung ist hier empfindlich gegenüber der relativen Permittivität, was in Grafik 10 deutlich an der Auffächerung der Spannung für verschiedene ϵ_r -Werte zu erkennen ist. Auch der Übergang zum dielektrischen Bereich ist eine Funktion der Frequenz und des Widerstandes. Je größer der Widerstand ist, desto eher bestimmt die Permittivität die Messergebnisse und ist selbst ermittelbar.

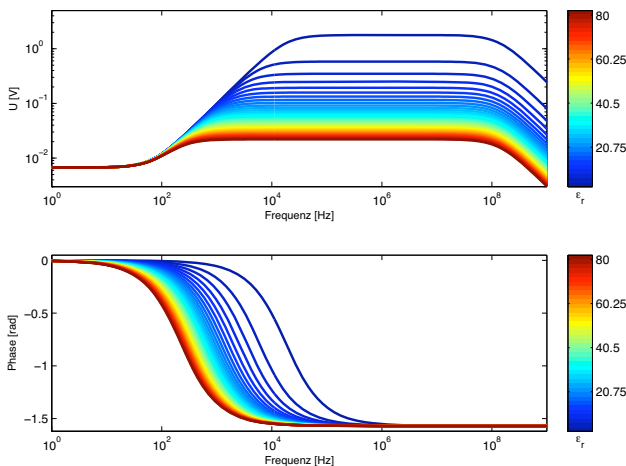


Abbildung 10: Betrag und Phase der an den Potentialelektroden gemessenen Spannung über der Frequenz in Abhängigkeit von der Permittivität. Der Widerstand ist zu $\rho = 10^6 \Omega m$ angenommen.

Der kritische Frequenzbereich für einen zu

erwartenden spezifischen Widerstand von $\rho = 10^6 \Omega m$ liegt bei ca. $10 kHz$. Um diese Frequenz sollten möglichst viele Messungen durchgeführt werden. Diese Erkenntnis deckt sich mit den Annahmen der ROSETTA-Mission, bei welcher auch bei einer Frequenz von $10 kHz$ eine Sensitivität zur Permittivität erwartet wird [Seidensticker et al., 2007]. Da der Messbereich des ERIC-Experimentes von $10 Hz$ bis $100 kHz$ geplant ist, sollten sowohl Messungen im widerstands-sensitiven- als auch im dielektrischen Bereich durchgeführt werden können. ERIC kann somit den Widerstand und die Permittivität des Phobos-Untergrundes bestimmen.

5 Der Stromfluss im Lander und im Boden

Der über die ERIC-Elektroden in den Phobos-Boden eingespeiste Strom ist für die Durchführbarkeit des Experimentes ein kritischer Parameter. Gleichzeitig muss der Stromfluss im Phobos-Lander auf Grund der einzelnen ERIC-Komponenten gering gehalten werden. Diese beiden kritischen Aspekte werden mit Hilfe des Ersatzschaltbildes in Abbildung 11 untersucht.

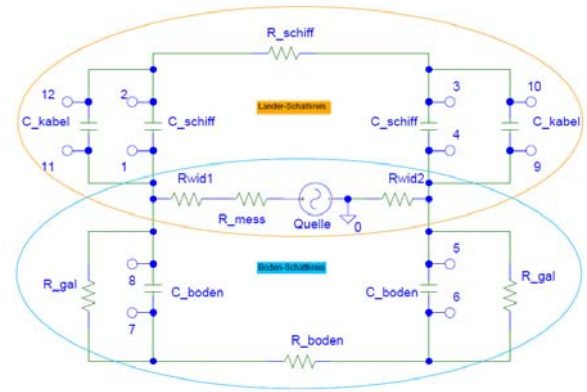


Abbildung 11: Ersatzschaltbild um den Stromfluss im Untergrund und im Lander zu ermitteln

Die orangefarbene Ellipse umschließt die Lander-Schleife, die blaue Ellipse die Boden-Schleife.

Nur wenn genügend Strom in den Untergrund gelangt, kann an den Potentialelektroden eine Spannung gemessen werden. Die Kapazität der ERIC-Elektroden C_{boden} zum Untergrund muss also möglichst groß sein. Um den Stromfluss im Phobos-Boden zu maximieren, werden die ERIC-Elektroden nicht isoliert und lassen so zusätzlich einen galvanischen Stromfluss zu (charakterisiert

durch R_{gal}).

Da die Lander-Füße miteinander verbunden sind, muss der galvanische Stromfluss zwischen Elektrode dem Lander-Fuß hingegen durch einen Isolator unterbunden werden. Dies verhindert auf Grund der verwendeten Frequenzen jedoch nicht, dass die Elektrode nicht nur einen Strom in den Boden einspeist, sondern auch kapazitiv mit den Lander-Füßen wechselwirkt. Elektrode und Lander-Fuß bilden somit einen Kondensator C_{schiff} mit dem Isolationsmaterial als Dielektrikum. Um den Stromfluss im Lander gering zu halten, wird hier eine 5 mm dicke PEEK-Vakuum-Konstruktion (PEEK: *Polyetheretherketon*) vorgeschlagen, welche ein geringe Permittivität, sehr wenig Gewicht besitzt und in Abbildung 12 dargestellt ist.

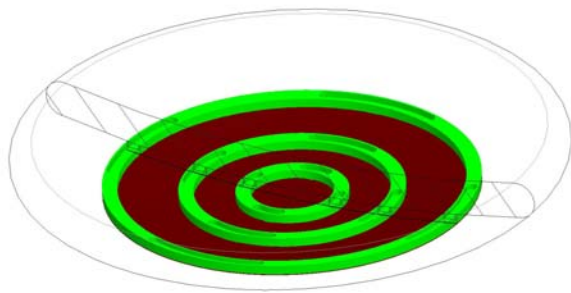


Abbildung 12: PEEK-Vakuum-Konstruktion zur Minimierung des Stromflusses im Phobos-Lander auf Grund kapazitiver Wechselwirkungen der Elektroden mit den Lander-Füßen. Grün: PEEK-Ringe, braun: ERIC-Elektrode.

Hinzu kommt noch eine Wechselwirkung der Koaxial-Kabel (C_{kabel}). Die Werte der Übergangswiderstände und Kapazitäten der Elektroden wurden in Abhängigkeit von den Eigenschaften des Untergrundes errechnet (vergleiche Hördt [2007]) und der Strom mit Hilfe des Programms PSPICE bestimmt. In Abbildung 13 ist sowohl der effektive Strom im Lander- als auch im Phobos-Boden für eine angelegte Spannung von $U_{Quelle} = 28 V$ dargestellt. Die Kabelkapazitäten wurden zu $C_{Kabel,1m} = 40 pF/m$ angenommen.

Um den Strom mit PSPICE zu ermitteln wurde der Strom in R_{boden} und R_{schiff} simuliert, was in Abbildung 13 dargestellt ist. Der eingekoppelte Strom im Boden ist auf Grund der größeren Kapazität der Elektroden zu den Lander-Füßen um ca. eine Größenordnung geringer als der Strom im

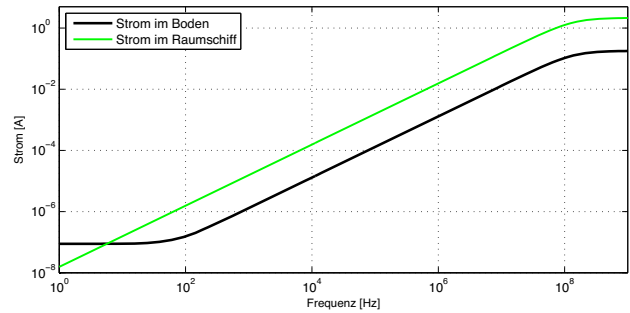


Abbildung 13: Strom als Funktion der Frequenz, berechnet mit dem Schaltbild in Abbildung 12.

Lander. Allerdings befindet sich der Stromfluss im Lander auf Grund des ERIC-Experimentes nicht in einem kritischen Bereich und es gelangt genügend Strom in den Boden, um Messungen durchführen zu können. Des Weiteren muss erwähnt werden, dass die Annahmen in dieser Arbeit konservativ sind, und somit in der Realität ein noch besseres Ergebnis erwartet wird. Die Messelektronik wird jedoch einen Stromfluss größer als $I \approx 10 mA$ nicht unterstützen, was eine frequenzlimitierung von $f < 10^6 Hz$ zur Folge hat. In diesem Frequenzbereich muss das ERIC-Experiment jedoch keine Messungen durchführen, da hier keine neuen Informationen über den Untergrund gewonnen werden (vergleiche Abbildung 9 und 10).

6 Vergleich mit Testmessungen an einem Versuchsaufbau

Im Verlaufe dieser Arbeit wurde eine Versuchsaufbau gebaut, um die theoretischen Betrachtungen durch Experimente zu ergänzen und zu verifizieren. Dieser Versuchsaufbau besteht aus einem Sandkasten, in welchen verschiedene Materialien eingefüllt werden können. Dieser Sandkasten ist komplett aus Holz gefertigt und so ausgelegt, dass auch Messungen in einer Eiskammer oder mit Wasser durchgeführt werden können (Abbildung 14). Auf diesem Bild ist am linken Rand noch die zweite Komponente des Versuchsaufbaus, ein Multikanal-Impedanzspektrometer zur Bestimmung der Impedanz zu erkennen.

Testmessungen sowohl mit kontrollierbaren Schaltkreisen, als auch am Sandkasten wurden durchgeführt. In Grafik 15 ist ein Vergleich zwischen theoretisch bestimmten Verläufen und einer Messung dargestellt. Die Elektrodenanordnung für diese Messung ist derjenigen von Abbildung 8



Abbildung 14: Sandkasten zum Untersuchen eines Probedodens.

nachempfunden. Der Abstand der Potentialelektroden wurde hier zu $r_{MN} = 10 \text{ cm}$, $r_{AB} = 80 \text{ cm}$ und $r_{AM} = 60 \text{ cm}$ und gewählt.

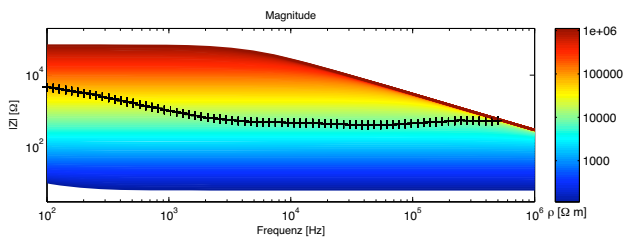


Abbildung 15: Impedanz als Funktion der Frequenz und des spezifischen Widerstandes. Die schwarzen Kreuze charakterisieren die Ergebnisse der Impedanzmessung mit kapazitiven Elektroden im Sandkasten.

Die Betragsmessung ist über einen großen Frequenzbereich konstant (von ca. $3 - 500 \text{ kHz}$). In diesem Bereich stimmt der gemessene Verlauf des kapazitiv ermittelten Betrages mit einem theoretisch zu erwartenden Ergebnissen von ca. $7 - 10 \text{ k}\Omega\text{m}$ überein (nicht dargestellt).

Weitere Messungen auf hochohmigen Untergründen (Eis, Sand) mit den ERIC-Komponenten müssen noch durchgeführt werden.

7 Schlußfolgerung

Mit Hilfe der kapazitiven ERIC-Elektroden ist es auch mit der Präsenz des Landers möglich, einen ausreichenden Strom in den Untergrund des Phobos-Mondes einzuspeisen. Hierzu wurde der Lander und der Boden des Phobos in einem Ersatzschaltbild simuliert. Das Spannungssignal an den Potentialelektroden ist auch bei Erhöhung

einer der vier Elektroden noch messbar.

Zwei Konstruktionen der Elektroden-Anbringung wurden diskutiert und mit beiden ist es möglich, die Untergrundparameter spezifischer Widerstand und dielektrische Permittivität im Rahmen der Phobos-Mission zu bestimmen. Hier ist die verwendete Frequenz von Wichtigkeit.

Erste Messungen ergeben eine gute Übereinstimmung mit dem zu erwartenden Verhalten kapazitiver Elektroden.

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Untersuchungen zur Verbesserung der Objektidentifizierung von Multifrequenz-EMI-Minensuchgeräten

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Zusammenfassung

Diese Arbeit befasst sich mit der Verbesserung der Objektidentifizierung von Metalldetektoren im Frequenzbereich (FD) für die Antipersonenminen-Suche durch Verwendung von Frequenzen zwischen $f = 1\text{ kHz}$ und $f = 100\text{ kHz}$. Die Objektidentifizierung basiert auf einem Inversionsalgorithmus, der einen homogenen Ellipsoiden an das vom Detektor gemessene Signal anpasst. Als Datengrundlage für die durchgeführten Untersuchungen werden eine Mine und zwei Clutterobjekte mit COMSOL-Multiphysics simuliert. Anhand dieser Daten werden die Einflüsse der einzelnen Frequenzen analysiert und die beste Frequenzkombination bestimmt.

1 Einleitung

Bei der Suche nach Antipersonen-Landminen werden überwiegend tragbare Metalldetektoren benutzt. Diese sind billig, einfach zu bedienen und sehr zuverlässig.

Die heute aktuellen Geräte sind in der Lage auch kleinste Metallteile ($r \approx 2.5\text{ mm}$) in den nötigen Tiefen (10 cm bis 15 cm) aufzuspüren (Guelle et al., 2006). Dies ist aber gleichzeitig der entscheidende Nachteil, da auch alle anderen metallischen Abfälle (engl. Clutter) angezeigt werden, ohne dass der Benutzer Informationen über die Art des Objektes erhält. Das führt zu einer hohen Fehlalarmquote, die die Minenräumung zeitintensiv und damit teuer macht.

Eine Möglichkeit der Unterscheidung zwischen Minen und Clutter wurde von Löhken (2007) untersucht. Er entwickelte ein Messverfahren und einen Inversionsalgorithmus, der mit bestehenden Metallsuchgeräten im Zeit- und Frequenzbereich eine Identifizierung der meisten Minen zulässt.

Dieser basiert auf der Anpassung des Signals eines Rotationsellipsoids an das vom Detektor aufgezeichnete Signal. Die berechneten Modellparameter des Ellipsoids geben dann Aufschluss über das verborgene Objekt.

Diese Arbeit befasst sich nun mit der Frage, ob ein Multifrequenz-System, welches nach dem elektromagnetischen Induktionsprinzip (EMI) arbeitet, gegenüber den herkömmlichen Zweifrequenz-EMI-Systemen genug zusätzliche Informationen liefert, um die Objektdiskriminierung mit diesem Inversionsalgorithmus im Frequenzbereich weiter zu verbessern, und damit die Fehlalarmquote aufgrund von metallischen Abfällen zu reduzieren. Dazu wird der Einfluss der einzelnen Frequenzen auf die Inversionsergebnisse untersucht, um die wichtigsten Frequenzen für die Objektidentifizierung zu bestimmen und die beste Frequenzkombination zu ermitteln.

Da zum Zeitpunkt dieser Arbeit kein Metalldetektor in der Lage ist bei verschiedenen Frequenzen im gesamten Frequenzbereich von $f = 1 - 100\text{ kHz}$ Messungen durchzuführen, werden die nötigen Daten synthetisch erzeugt. Dazu wird das Programm COMSOL-Multiphysics verwendet. Mit diesem wird der Zündmechanismus einer Mine, sowie zwei verschiedene Clutterobjekte zum Vergleich simuliert.

2 Grundlagen

2.1 Funktionsweise eines Metalldetektors im Frequenzbereich

Der Signalgenerator des Metalldetektors erzeugt einen periodischen Strom $I_P(t) \propto e^{i\omega t}$ einer bestimmten Frequenz. Dieser Strom in der Primärspule bewirkt die Ausbildung des primären Magnetfeldes $\underline{H}_P(t)$. Dieses zeitlich veränderliche Magnetfeld induziert einerseits direkt eine Spannung

in der Empfangsspule, und wirkt andererseits auch auf den Boden und auf das in diesem liegende Objekt. Abhängig von den Materialeigenschaften kommt es bei beiden zu einer induzierten Magnetisierung bzw. zu einem elektrisch induziertem Magnetfeld, erzeugt durch induzierte Wirbelströme.

Die beiden Effekte erzeugen des zeitabhängige sekundäre Magnetfeld $\underline{H}_S(t)$. Da für Magnetfelder das Superpositionsgesetz gilt, ist das in der Empfangsspule induzierte Signal $I_S(t)$ von dem Gesamtfeld $\underline{H}_{ges}(t) = \underline{H}_P(t) + \underline{H}_S(t)$ abhängig. Dieses Signal wird in einen Realteil und in einen Imaginärteil zerlegt und sorgt nach verschiedenen Verarbeitungsschritten für einen Warnton, dessen Amplitude proportional zu der Stärke des sekundären Signals ist.

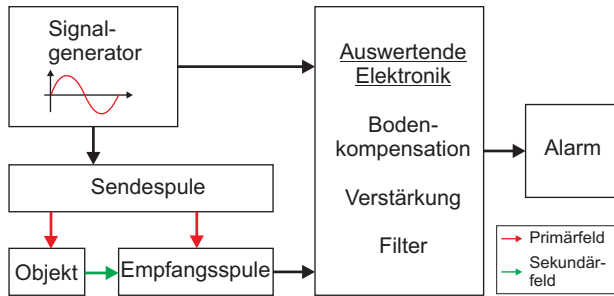


Abbildung 1: Schematische Darstellung der Funktionsweise eines Metalldetektors.

2.2 Vorwärtsrechnung

Für die Vorwärtsrechnung der Inversion wird die Signalantwort eines homogenen Rotationsellipsoids benötigt, der sich in einem periodischen Magnetfeld befindet. Die analytische Lösung nach Ao (2001) oder Barrows (2004) benötigt zu viel Rechenzeit und ist für eine Auswertung der Daten im Feld zu aufwendig.

Um die Rechenzeit zu minimieren wird eine Näherung des Ellipsoids durch zwei Kugeln benutzt. Hanstein et al. (2005) berechnet die Radien so, dass die magnetischen Momente der Kugel gerade den magnetischen Momenten des Ellipsoids in achsialer bzw. transversaler Richtung entsprechen.

Die Radien der Kugel a' und b' ergeben sich zu

(Löhken, 2007):

$$a' = \left(\frac{1}{3} a^2 b \frac{\mu_r + 2}{1 + A_t(\mu_r - 1)} \right)^{1/3} \quad (1)$$

$$b' = \left(\frac{1}{3} a^2 b \frac{\mu_r + 2}{1 + A_z(\mu_r - 1)} \right)^{1/3} \quad (2)$$

a und b sind die Halbachsen des Ellipsoids in transversaler, bzw. achsialer Richtung. μ_r ist die relative magnetische Permeabilität und A_t , bzw. A_z sind die Depolarisationskoeffizienten, wie sie in Sivester und Omeragic (1995) eingeführt werden.

Unter der Annahme, dass das primäre Magnetfeld homogen am Ort der Kugel ist, ergibt sich in Kugelkoordinaten die Φ -Komponente des sekundären elektrischen Feldes zu (Kaufman und Eaton, 2001):

$$E_{S,\phi} = i\omega\mu_0\mu_{r,2} \frac{Da'^3}{2r^2} H_P \sin\theta, \quad (3)$$

mit dem komplexen Reflektionsparameter D (Wait, 1951):

$$D = \frac{(2\mu_{r,1}/\mu_{r,2} + 1)\alpha - (2\mu_{r,1}/\mu_{r,2} + 1 + \alpha^2) \tanh\alpha}{(\mu_{r,1}/\mu_{r,2} - 1)\alpha + (1 - \mu_{r,1}/\mu_{r,2} + \alpha^2) \tanh\alpha}. \quad (4)$$

$\mu_{r,1}$ ist die relative Permeabilität der Kugel, $\mu_{r,2}$ die des umgebenen Mediums.

Die Induktionszahl α ist das Produkt aus Kugelradius a und Wellenzahl k_1 im Medium der Kugel. Mit der Annahme der quasistatischen Näherung folgt diese zu:

$$\alpha = a k_1 = a \sqrt{i\omega\sigma_1\mu_0\mu_{r,1}}. \quad (5)$$

Aus $U = \int \underline{E} d\underline{l}$ folgt durch Integration über den Kreisumfang die induzierte Spannung in der Empfängerspule durch das Sekundärfeld.

2.3 Inversion

Die Inversion beruht auf der gewichteten Anpassung der kleinsten Quadrate. Die zu minimierende Funktion ergibt sich also zu:

$$\chi^2 = \frac{1}{n} \left\| \mathbf{W} (\underline{d} - \underline{F}(\underline{m})) \right\|_2^2 \quad (6)$$

$\mathbf{W}$ ist die Wichtungsmatrix, die die Messfehler beinhaltet:

$$W_{ij} = \delta_{ij} \cdot 1/\Delta d_i \quad \text{mit} \quad \delta_{ii} = 1, \delta_{i \neq j} = 0 \quad (7)$$

$\underline{d}$ ist der n -dimensionaler Messdatenvektor, $\underline{m}$ der Modellparametervektor, der die Eigenschaften des Rotationsellipsoids beschreibt und $\underline{F}(\underline{m})$ ist der linearisierte Vorwärtsoperator.

$$\underline{F}(\underline{m}) \approx \underline{F}(\underline{m}_0) + \mathbf{J}\Delta\underline{m} \quad (8)$$

$\underline{m}_0$ ist das Startmodell und $\mathbf{J}$ ist die Jakobiematrix.

$$J_{ij} = \left. \frac{\partial F_i}{\partial m_j} \right|_{\underline{m}_0} \quad i = 1, \dots, n, j = 1, \dots, m \quad (9)$$

Aus Stabilitätsgründen wird die Inversion mit dem Marquardt-Levenberg-Verfahren 2.Ordnung (Jupp und Vozoff, 1975) gedämpft. Damit ergibt sich die Modellverbesserung eines Inversionschrittes zu:

$$\Delta\underline{m} = (\mathbf{G}^T \mathbf{G} + \beta^2 \mathbf{I})^{-1} \mathbf{G} \underline{e} \quad (10)$$

Mit den gewichteten Größen $\mathbf{G} = \mathbf{WJ}$ und $\underline{e} = \mathbf{W}(\underline{d} - \underline{F}(\underline{m}_0))$.

Zur schnellen und stabilen Berechnung der Pseudoinversen wird die generalisierte Eigenwertzerlegung (SVD) genutzt.

$$\mathbf{G} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^T \quad (11)$$

Damit folgt (Jupp und Vozoff, 1975):

$$\Delta\underline{m} = \mathbf{V} \mathbf{\Lambda}^* \mathbf{T} \mathbf{U}^T \underline{e}, \quad (12)$$

mit $\mathbf{\Lambda}^* = (\mathbf{\Lambda}^T \mathbf{\Lambda})^{-1} \mathbf{\Lambda}^T$ und der Dämpfungsmatrix $\mathbf{T}$ aus den Eigenwerten λ :

$$T_{i,i} = \frac{\lambda_i^4}{\lambda_i^4 + \beta^4}. \quad (13)$$

Als Abbruchkriterium für den Inversionsalgorithmus wurde eine RMS¹-Verbesserung zwischen zwei Iterationsschritten von 0.01% gewählt. Damit wird ein Ellipsoid bestimmt, welcher die simulierten Daten am genauesten beschreibt. Dieses Abbruchkriterium ist nötig, wenn man wie in Abschnitt 3.2 eine Aussage über das Objekt mittels dem RMS-Wert machen möchte.

Die SVD-Zerlegung ermöglicht außerdem die quantitative Auswertung der Inversionsstatistik. Die wichtigsten Informationen liefern die Auflösungs- und Informationsdichtematrix

($\mathbf{R}$, bzw. $\mathbf{S}$), sowie die Kovarianzmatrix der Modellparameter $\mathbf{C}_m$:

$$\mathbf{R} = \mathbf{V} \mathbf{T} \mathbf{V}^T \quad (14)$$

$$\mathbf{S} = \mathbf{U} \mathbf{T} \mathbf{U}^T \quad (15)$$

$$\mathbf{C}_m = \mathbf{V} \mathbf{\Lambda}^* \mathbf{T} \mathbf{T}^T \mathbf{\Lambda}^{*T} \mathbf{V}^T \quad (16)$$

Die Diagonalelemente der Auflösungsmatrix geben die Wichtigkeit der entsprechenden Modellparameter für die Inversion an. Zusammen mit den Nichtdiagonalelementen ergibt sich die Korrelation der Parameter untereinander (Weidelt, 1991). Dieselbe Funktion erfüllt die Informationsdichtematrix für die Datenpunkte.

Eine Abschätzung der Fehler der berechneten Modellparameter ist mit der Kovarianzmatrix möglich. Die Wurzel aus den Diagonalelementen gibt die Standardabweichung der Parameter.

Eine genauere Betrachtung der Inversionsstatistik findet sich unter anderem in Müller (1991) und Weidelt (1991).

2.4 Simulation

Um die synthetischen Daten zu erzeugen mit denen später die Einflüsse der Frequenzen auf die Inversion untersucht werden sollen, wurden drei Objekte simuliert. Das erste ist ein Zündmechanismus einer Landmine bestehend aus einer Stahlfeder, einem Stahlschlagbolzen und einem Zündhut aus Aluminium.

Das zweite Objekt ist ein V-förmig gebogenes Drahtstück aus magnetisierbarem Stahl. Das letzte Objekt ist ein asymmetrisches Blechstück aus Aluminium.

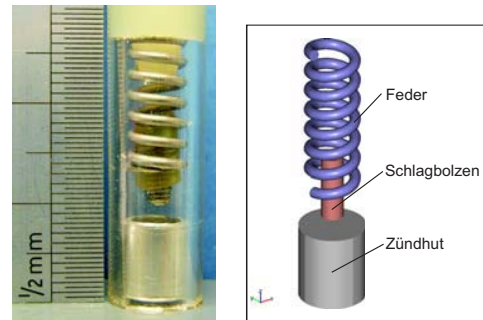


Abbildung 2: links: Foto des nachgebauten Zündmechanismus der Mine M1A, Quelle: Löhken (2007), rechts: Skizze des simulierten Zündmechanismus der Mine M1A. blau: Spiralfeder aus Stahl, rot: Schlagbolzen aus Stahl, grau: Zündhut aus Aluminium

¹RMS=Root Mean Square= $\sqrt{\chi^2}$

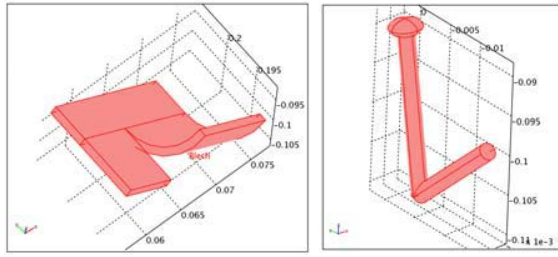


Abbildung 3: Simulierte Clutterobjekte. links: Draht aus Stahl, rechts: Blech aus Aluminium

Für jedes dieser Objekte wurden 63 Simulationen pro Frequenz für verschiedene Positionen durchgeführt. Die Abstände zur Spulenmitte sind $y = (-0.2, -0.18, \dots, 0.2) \text{ m}$ und $x = (-0.067, 0, 0.067) \text{ m}$. Die simulierten Frequenzen sind $f = (1, 2.4, 5, 10, 19.2, 30, 50, 75, 100) \text{ kHz}$.

Für eine nähere Beschreibung der Simulation und der Objekte siehe Virgil (2008).

3 Auswertung

3.1 Gegeninduktion

Ein Unterschied zwischen Landminen und den Clutterobjekten ist deren Komplexität. Während Clutterobjekte meist nur aus einem Teil bestehen (Granatsplitter, Projektile, Draht, etc.), setzen sich die Zündmechanismen der Minen häufig aus mehreren metallischen Teilen in unmittelbarer Nähe zusammen. In einem zeitlich veränderlichen elektromagnetischen Feld beeinflussen sich diese Teile untereinander durch Gegeninduktion.

Um die Frequenzabhängigkeit dieser Gegeninduktion zu untersuchen wurden Simulationen der Einzelteile der Mine mit Simulationen der gesamten Mine verglichen. Ein solcher Vergleich für den magnetisierbaren Schlagbolzen aus Stahl und den nichtmagnetisierbaren Zündhut aus Aluminium ist in Abbildung 4 zu sehen.

Der Effekt der Gegeninduktion wird in Abbildung 5 verdeutlicht. Hier ist die relative Differenz zwischen den addierten Einzelsimulationen und der gemeinsamen Simulation über der Frequenz aufgetragen. Die relativen Differenzen des Imaginärteils sind mit ca. -12% (bezogen auf die gemeinsame Simulation) für den niedrigen, bzw. bis zu 22% im hohen Frequenzbereich schon für diese einfache Geometrie aus zwei Objekten eine ernst zunehmende Beeinflussung des Signals. Bei dem Realteil ist der Einfluss der Gegeninduktion offensichtlich nur für Frequenzen in der Nähe des

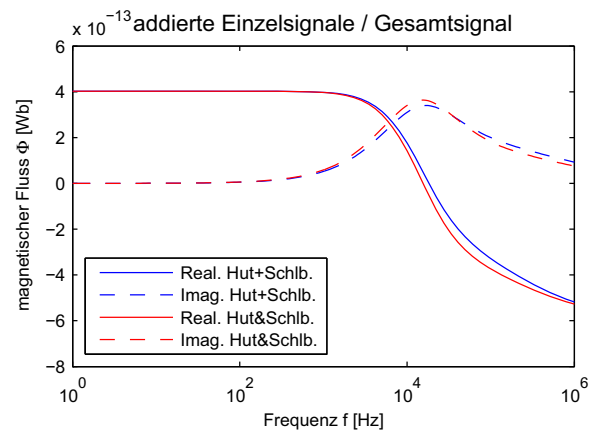


Abbildung 4: Vergleich der Real- und Imaginärteile der addierten Einzelsimulationen (+) und der gemeinsamen Simulation (&). Betrachtete Größe: magnetischer Fluss durch die Empfangsspulen.

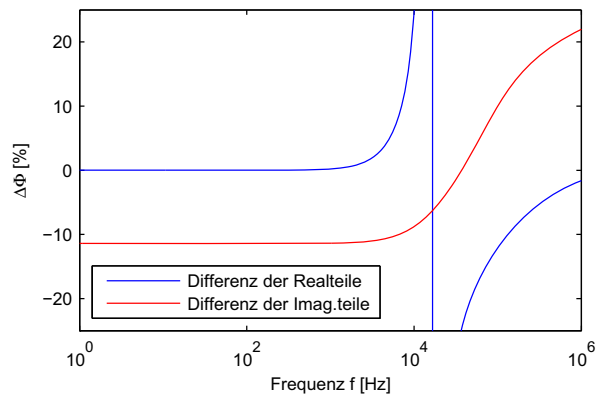


Abbildung 5: Relative Abweichung der Real- und Imaginärteile des magnetischen Flusses zwischen den Einzelsimulationen und der gemeinsamen Simulation aufgrund von Gegeninduktionseffekten

Nulldurchganges des Realteils des magnetischen Flusses (siehe Abbildung 4) wichtig. Da dieser Bereich aber für alle Objekte unterschiedlich ausfällt, ist der Einfluss auf die Inversion schwer abzuschätzen.

Wichtig für die spätere Auswertung der Inversionsergebnisse ist der steigende Einfluss der Gegeninduktion mit wachsender Frequenz für das gesamte Signal. Dieser macht sich im Verhalten des RMS-Wertes bemerkbar (s.u.).

3.2 Einfluss der Frequenzen

Um den Einfluss der hohen bzw. tiefen Frequenzen auf die Inversionsergebnisse zu untersuchen, wurde die Inversion für verschiedene Kombinationen

von Simulationsdaten durchgeführt. Tabelle 1 ordnet jeder Kombination von Frequenzen eine Konfigurationsnummer zu. Bei dieser Konfigurationswahl verschiebt sich der Schwerpunkt der beteiligten Frequenzen von mittleren zu hohen (Konfigurationsnummer 1-8) bzw. zu niedrigen Frequenzen (Konfigurationsnr. 9-16).

Konfigurationsnr.	Frequenzen in kHz
1	1,2,4,5,10,19.2,30,50,75,100
2	2,4,5,10,19.2,30,50,75,100
3	5,10,19.2,30,50,75,100
4	10,19.2,30,50,75,100
5	19.2,30,50,75,100
6	30,50,75,100
7	50,75,100
8	75,100
9	1,2,4,5,10,19.2,30,50,75,100
10	1,2,4,5,10,19.2,30,50,75
11	1,2,4,5,10,19.2,30,50
12	1,2,4,5,10,19.2,30
13	1,2,4,5,10,19.2
14	1,2,4,5,10
15	1,2,4,5
16	1,2,4

Tabelle 1: *Tabelle der Konfigurationsnummern*

Die Inversionen wurden mit dem in Kapitel 2.3 beschriebenen Inversionsprogramm durchgeführt. Die freien Parameter, also die, die von dem Algorithmus verändert werden dürfen, sind: Tiefe T , Position x und y des Mittelpunktes des Ellipsoiden, Halbachsen a und b , Deklination D , Inklination I , elektrische Leitfähigkeit σ und relative magnetische Permeabilität μ_r .

Ein deutlicher Unterschied zwischen den Inversionsergebnissen der drei simulierten Objekte ist der RMS-Wert. In Abbildung 6 ist dieser über der Konfigurationsnummer aufgetragen. Wie man sieht liegt der RMS-Wert der Mine bei hohen Frequenzen deutlich über den RMS-Werten der anderen Objekte. Außerdem ist er in diesem Bereich fast konstant. Dies ist ein Effekt der Gegeninduktion zwischen den drei Einzelteilen der Mine. Je größer der RMS-Wert ist, um so schlechter erklärt unser Modell das tatsächliche Objekt. Das bedeutet aber, dass das verwendete Modell zu einfach ist, und man dadurch Informationen über das Objekt verliert. Angestrebt werden demnach Konfigura-

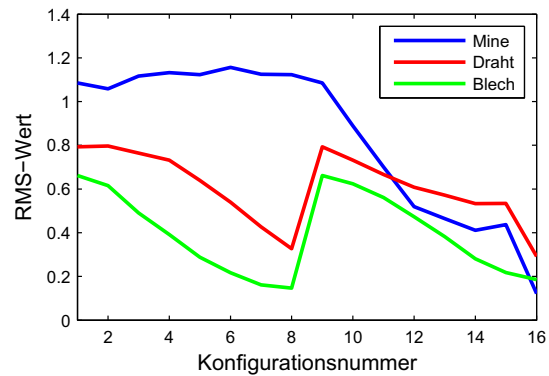


Abbildung 6: *RMS-Wert als Maß der Anpassung über den für die Inversion verwendeten Frequenzkonfigurationen.*

tionen mit einem möglichst großem RMS-Wert, da diese mit einem besseren Modell als einem einfach Ellipsoiden mehr Informationen enthalten. Somit lassen sich komplexe Objekte, deren Einzelteile eng beieinander liegen, von einfachen Objekten unterscheiden.

Um die Wichtigkeit der einzelnen Frequenzen quantitativ zu untersuchen wurde die Informationsdichtematrix (Gleichung (15)) ausgewertet. In Abbildung 7 sind die gemittelten Wichtigkeiten der Daten für jede Frequenz aufgetragen. Die großen und kleinen Frequenzen haben offen-

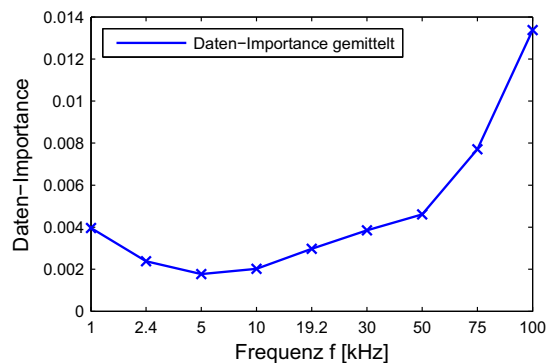


Abbildung 7: *Wichtigkeit der Frequenzen über den Frequenzen aufgetragen.*

sichtlich einen höheren Informationsgehalt, als die mittleren. Bei der Suche nach der besten Frequenzkombination wurde sich entsprechend auf diese Frequenzen konzentriert.

4 Beste Frequenzkombination

Um die beste Frequenzkombination zu finden, muss zunächst definiert werden, was diese

auszeichnet. Eine gute Konfiguration muss zuverlässige Modellparameter erzeugen. Das bedeutet, dass die Fehler und die Korrelation der Parameter möglichst gering, während die Wichtigkeit der Modellparameter und der Daten möglichst groß sein sollte.

Wenn diese Voraussetzungen für mehrere Kombinationen ähnlich gut erfüllt werden, ist der RMS-Wert der Anpassung ein weiteres Kriterium. Da bei den Inversionsrechnungen als Abbruchbedingung eine quasi stabile Lösung vorausgesetzt wird (RMS-Verbesserung $< 10^{-2}\%$), entspricht der berechnete Ellipsoid der besten, mit diesem Modell möglichen, Anpassung an die Daten.

Als letztes Kriterium dient die Anzahl der Frequenzen. Um den technischen Aufwand und damit auch die Kosten der Herstellung der Metalldetektoren zu minimieren, aber auch um die Rechenzeit der Inversion im Feld zu reduzieren, sollte diese möglichst klein sein.

Um nach diesen Kriterien die beste Frequenzkombination für die Detektion und Identifikation von Antipersonenlandminen zu finden, wurden verschiedene Kombinationen getestet. Die Konfiguration aus $f = 1\text{ kHz}$, $f = 19.2\text{ kHz}$ und $f = 100\text{ kHz}$ erzeugt insgesamt die besten Ergebnisse. Das wichtigste Kriterium sind große Wichtigkeiten für die Modellparameter, insbesondere der Suszeptibilität. Für diese Frequenzkombination ist außerdem die Korrelation zwischen den einzelnen Parametern fast minimal, so dass diese bestmöglich aufgelöst werden.

Da in dieser Arbeit eine mögliche Verbesserung zu bereits bestehenden Metalldetektoren untersucht werden soll, werden nun die Inversionsergebnisse der Mine für die bisher verwendete Konfiguration $f = (2.4, 19.2)\text{ kHz}$ und der neu bestimmten $f = (1, 19.2, 100)\text{ kHz}$ verglichen. Dazu sind die jeweiligen Inversionsergebnisse und, falls möglich, auch die Parameter des simulierten Objektes in Tabelle 2 aufgelistet.

Ein wesentlicher Vorteil der neuen Konfiguration ist der große RMS-Wert. Hier besteht also ein größeres Verbesserungspotential durch komplexere Modelle, als bei originalen Konfiguration.

Die neue Konfiguration erlaubt es außerdem die

Parameter mit einem geringeren Fehler zu bestimmen. Die Parameterfehler, die sich aus der neuen Frequenzkombination ergeben sind alle kleiner² als die Fehler der originalen Konfiguration. Auch die Position ist, wenn auch nur minimal, besser bestimmt. Die x/y-Position und die Tiefe sind im Rahmen der Fehler mit der neuen Frequenzkonfiguration näher an den Werten der Simulation.

In Tabelle 3 sind die Wichtigkeiten der einzelnen Modellparameter für beide Frequenzkonfigurationen verglichen.

Die neue Konfiguration hat eine etwas kleinere Importance der Leitfähigkeit. In Verbindung mit der Korrelation zwischen σ und κ ergibt sich damit eine geringere Auflösung dieser Parameter. Gleichzeitig ist die Importance der Suszeptibilität aber fast doppelt so groß wie die der originalen Konfiguration. Damit ist die Auflösbarkeit der Suszeptibilität auf Kosten der weniger zuverlässigen Leitfähigkeit erhöht.

Die beiden Clutterobjekte zeigen dasselbe Verhalten. Auch hier sind die Wichtigkeiten der Modellparameter maximal und die Korrelation minimal für die Frequenzkombination $f = (1, 19.2, 100)\text{ kHz}$. Somit ist diese Frequenzkombination für alle drei Objekte optimal, um mit einer minimalen Anzahl an Frequenzen maximal viel Informationen zu erhalten.

5 Diskussion

Das Ziel dieser Arbeit war, die Objektidentifizierung anhand eines größeren Frequenzbereiches, bzw. mit mehr Frequenzen zu verbessern.

Der einzige systematische Unterschied zwischen der Mine und den Clutterobjekten ist das Verhalten des RMS-Wertes, wenn sich der Schwerpunkt der verwendeten Frequenzen zu höheren Frequenzen verschiebt. Aufgrund der Komplexität des Zündmechanismus und der damit einhergehenden Gegeninduktion bleibt der RMS-Wert in diesem Bereich nahezu konstant. Um diese Information auszuwerten, benötigt man aber zum einen sehr viele Frequenzen (in diesem Beispiel 9) und zum anderen eine längere Zeit im Feld (einige Minuten), da für viele verschiedene

²Bezogen auf den jeweiligen Parameterwert

Parameter	$f = (2.4, 19.2) \text{ kHz}$	$f = (1, 19.2, 100) \text{ kHz}$	Simulation
Leitfähigkeit	$(4.38 \pm 0.07) \text{ MS/m}$	$(5.26 \pm 0.04) \text{ MS/m}$	$5.5/30 \text{ MS/m}$
rel. Permeabilität	9.31 ± 0.14	18.87 ± 0.29	30/1
Halbachse a	$(1.94 \pm 0.04) \text{ mm}$	$(2.03 \pm 0.03) \text{ mm}$	–
Halbachse b	$(3.64 \pm 0.04) \text{ mm}$	$(3.30 \pm 0.02) \text{ mm}$	–
x-Position	$(-0.09 \pm 1.40) \text{ mm}$	$(-0.08 \pm 0.87) \text{ mm}$	0.0 mm
y-Position	$(-0.06 \pm 0.52) \text{ mm}$	$(-0.19 \pm 0.38) \text{ mm}$	0.0 mm
Tiefe	$(80.73 \pm 0.66) \text{ mm}$	$(82.60 \pm 0.59) \text{ mm}$	$(82 - 107) \text{ mm}$

Tabelle 2: Vergleich der Inversionsergebnisse der neuen und originalen Frequenzkonfiguration mit den Werten der Simulation.

Parameter	$f = (2.4, 19.2) \text{ kHz}$	$f = (1, 19.2, 100) \text{ kHz}$
Importance von σ	0.440	0.362
Importance von κ	0.275	0.529
Importance von a	0.693	0.847
Importance von b	0.920	0.900
Importance von x	1.000	1.000
Importance von y	1.000	1.000
Importance von T	0.901	0.914
Korrelation von σ und κ	-0.201	-0.272
RMS-Wert	0.487	0.982

Tabelle 3: Vergleich der Wichtigkeiten der Modellparameter der bisherigen Frequenzkonfiguration mit der denen der neuen Konfiguration.

Konfigurationen Inversionsrechnungen durchgeführt werden müssten. Ob der Aufwand für diesen Ansatz gerechtfertigt ist, müssen weitere Tests mit anderen Minentypen und Clutterobjekten zeigen.

Allerdings gibt es eine Kombination aus nur drei Frequenzen, die wesentlich mehr Informationen als die zwei bisher verwendeten Frequenzen $f = (2.4, 19.2 \text{ kHz})$ enthält und eine zuverlässigere Bestimmung der Objektparameter erlaubt.

Für die Identifizierung der Landminen ist eine Bibliothek der Modellparameter der einzelnen Minen nötig. Diese wird am besten durch Messungen unter Laborbedingungen und anschließender Inversion erstellt. Die Parameter können dann mit dem angepassten Modell der Felddaten verglichen werden um so eine mögliche Mine zu identifizieren (Löhken, 2007).

Wenn im Feldeinsatz dann ein unbekanntes Objekt detektiert wird, welches keine Entsprechung in der Bibliothek besitzt, kann der Inversionsalgorithmus dazu eingesetzt werden, um die Position des Objektes und dessen grundlegende Eigenschaften wie zum Beispiel die ungefähre Größe zu

bestimmen. Eine weitere wichtige Information für den Benutzer des Detektors ist, ob das Objekt magnetisierbar oder nichtmagnetisierbar ist. Diese Informationen können mit der in dieser Arbeit ermittelten Frequenzkombination sehr sicher bestimmt werden.

Alle diese Zusatzinformationen können dem Nutzer auf einfache Weise grafisch dargestellt werden, so dass beim Bergen des Objektes zum einen die Sicherheit und zum anderen die Geschwindigkeit erhöht werden. Damit wäre eine wesentliche Verbesserung bei der Räumung von Landminen erreicht.

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GEOLORE: Migration from an Experiment to a versatile Instrument

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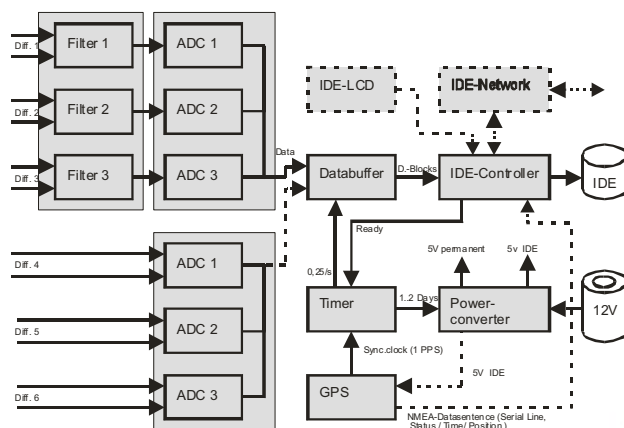
Introduction

Originally the datalogger GEOLORE (**Geophysical-longtimerecorder**) was designed for recording of electrical fields on a lake bottom in Iceland. The system design has been strongly focused on low power consumption to allow long time recording with standard batteries. The principle was presented at the EMTF-meeting in 2003 /1/ and in an electronic magazine /2/. The successful practical operation was been presented in the poster /3/. During the last four years other applications came up and the technology of the datalogger was improved extensively.

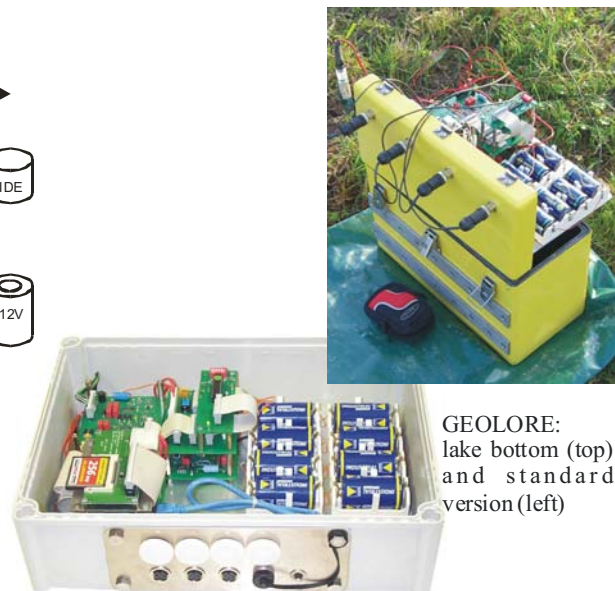
All hard- and software components were individually developed and optimized during the last years, and are presented in this poster.

System Overview and Principle of Operation

The datalogger GEOLORE is designed for logging voltages over long time intervals typically for several months without service. All components were optimized for low power consumption, using modern microsystem technology. For easy upgrading or modification the system is divided up into separate units, each on a printed circuit board.



Blockdiagram of GEOLORE, the basic system is outlined, the options are dotted. Each printed circuit board is shaded grey.



GEOLORE:
lake bottom (top)
and standard
version (left)

The datalogger is divided into two main units. The interaction between the units is controlled by the timer module. The timer and aquisition unit are powered continuously. The power intensive components are operated temporally on demand. The timer module generates the sample clock and the power management signals to control the special designed DC/DC-converter. The input signals are digitized by a 24-bit ADC and are stored intermediately in the SRAM databuffer unit. When the buffer

is filled (or on user request by a push button) the intelligent IDE-controller is switched on and the data are copied to the CF[®]-card. The IDE-controller is shutdown after the copy procedure.

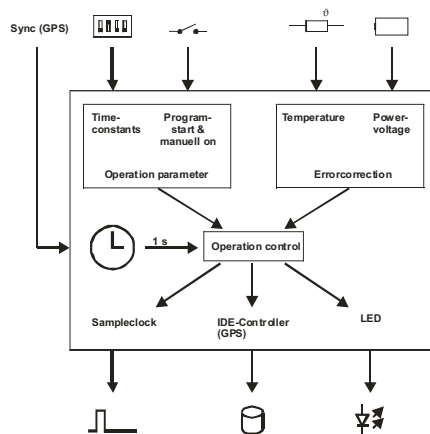
Addon 6 Input Channels

In the first experimental setup GEOLORE was equipped with a databuffer of 1 Mbyte and one ADC-board comprising three input channels. Because the 3-channel board was well approved and the RAM-chips of the first version of the databuffer are obsolete a new databuffer with 2 input connectors for 2 ADC-boards was designed. The buffersize, equipped with modern SMD-chips, has been extended to 2 Mbyte to avoid modifications in the timer system and keep interchangeability. Now GEOLORE can be equipped with one or two ADC-boards without configuration in firmware.

For a planned experiment in Iceland also a new active high impedance input and lowpass filter were designed.

New Timersystem

The sampleclock is generated by the microcontroller, including the temperature error compensation. The sample rate can be adjusted via a DIP-Switch in steps up to 8 Hz. The powerconsumption is proportional to the selected samplerate.

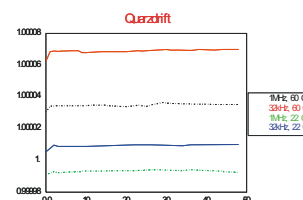


The timer is driven by an external crystal with a frequency of 32.768 kHz (for lowest power consumption) or 1 MHz. The deviation from the nominal frequency results from three major items: **deviation from nominal** resonance frequency, temperature drift and long term stability. The first two parameters can be compensated by the timers firmware. The last one results from fabrication process [and can only be influenced by the manufacturer].

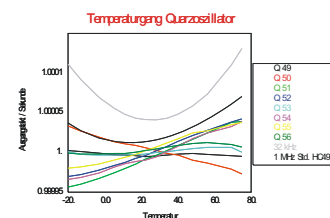
To enhance clock accuracy of two options are available. The standard crystal can be replaced by a special fabricated custom specific AT-cut crystal with higher accuracy and well defined minimum ageing (only with > 1 MHz available). The highest accuracy can be achieved by the external synchronization with an external high precision clock, e.g. a commercially available GPS-receiver, which is more power consuming. In our system the timebase is resynchronized periodically once an hour.



Intelligent crystal based Timebase



Ageing of a standard consumer crystals with 32kHz (tuning fork) and 1 Mhz (AT-cut). With high precision AT-cut crystal (fabricated by KVG) no ageing could be observed.



Drift versus temperature of typical (uncompensated) crystal oscillators

Interfacing external Devices

The CF[®]-card is connected to the system via the IDE-bus (known as a harddisk interface from PC's). The DOS-Filesystem FAT16, used in standard PC's, has been implemented to keep compatibility. The DOS code has been optimized in program (ROM) and working (RAM) space with respect to the limited resources of the microcontroller. The IDE-bus has been chosen to avoid changes in hardware for easy integration of new data devices also.

Disk Operating System

Up to now a filesystem based on the FAT16-structure is used for easy interchanging data. This filesystem is commonly used in PC-systems and allows the handling of datafiles up to 2 Gbyte. In the originay system the recording file had to be created prior to operation. This procedure was optimized by implementing a lean real disk operating system to establish easy handling of the system.

Tasks which create, write or delete FAT based data files, modify the directory entry or FAT. Therefore a RAM-memory with significant more then 512 Byte is needed, because sectors to be modified have to be buffered during the update process in the RAM. A higher sophisticated software system has been implemented with the new pin compatible microcontroller of type PIC18F252. The codesize, including the BIOS for interfacing the memory card, amounts to approximate by 10 kBytes. In the present system a new, unprepared CF[®]-card can be inserted and the system can be started for acquiring the data. The operating system can manage a filesize up to 2 Gbyte.

LC-Display

Optionally the network and GPS can be controlled by an LC-display.

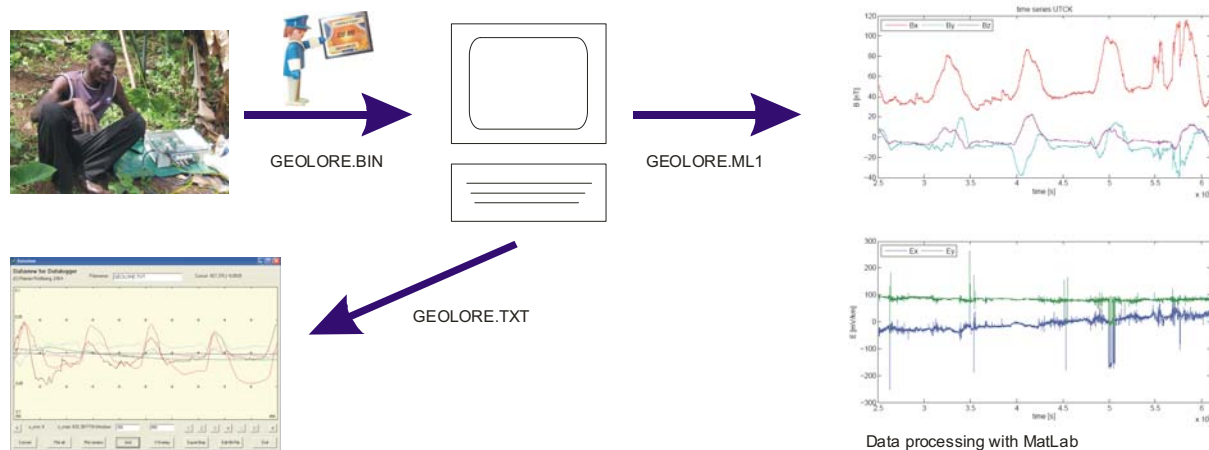
Network Interface

A network server, based on the ethernet and IP, has been added for remote monitoring. The most common IP protocols HTTP (for monitoring via standard browser) and TFTP (for bidirectional filetransfer) are implemented.

Application and Data Management

Longtime recordings produce large volume of data, typically binary filesize of more then 500 Mbyte have to be managed. GEOLORE is designed to handle most of the software tasks with standard software packages, supplied by the modern operation system of standard personal computers. Interchanging of data can be done easily and rapidly with a standard USB-cardreader.

ADC-data are stored in a compact binary format. For first verifications of the recordings in the field a visualizing software tool DATAVIEW, running under Windows, has been programmed to check data quality in field. The software displays the complete recording without limits in size. Using a zoom function the recording can be studied in more detail.



First Inspection of Data

Conclusion

With the integration of new functions the datalogger GEOLORE has been developed to a multifunctional easy to handle instrument. The system has been used successfully in various geological field campaigns for recording passive and active induced EM-fields and will be employed in future experiments. An add on for remote controlling is planned in the future.

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Long Period Telluric – Magnetotelluric Measurements in the North East of the Rwenzori Mountains, Uganda

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Introduction

The survey is part of the DFG funded research project RiftLink, which addresses the causes of rift-flank uplift in the East African Rift since the late Miocene, its impact on climate changes in Equatorial Africa, and the possible consequences for the evolution of hominids. The immediate objective is to gain a process understanding of rift-flank uplift by investigating the origin of the more than 5000 meter high Rwenzori Mountains, which are located within the Ugandan part of the East African Rift (Fig.1).

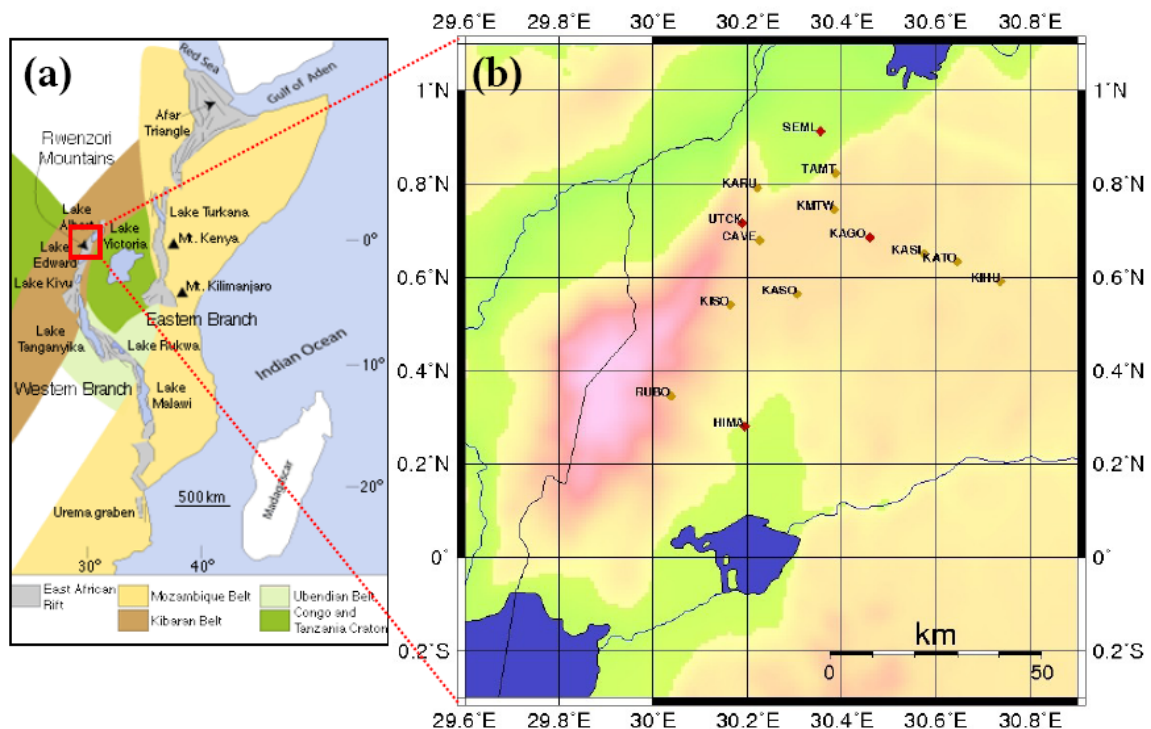


Fig. 1: (a) Geological situation, the red square shows the area of interest, (b) site distribution: orange diamonds refer to telluric sites, red diamonds to magnetotelluric sites

RiftLink is subdivided into four scientific themes including eleven projects.

Theme A: Lithosphere/ Asthenosphere Processes

Theme B: (Near-) Surface Processes

Theme C: Surface/ Atmosphere Processes

Theme D: Modeling

Within the project A2 the conductivity structure of the crust and upper mantle beneath and East of the Rwenzori Mountains was investigated with the magnetotelluric method. In the long run the measurements will give a detailed 3D image of conductive features from the upper crust down to several hundred kilometres into the upper mantle.

The following goals will be pursued:

- a) Investigation of the geometry and depth range of conductive faults,
- b) Correlation of conductive structures with the seismogenic zone,
- c) Correlation of electrical anisotropy with fault plane solutions,
- d) Mapping of the electrical asthenosphere including anisotropy,
- e) Investigation of conductivity mechanisms, especially the influence of water, using petrologic results,
- f) Constraining geodynamic modelling.

In this contribution we show the measurement set up and first results.

Field Set Up

During our first field experiment we wanted to investigate the Rwenzori Mountains - Rift Shoulder Connection (RMRSC, Fig. 2). Thus 14 instruments were set up at an area around the

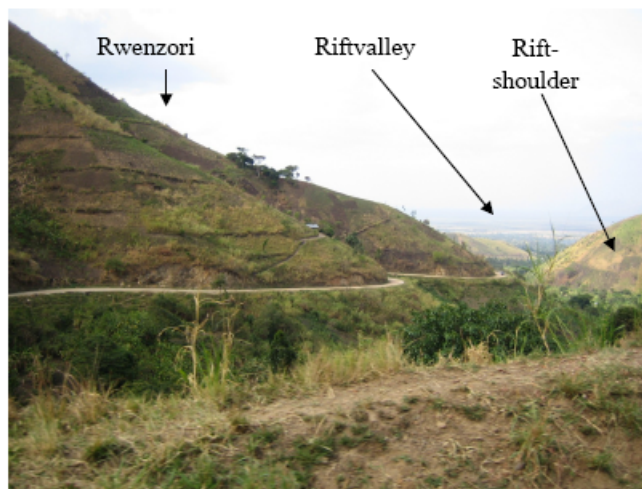


Fig. 2: View to the North into the Riftvalley, view point close to site UTCK (Fig. 1), at the Rwenzori Mountains- Rift Shoulder Connection (RMRSC).

North-Eastern Edge of the Rwenzori Mountains from April to July 2007, the distance between the sites varied from 10-15 km (Fig. 1). At all locations time variations of the natural telluric field were recorded continuously with 4 Hz sampling rate, at 4 sites additionally time variations of the magnetic field were observed by 3-component fluxgate magnetometers (GEO-MAG 01, MAGSON). The horizontal telluric field was calculated from the voltage difference between pairs of electrodes ($\text{Ag}/\text{AgCl}/\text{KCl}(\text{aq})$) buried in the ground and separated by 20-40m. The electrodes are installed in a saturated KCl solution within a PVC tube; small ceramic

diaphragms guarantee electric contact to the soil. All the field components were recorded with the GEOLORE data logger (Roßberg, 2007); the internal clock was triggered by a GPS signal to enable synchronization of the field records.

For the telluric sites all cables were buried in the ground and the logger was wrapped in a waterproof PVC bag, while for the Magnetotelluric sites the electronic device was stored in a ZARGES box and the magnetometer was buried in the ground.

Logging the telluric field is less power consuming and the equipment is far less expensive than that for magnetic field observations – therefore the number of telluric sites is much higher than that of the magnetotelluric sites.

Time Series

Fig. 3 shows the simultaneous time series of the horizontal magnetic field components at the 3 Northern sites for 4 days. There is no influence by the equatorial electrojet on the daily variations as the dip equator is about 10° N (cf. Kuvshinov et al., 2007). Despite some spikes of artificial origin there is almost perfect correlation between the data for each component respectively. The spatial homogeneity of the magnetic field is also demonstrated for short oscillations of several minutes period. There is also a high correlation between the orthogonal tel-

luric and magnetic field variations, although the amplitudes of the telluric fields vary significantly between different sites, reflecting the influence of lateral near surface variations of the electric conductivity.

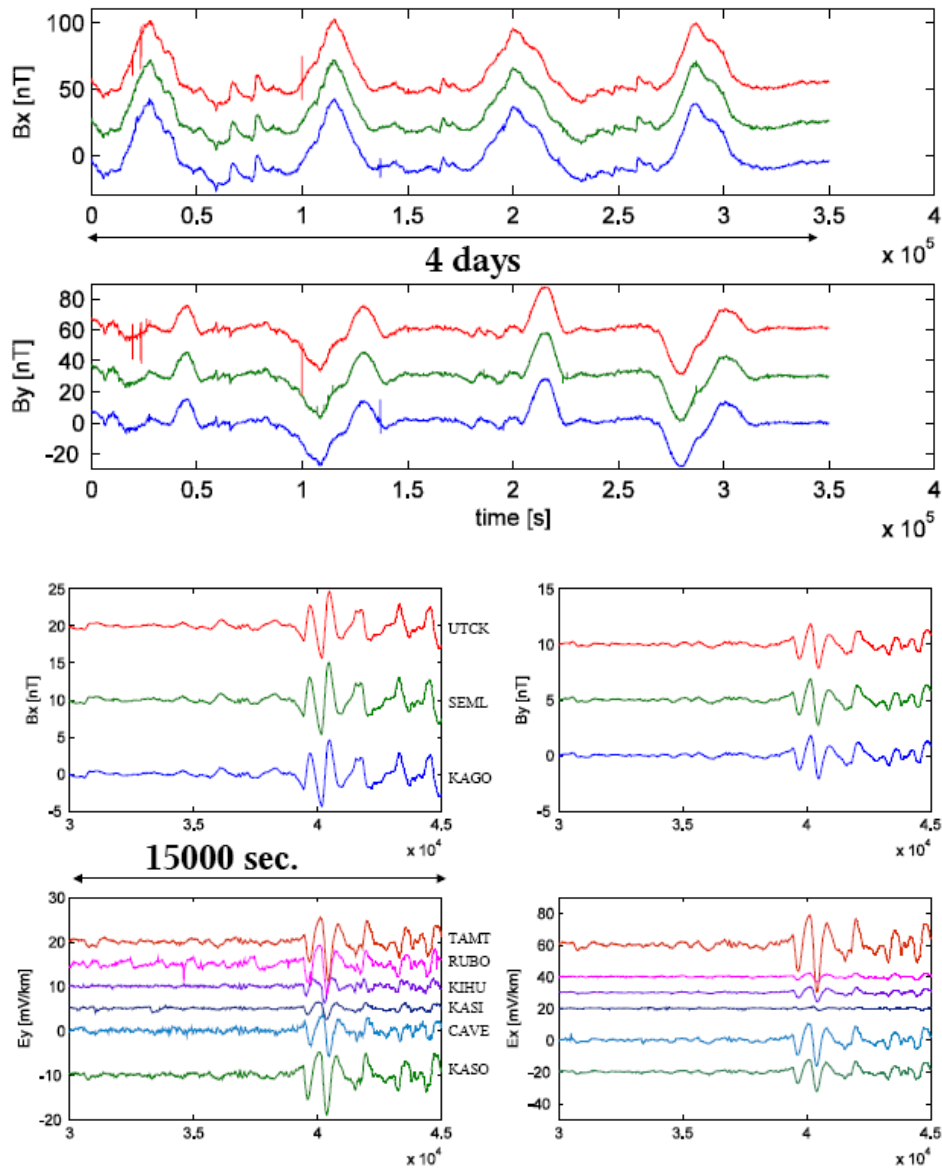


Fig. 3: Examples for time variations of magnetic and telluric fields at different sites: (Top) The horizontal magnetic field at 3 sites for a 4 days time interval, (Bottom) Magnetic and telluric fields at different sites for a 4 hours time interval.

Transfer Functions

Using the bivariate approach $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix}$ in the frequency domain, the frequency dependent complex elements of the impedance tensor $\mathbf{Z}$, i.e. the transfer functions, contain the information about the conductivity structure of the subsurface. Fig. 4 shows the transfer functions at sites TAMT and CAVE (cf. Fig. 1). The complex values are displayed as apparent resistivity and phase. Note the significant differences between the 2 tensor elements

and between the 2 sites with phases far above 90° . We take the high phases as an indication for an extreme current distortion due to high lateral conductivity changes in the vicinity of the sites.

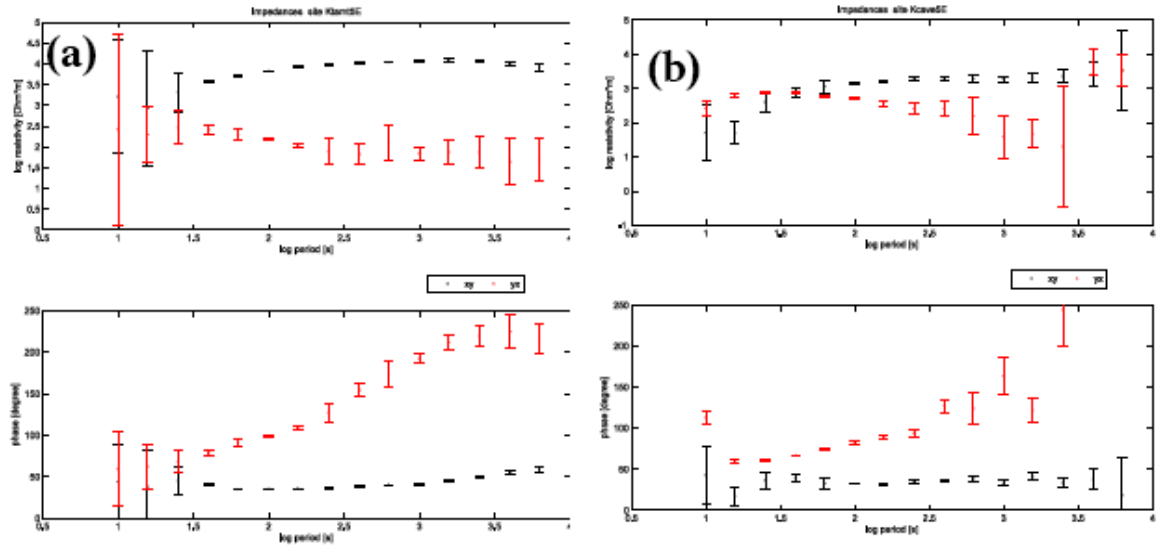


Fig. 4: Examples for transfer functions of the impedance tensor elements Z_{xy} (black) and Z_{yx} (red) at sites TAMT (a) and CAVE (b). The values are displayed as apparent resistivity (top) and phase (bottom).

Phase Tensor

The telluric field and thus the transfer functions' magnitudes can be influenced by near-surface conductivity inhomogeneities, giving rise to strongly distorted estimates of the conductivity distribution (static shift). A more robust estimate is the phase tensor Φ (Caldwell et al., 2004) which is derived from the imaginary and real part of the impedance tensor by

$$\Phi = \begin{pmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{yx} & \Phi_{yy} \end{pmatrix} = \Im \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} / \Re \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \quad \text{and} \quad \phi = \tan^{-1} \Phi.$$

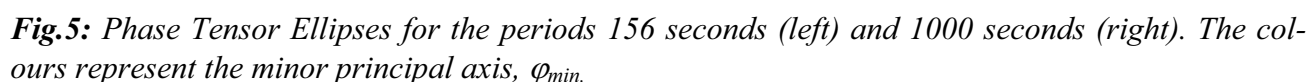
There are 3 rotational invariants of Φ , which are the principal axis, $\phi_{\min}$ and $\phi_{\max}$, and the skew

$$\beta = \frac{1}{2} \tan^{-1} \left(\frac{\Phi_{xy} - \Phi_{yx}}{\Phi_{xx} + \Phi_{yy}} \right),$$

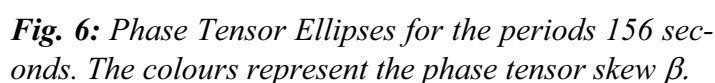
which is a measure of the tensor's asymmetry. The aspect ratio of

the ellipse is a measure for the influence of lateral conductivity changes on the data, whereas the skew angle β is an indicator for the existence of a 3 dimensional conductivity structure ($\beta=0$: 1D, 2D, $\beta \neq 0$: 3D), appearing within the range of the skin depth of the variation fields.

It is most convenient to represent Φ in form of an ellipse which is normalized by its major principal axis $\phi_{\max}$. Fig. 5 displays the phase tensor ellipses at some locations for the periods of 156 and 1000 seconds. The area of the ellipse is colour coded by the value of the minor axis, $\phi_{\min}$, using a colour scale sensitive to the frequency of occurrence of the values. Thus the colours reflect the change of $\phi_{\min}$ with period resp. depth. Each period reveals a spatially consistent pattern of the ellipses orientation and their colours, however, note the significant phase changes for sites close to and far from the RMRSC. In general there is an increase of $\phi_{\min}$ towards longer periods (cf. Fig. 4). Furthermore the overall orientation of the ellipses is oblique to the Rift axis, but parallel to the well known geological strike direction of the main



The skew angle β is displayed in Fig. 6 for the period of 156 seconds. Its pattern is spatially very consistent and it shows very impressively the increase of β at the RMRSC, thus pointing at the origin of a significant three dimensional distortion of the electric currents.



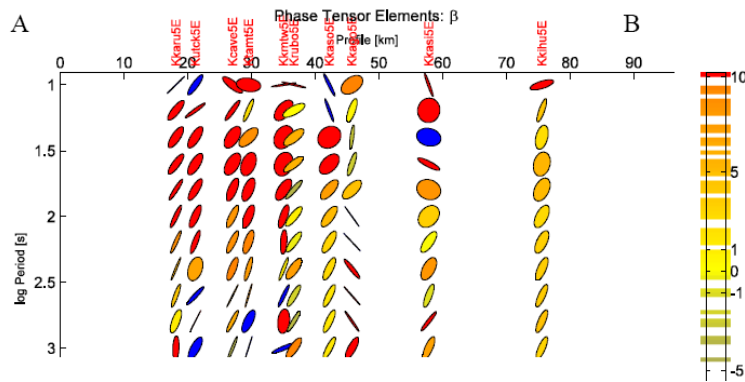


Fig. 7: Phase Tensor ellipses coloured with the skew angle β along the profile AB (Fig. 6) with all sites. Note the low values of β at the sites KIHU and RUBO (B) vs. high values at the sites near the RMRSC (A).

Fig. 7 presents a cross section of the phase tensor ellipses of different sites projected onto the profile AB (cf. Fig. 6) for the periods observed. The ellipses colour reflects the skew β . Despite the partly heterogeneous picture, it is evident that the high values of β concentrate towards A for shorter periods. Thus it may be concluded that the 3D distortion is limited to the upper crust.

Conclusion

This contribution gives first results: The spatial and frequency dependence of the transfer functions' amplitude and phase reveal information for the conductivity structure underneath the Northern Section of the Rwenzori Mountains, best shown by phase tensor ellipses. There is evidence for a strong distortion of the induced currents North of Fort Portal where the most Northern Part of the Rwenzori Mountains contacts the Eastern Part of the Rift Shoulder.

The preferential direction of the electric currents matches the direction of the major fault system to the East of the Rwenzori Mountains, whereby there is a slight clockwise rotation of the preferential direction for greater depths.

These findings open up the question, whether the current deviation is caused basically by the high resistive body of the Rwenzori Mountains, which at least partly block the high conductive sediments of the Rift Valley, or if there is a wide spread electrical anisotropy within the Earth Crust.

Therefore it is planned to extend the site array towards the Southern part of the Rwenzori Mountains in a 2nd field survey during spring 2008 to test these hypotheses.

Acknowledgments

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Tufa Deposits in the Mygdonian Basin (Northern Greece) studied with RMT/CSTAMT, VLF & Self-Potential

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SUMMARY

During 2007, near surface EM geophysical soundings have been conducted to study tufa outcrops in the Mygdonian Basin. The presence of carbon rich hot springs in the eastern lake (Volvi) and various tufa outcrops in the northern rim of the basin supports our idea that the tufa genesis is connected down to lineament faults in the basement. More precisely, those deposits are preferentially located along fracture traces, either immediately above extensional fissures or in the hanging wall of normal faults. Hence, the distribution of the tufa at surface may delineate the fault distribution at depth and/or shows areas with increased basement fracturation. Analysing the present day geothermal regime allows to sketch a hydrogeological model based on neotectonic seismic activities in the past.

Keywords: Travertine, Tufa, Self-Potential, streaming electrical potential, RMT, CSTAMT, VLF, Mygdonian basin, geothermal area, normal faulting

INTRODUCTION

The Mygdonian basin, situated between the two lakes Volvi and Lagada ca. 45 km east of Thessaloniki (Fig. 1), is a neotectonic graben structure (5 km wide) with increased seismic activity along distinct normal fault patterns (Papazachos et al., 1979; Karagianni et al., 1999; Goldsworthy et al., 2002). Fluvio-terrestrial and lacustrine sediments (approx. 350-400 m thick) are overlying the basement consisting of gneiss with several marble bands embedded between amphiboles, limestones, quartzites or phyllites.

The presence of carbon rich hot springs in the eastern lake (Volvi) and various tufa outcrops in the northern rim (Fig. 2) of the basin supports our idea that the tufa genesis is connected down to lineament faults in the basement. More precisely, tufa deposits are preferentially located along fracture traces, either immediately above extensional fissures or in the hanging wall of normal faults (Hancock et al., 1999; Piper et al., 2007). Hence, the distribution of the tufa at surface may delineate the fault distribution at depth and/or shows areas with increased basement fracturation.

Based on this hypothesis, a combined geophysical survey has been conducted to answer the following questions:

- i) The tufa morphology (cones, pinnacles or bedded elongated structures along fissure ridges)
- ii) The present day hydrothermal regime/activity
- iii) Depth ranges of the tufa deposits

TUFA & TRAVERTINE

Referring to Hancock et al. (1999), tufa and travertine are both 'freshwater' limestone deposits originated from springs or other waters that are supersaturated with calcium carbonate. There are gradations among the two rock types. Based on their texture they can be identified: tufa is a name that is usually used to describe a porous and bedded deposit originated from either cold springs or accumulating in veins or lakes. Travertine is a limestone of compact and white texture generated by hydrothermal hot-springs (Hancock et. al, 1999). Due to Hancock's (1999) analysis of deposits in the Mygdonian Basin in the same area, we name this outcropping rock type tufa. Tufa cones as shown in Fig. 2 form in and on unconsolidated sediments of past and present lake floors where the sediments overly fissure (Fig. 3) in bedrocks (Hancock et. al, 1999). In regions of neotectonism, tufa/travertine preserves a layered signature reflecting the past earthquake activity (Piper et al., 2007)

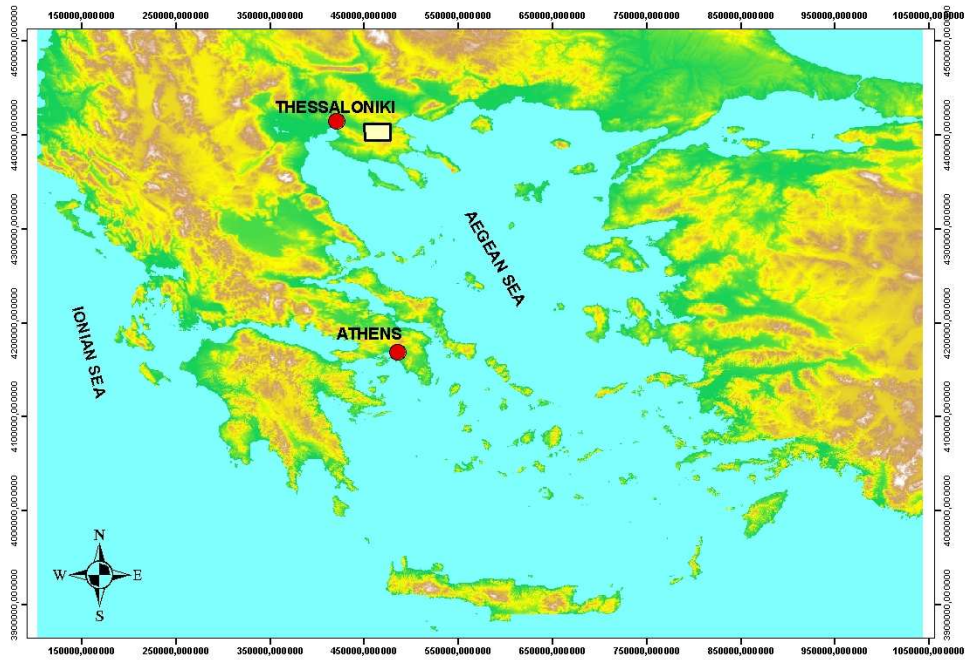


Figure 1: Map of the Aegean See with the location of the survey area (white box).



Figure 2: Tufa cones in the northern part of the Mygdonian Basin. View to the northeast with the village Profitis in the background.

GEOPHYSICAL SURVEY

Limestone deposits such as tufa and travertine exhibit low magnetization (Piper et. al, 2007) but can show high resistivities compared to their host sediments. Especially the lacustrine tufa is generally so porous that if it is above the water table it is almost dry and is hence very high resistive (Ian Hill, pers. comm., 2007).

The analysis of VES soundings (EUROSEISTEST, 1995) along the profile Stivos-Profitis already stated shallow high resistive thin and inhomogeneous layers that have been associated with travertine (tufa) and causes noise in the data acquisition. Hence, near surface resistivity methods are promising tools to detect these rock types.

During 2007, near surface EM geophysical soundings (Tab. 1) have been conducted to study tufa outcrops in Mygdonian Basin. Figure 4 shows the RMT/CSTAMT site location along two Profiles (1&2). Profile 1 is part of the Profile Stivos-Profitis, as referred in the EUROSEISTEST (1995) report for this area. In order to investigate near surface structures with high resolution and the present day geothermal regime, additional VLF (Müller, 1982b; Stiefelhaven, 1998) and self-potential (SP) data were obtained along a road that defines Profile 1.

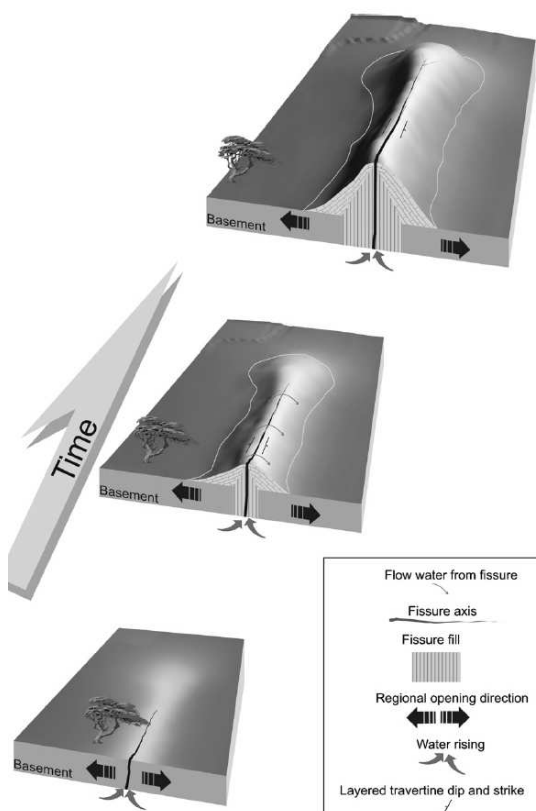


Figure 3: Successive stage of the formation of a travertine mound above an extensional fissure (Piper et. al., 2007; Mesci, 2004)

Method	Sampling	Device	Freq. range
SP	dx =10 m	Ag/AgCl electrodes Gurk (2007)	
RMT/CSTAMT	dx=50 m or more	ENVIROMT Bastani (2001)	1-250 kHz
VLF	10 Hz	CHYN Müller (1982a)	19.6 kHz

Table 1: List of sounding parameter.

RMT/CSTAMT soundings:

The soundings were carried out along both profiles. A remotely controlled double horizontal magnetic dipole transmitter that is located at certain distances provides the signals for the CSTAMT data. Time series in the RMT and CSTAMT band are processed in the field up to sounding curves allowing to reject data and to repeat the measurement if necessary. Generally data are collected every 50 m along the profile 1. On profile 2, this distance is extended or shortened, if necessary.

Continuously VLF sounding:

Prior to the soundings, we set up marks every 50 m along the profile to assure a good reference to the RMT/CSTAMT measurements. A 19.6 kHz transmitter located at an azimuth of 270°N was chosen to collect VLF data along profile 1. Depending on the walking speed, the sampling rate of 10 Hz will allow for a lateral resolution of about 50 cm. The skin depth is of about 23 m assuming a half space of 40 Ω m.

SP data:

For this study we used Ag/AgCl electrodes arranged on a spade stick. One electrode is kept fixed, whereas the other electrode sampled the potential field on profile 1 every 10 m.

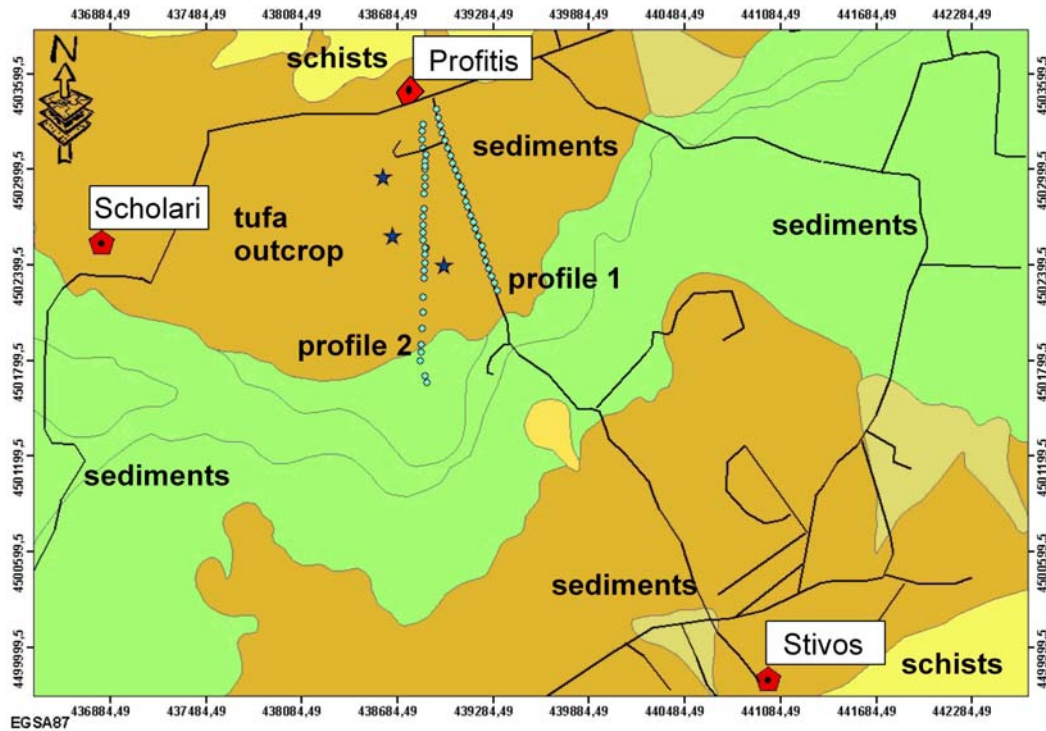


Figure 4: Map of the study area with tufa outcrops (stars) and RMT&CSTAMT site locations (dots). Black lines indicate major roads.

DATA PROCESSING & ANALYSIS

RMT/CSTAMT data are preprocessed in the field. Normally, no further processing steps are required. The data are available as EDI MT SECT files (Wight, 1987) including GDS data.

To invert the data within the 2D approach we followed the strategy suggested in (Pedersen & Engels, 2005). At this stage, we confine ourselves to analyse Berdichevsky Invariant data (Berdichevsky et al., 1976) which has advantages compared to bi-modal data in 2D inversion. Any variability in strike direction with period does not affect the results as much as for bi-modal inversion, when the proper mode decomposition is vital.

The 2D inversion routine by Siriponvaraporn and Egbert (Siriponvaraporn & Egbert, 2000) has been used to invert the Berdichevsky Invariant data without any topographic correction. As the error floor, we used the standard deviation of the impedance tensor.

The SP data are drift corrected and the VLF data files are corrected with respect to their location along the profile. Since the quadrature of the VLF soundings are less effected by shaking the antenna while walking, we present these data in the following text.

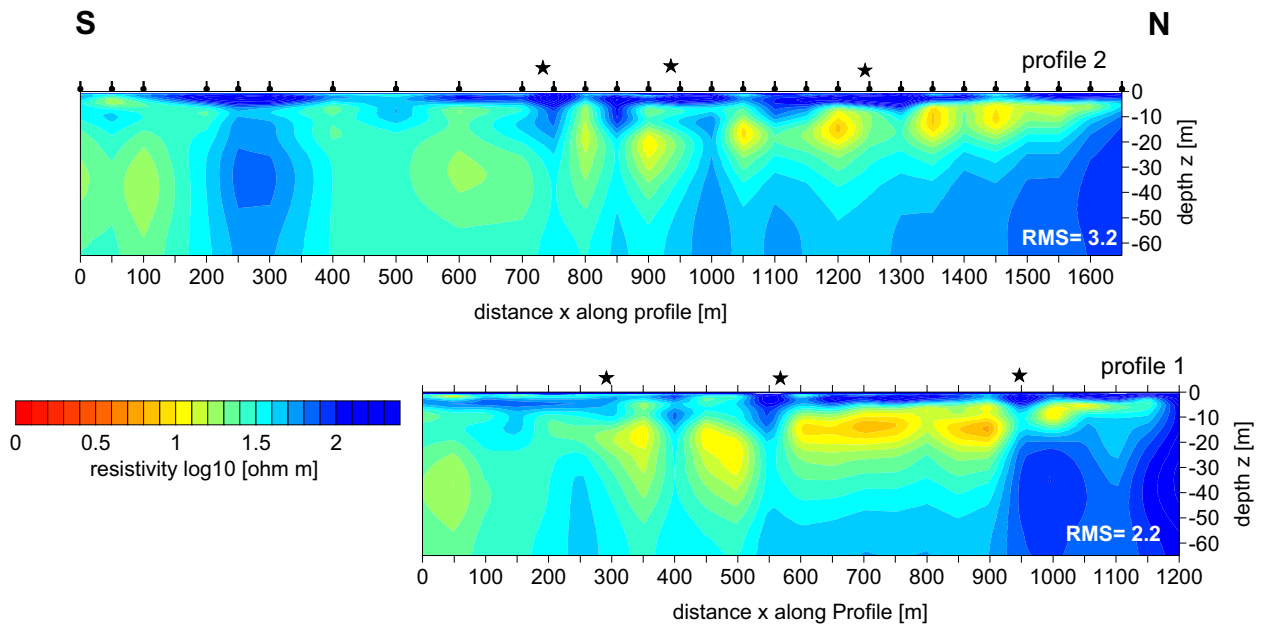


Figure 5: 2D resistivity (Berdichevsky Invariant) cross sections of profile 1&2. Stars indicate projected location of tufa outcrops onto the profiles.

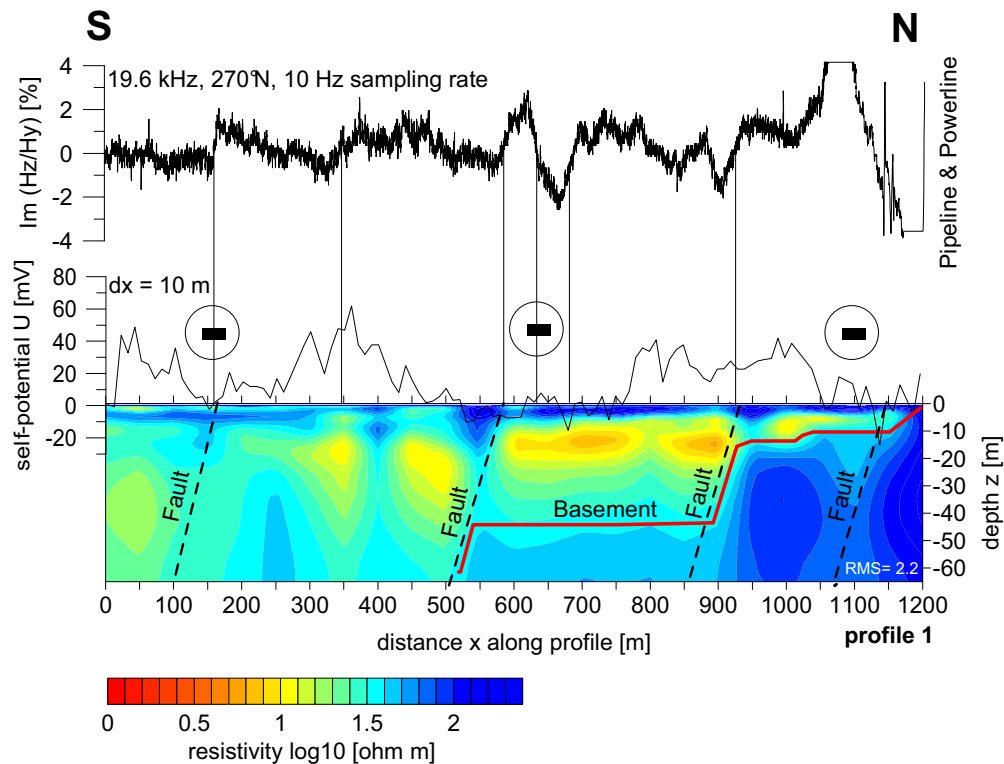


Figure 6: 2D resistivity (Berdichevsky Invariant) cross section of profile 1 together with VLF and SP data. Possible fault locations are presented with dashed lines and assumed basement topography with red lines.

Figure 5 shows results of the 2D inversion for both profiles. Near to the surface (0 to -10 m), we see a series of conductive and high resistive thin layers uneven distributed along the profile. At this stage of investigation, we directly associate the resistive layers with the travertine/tufa rocks and the conductive layers as quaternary sediments. At larger depth, good conductive structures, sometimes interrupted, are predominant followed by the resistive basement rock. Due to the limited skin depth of the EM fields used, the basement is only recognisable at the northern part of the profile where it actually crops out.

Unlike to what we would expect on a consolidated former lacustrine floor, the resistivity structures and therefore the bedding of the sediments are disrupted – and we think, because of the evolving tufa rocks. This idea is supported by the VLF and SP data as sketched out in Fig. 6. Major inflection points in the VLF sounding are superimposed to structural discontinuities in the resistivity cross-section of line 1 indicating strong lateral contrast in conductivity. Those contrasts are likely to be caused by vertical structures such as travertine/tufa pinnacles and faults as sketched out in Fig. 6.

Following the model of Revil & Pezard (1998) in Fig. 7, we are able to identify areas of negative (admittedly, quite weak) self potentials (Fig. 6) on profile 1. They can be addressed to recharge areas at locations where near surface resistive structures are modeled and where major VLF anomalies are present.

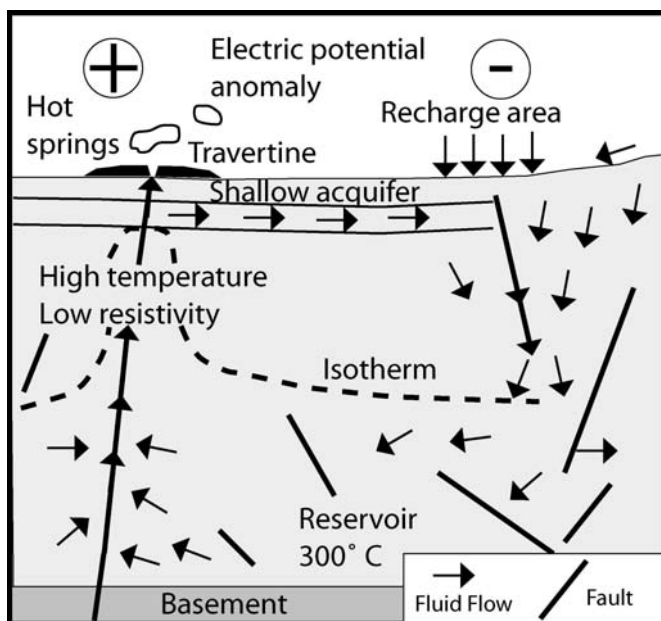


Figure 7: Sketch of a natural thermo-electrokinetic battery (after Revil & Pezard, 1998)

CONCLUSIONS

Supersaturated calcium carbonate hot waters (maybe generated from the marble bands) have followed fissures and fault pathways in the basement and in the unconsolidated lacustrine sediments to create tufa cones and patches of tufa outcrops at surface (Fig. 7). Repeatedly, they became covered by new lacustrine and/or alluvial sediments. Due to a neotectonic seismic event, the geothermal regime changed and the faults that allowed hot water to rise on surface in the past became now preferred pathways for groundwater recharge.

Regarding the morphology of the tufa rocks, we conclude that they cover areas as embedded layers above fissures with possible pinnacle like roots at depth which remain as a more competent rock after erosion processes. The present day depth range of the embedded tufa structures is approx. 0 to -10 m. It is possible that subsequent travertine/tufa layers report revolving changes in the geothermal regime caused by seismic events. Tufa outcrops do indicate areas with increased basement fracturation and may be used to delineate lineament structures at depth.

From a hydrogeological point of view, recharge areas as indicated in Fig. 6 are potential drill locations for groundwater production (e.g. at profile location $x = 600$ m and 900 m).

ACKNOWLEDGEMENTS

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A 3D magnetotelluric study of the basement structure in the Mygdonian Basin (Northern Greece)

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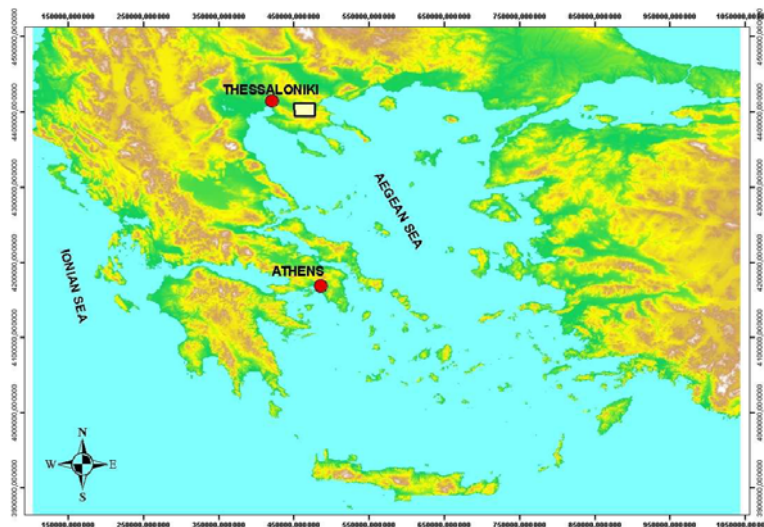
SUMMARY

During 2006 and 2007 a total number of 92 MT/GDS sites were deployed in the Mygdonian basin (Northern Greece) to gain knowledge about the basement structure by means of 2D and 3D MT data inversion and to give information about the top-of-basement depth for seismic wave propagation models. The structure of the basement is fairly well revealed by the data.

Keywords: MT, GDS, Mygdonian basin, top-of- basement, 2D inversion, 3D inversion

INTRODUCTION

The Mygdonian basin, situated between the two lakes Volvi and Lagada ca. 45 km northeast of Thessaloniki (Fig. 1&2), is a neotectonic graben structure (5 km wide) with significant seismic activity along distinct normal fault patterns (Papazachos et al., 1979; Karagianni et al., 1999; Goldsworthy et al., 2002). Fluvio-terrestrial and lacustrine sediments (approximately 200-600 m thick) are overlying the basement consisting of gneiss-schists.



During the project “Euroseistest Volvi-Thessaloniki”, a European test site for Engineering Seismology was installed in order to study velocity cross sections across the funnel shaped valley. With the actual EM survey we intend to map the top-of-basement and to contribute to seismic wave propagation modelling process and site effect assessment (Jongmans et al. 1998; Manakou et al., 2004; Savvaidis et al., 2004; Raptakis et al., 2005).

Figure 1: Map of the Aegean Sea with the location of the survey area (white box).

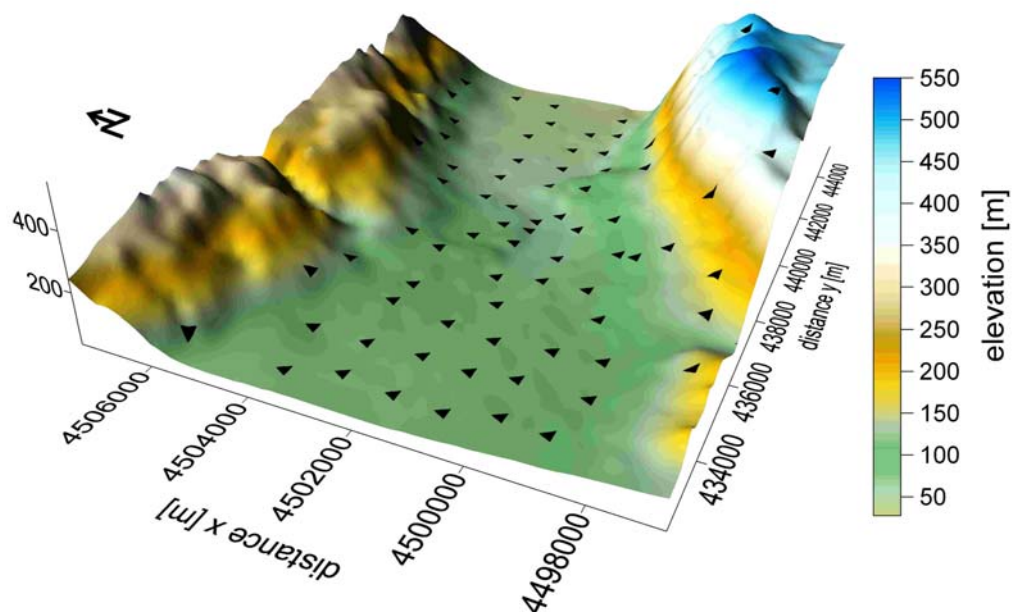
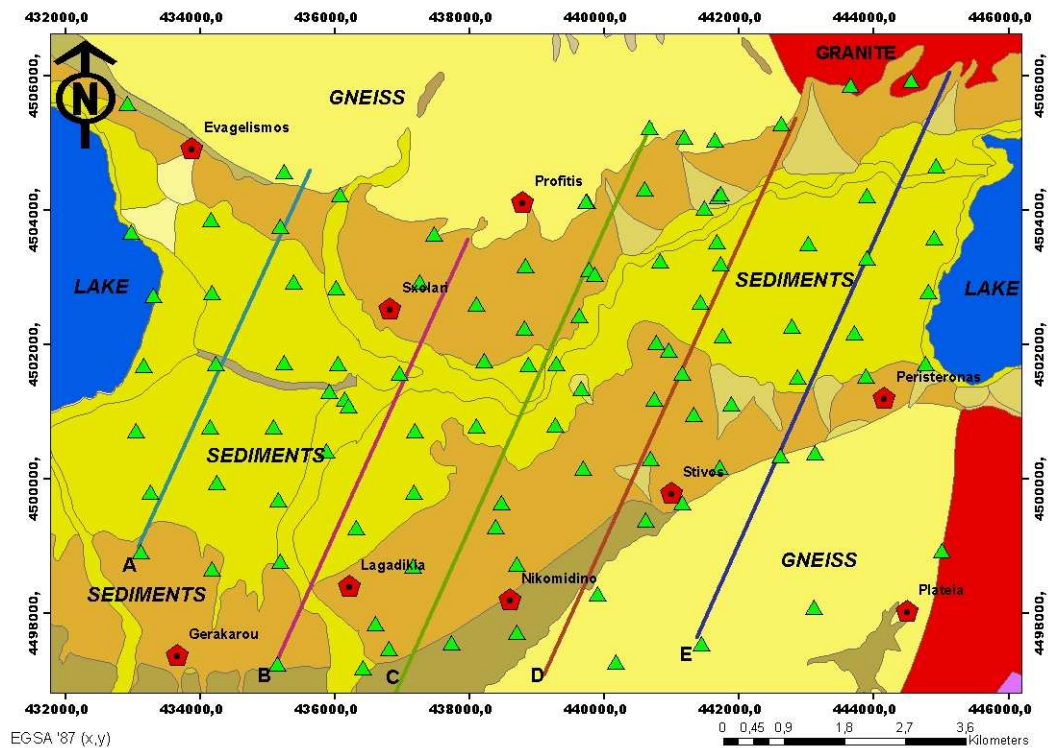


Figure 2: Map of the study area with MT site locations (triangles), Villages (red polygon), 2-D lines and geological outlines (above). Bird view towards the North-East, showing topography and MT site locations (below).

MT/GDS SURVEY

During 2006/2007 a total number of 92 MT/GDS sites were installed in the Mygdonian basin (Fig. 2). The sites were measured roughly on a regular grid (North-South and East-West) reflecting the predominant East-West orientation of many normal faults in this area. The site spacing on this grid is approximately 1 km. Some areas in the mountain and around villages are not covered due to the increased EM noise or due to inaccessibility.

MT and GDS data were collected using three MTU-2000 instruments (utilizing Earth Data PR6-24 loggers, <http://www.earthdata.co.uk/prod.html>) combined with Metronix MFS05 coils from Uppsala University (<http://88.198.212.158/mtxweb/index.php?id=53>). One of these instruments was used to collect remote reference data during the entire survey time. Due to the limited amount of induction coils, we were not able to collect the vertical magnetic field at each site. The horizontal electric field components E_x and E_y were measured with grounded non-polarisable Pb/PbCl electrodes. Where possible, the electrode spacing was extended to a maximum of 100 m using a symmetric cross shaped configuration, having the ground electrode in its centre (differential input). Generally, time series data were recorded in four frequency bands with the sampling frequencies and sample times shown in Table 1. In the following we display all spatial data in the metric Greek coordinate system EGSA-87.

	Sample frequency
Burst mode during night, 120 minutes	1 kHz
Night and day recording	20 Hz
Day recording ca. 120 minutes	3 kHz
Day recording ca. 120 minutes	120 Hz

Table 1: Sampling frequencies and recording times

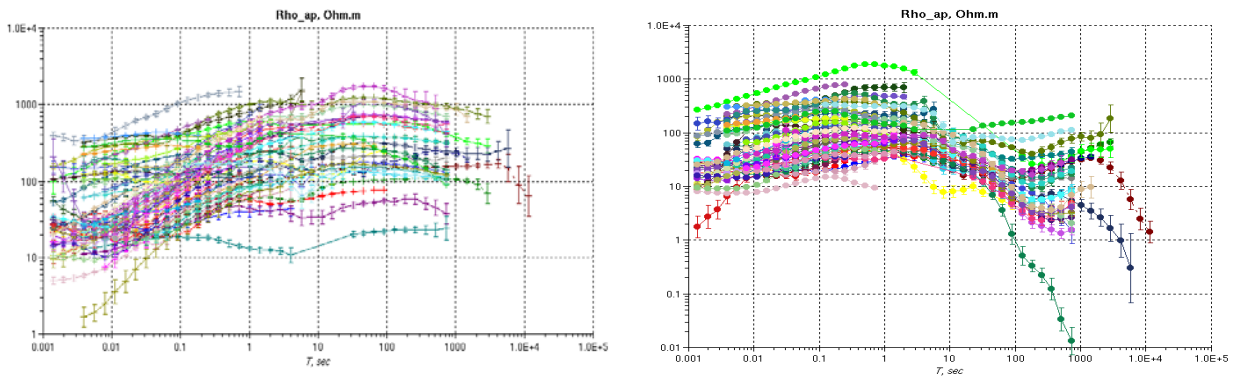


Figure 3: Unrotated apparent resistivities versus period for all sites (left: pa-xy, right: pa-yx).

DATA PROCESSING & ANALYSIS

Based on this survey design we obtained reliable estimates of the MT transfer function for 92 sites in the period range from $T = 0.001$ s to 1 s for day time recording and from $T = 0.001$ s to 1000 s for the sites where both day and night recordings were available (ca. 70% of the data).

The time series were processed with the robust remote reference code of Smirnov (2003). All permutations of remote reference sites from inside and outside of the study area were used to estimate transfer functions. Also several time segments for 1 kHz and 3 kHz recordings were processed independently and thereafter averaged together with long period data to obtain the final estimates of the MT transfer functions. During robust averaging using the reduced M-estimator we calculated error bars based on bootstrap method.

Additionally, MT/GDS data were available from a 1995 survey (Savvaidis et. al., 2000; Makris et al, 2002) measured with a seven channel S.P.A.M. MkIII system (Ritter et al., 1998) coupled with Metronix MFS05 magnetic sensors. The original time series were reprocessed with the Egbert code (Egbert, 1997) using standard 128 s time windows for the distinct frequency bands. From this 1995 survey, 9 MT sites in a period range of $T = 0.008$ s – 100 s were used for the 2D and 3D inversion.

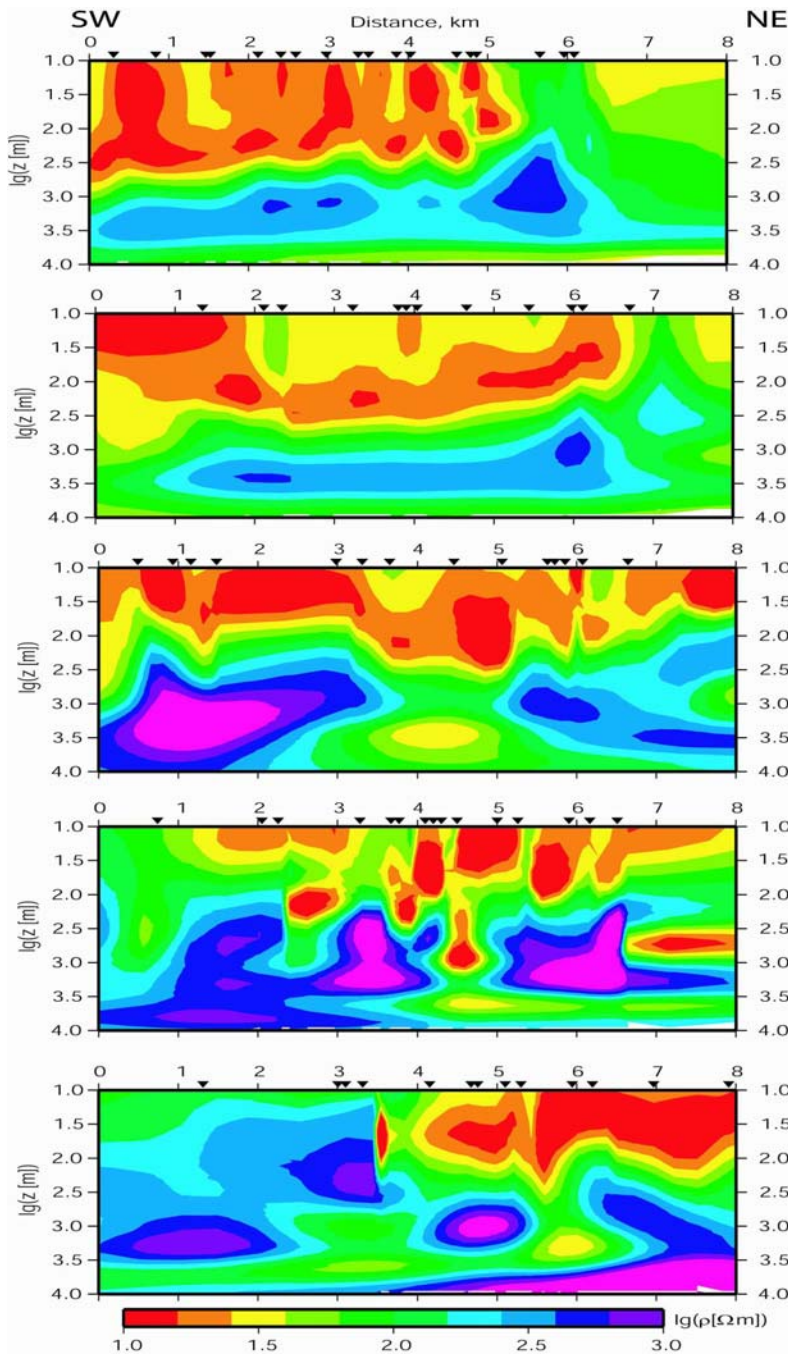


Figure 4: 2D resistivity models of line A-E (from top to bottom). Triangles indicate MT site locations.

All selected sounding curves used for this study are presented in Figure 3. The 2D inversion was performed along five parallel profiles (A-E) striking N30°E at quasi equal distances. Each MT site was then projected on the according profile. In order to perform the 2D inversion, we followed the strategy of Pedersen & Engels, (2005) inverting only Berdichevsky Invariant data. Since the Berdichevsky Invariant is by definition rotationally invariant, any variability in strike direction with period does not affect the results as much as for bi-modal inversion, when the proper mode decomposition is vital.

The 2D inversion routine by Siripunvaraporn and Egbert (2000) including the modifications made by Pedersen and Engels (2005) allow for inversion of the Berdichevsky Invariant. Error floors equal to 2% for the impedance phase (corresponding to an absolute error of 1.2°) and 50 % for the apparent resistivities were adopted. Since there was no

In order to validate the 2D inversion, we carried out strike and dimensionality analysis. The strike analysis of the MT data revealed two predominant strike directions: For short periods up to ca. $T = 3$ s, a local strike of approximately $0^\circ / 90^\circ$ is found, whereas for longer periods a regional strike of about $135^\circ / 45^\circ$ can be deduced from the MT data. The regional and local strike is consistent with previous results of MT data from this area (Makris et al, 2002). For longer periods, real parts of induction arrows are pointing towards the South-West showing a regional strike direction of about N120°E. The overall GDS data quality from the 2006/2007 survey is poor thus we did not use them for any 2D and 3D inversion tasks. At this time this behaviour is not well understood, since GDS data from the 1995 survey (Savvaidis et. al., 2000) and a recent GDS instrument test (Gurk, pers. comm.) show good GDS results. Dimensionality parameter did not show significant 3D effects in the data, allowing us to use 2D inversion techniques.

2D INVERSION

Generally, the MT data are of good quality. However before inversion, each MT transfer function has been carefully examined. The main impedance components as well as the Berdichevsky impedance tensor determinant average data (short: Berdichevsky Invariant; Berdichevsky et. al., 1976), were tested for consistency by using 1D inversion. After this procedure, MT data from 74 sites (out of 92 measured) have been chosen for the 2D and 3D modelling.

other information in order to constrain the static shift effect and the site spacing is relatively small, we have chosen a higher error floor for the resistivity data compared to the phases to allow the inversion procedure to have more freedom to compensate for these effects.

Finally, a homogeneous halfspace of 100 Ωm was used as the starting model for all 2D inversions. This procedure resulted in an overall good fit of the measured data to the model (RMS 1). The series of the 2D inversion results shown in Figure 4 indicate an increase in complexity of the conductivity distribution towards the East that is likely to be caused by a more complicated 3D geological setting in this part of the survey area. Hence a simplified 2D analysis of the MT transfer function can be misleading in these regions. Consequently we proceed with a 3D inversion of the data set.

3D INVERSION

The same data set, consisting of 74 pre-selected good quality MT sites, was used for the 3D inversion (Siriponvaraporn et al., 2005). Until now, the code does not account for topographic information, nor for vertical magnetic fields. Generally, MT data were measured on a more or less regular grid in the 12 km x 6 km survey area with an approximate site distance of about 1 km.

After choosing the origin of the model domain at the location of MT site a1309 it was necessary to shift the MT sites accordingly to their new position in the model domain. To get an even site distribution we allowed for relatively large lateral shifts of those sites that were measured directly on the outcropping basement rocks, e. g. in the Northern part of the area (Fig. 5). The model grid itself was designed to have 4 cells between each site. During the first inversion trial we have selected only 3 periods in the range from $T = 1 \text{ s} - 0.01 \text{ s}$. Therefore, the grid was constructed to be a 34 x 34 x 21 matrix including additional 5 horizontal outer cells to extend the grid boundaries to 60 km.

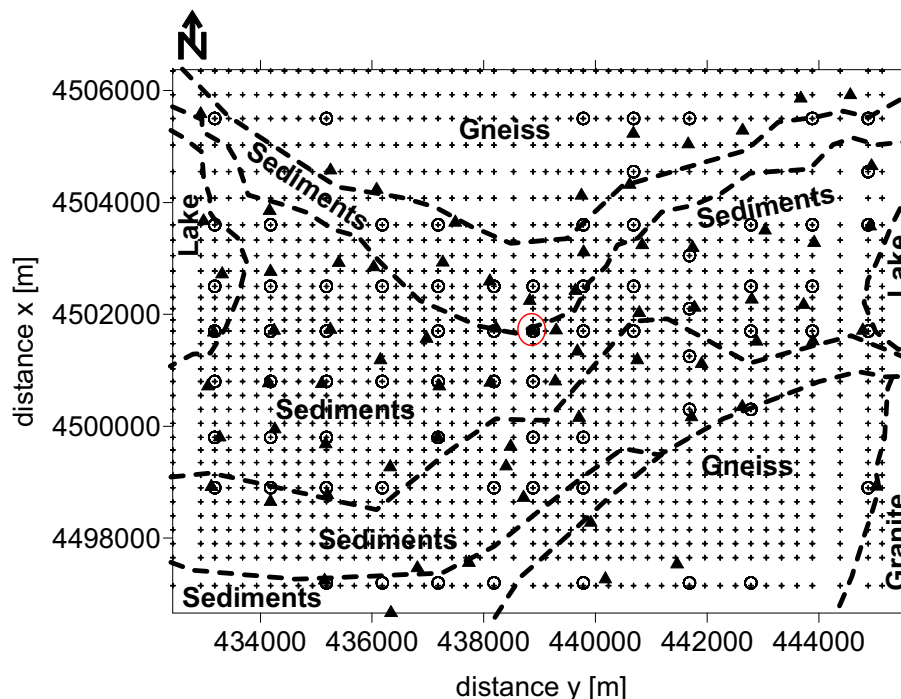
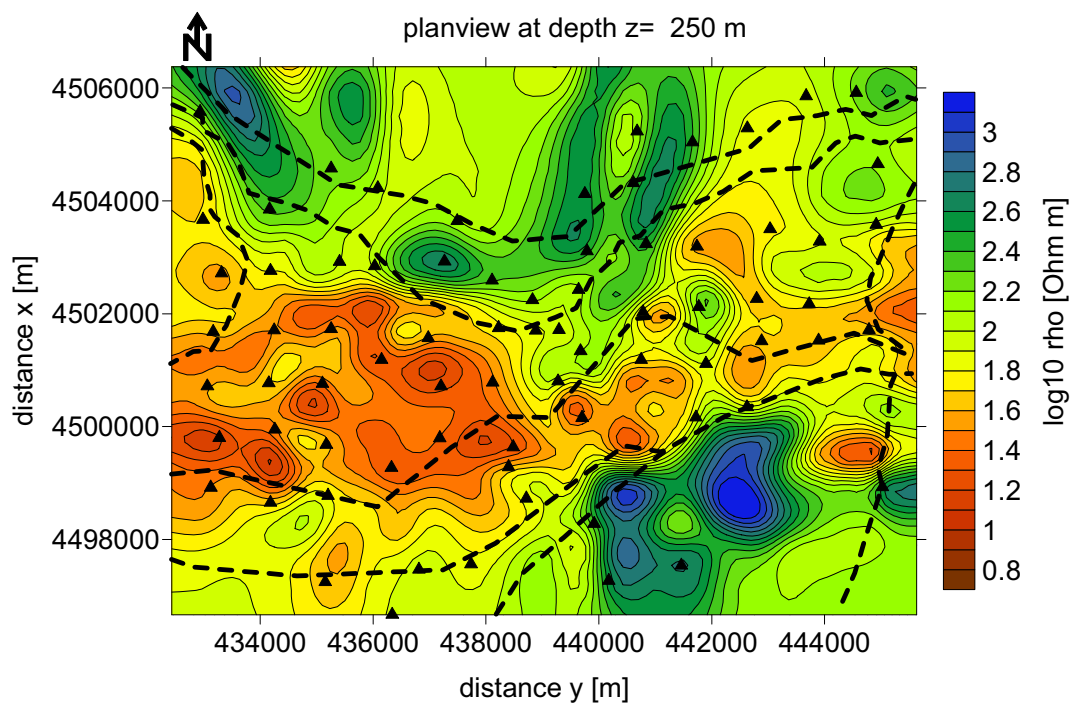
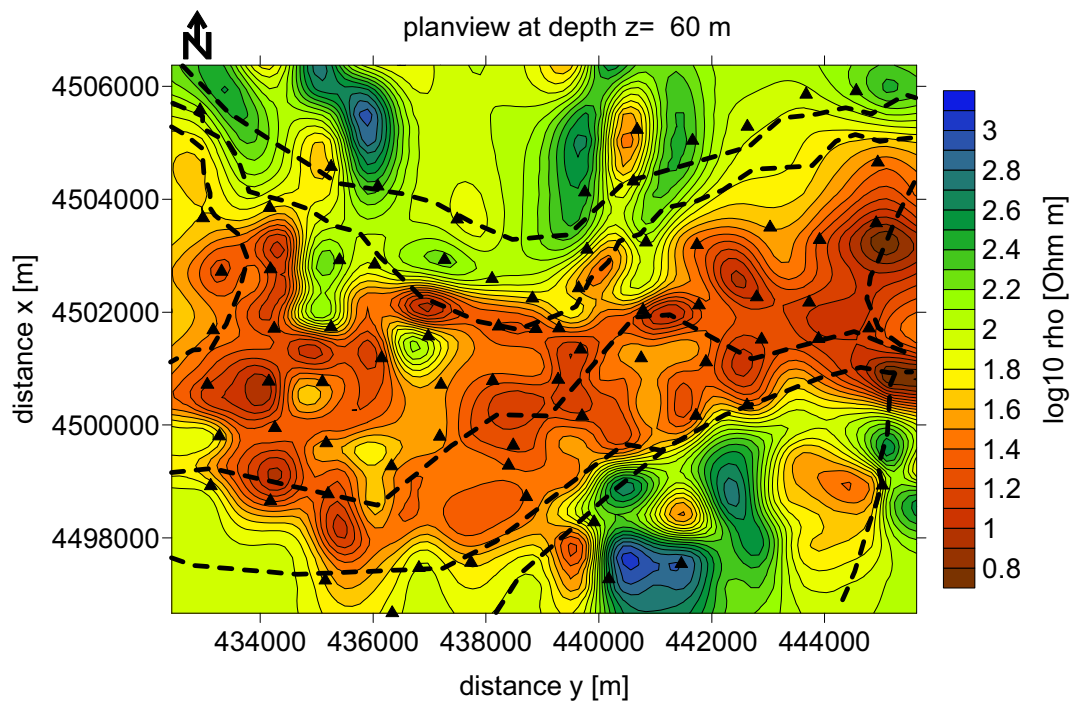


Figure 5: Plan view of the central part of the model grid (denoted as crosses) together with the MT site location (triangles), the MT site location in the model domain (circles) and geological outlines. The red circle indicates the centre of the grid.

The second inversion run included 8 periods in the range of $T = 10 \text{ s} - 0.01 \text{ s}$, using the same model grid, whereas for the third inversion the number of cells in-between sites were increased and a better vertical discrimination at target depths (42 x 54 x 26 cells) was applied. For the 3D inversion we have used the complete impedance tensor (Z_{xx} , Z_{xy} , Z_{yx} & Z_{yy}). After 4 successful iterations of the 3D routine the inversion code reached RMS values of approximately 10.



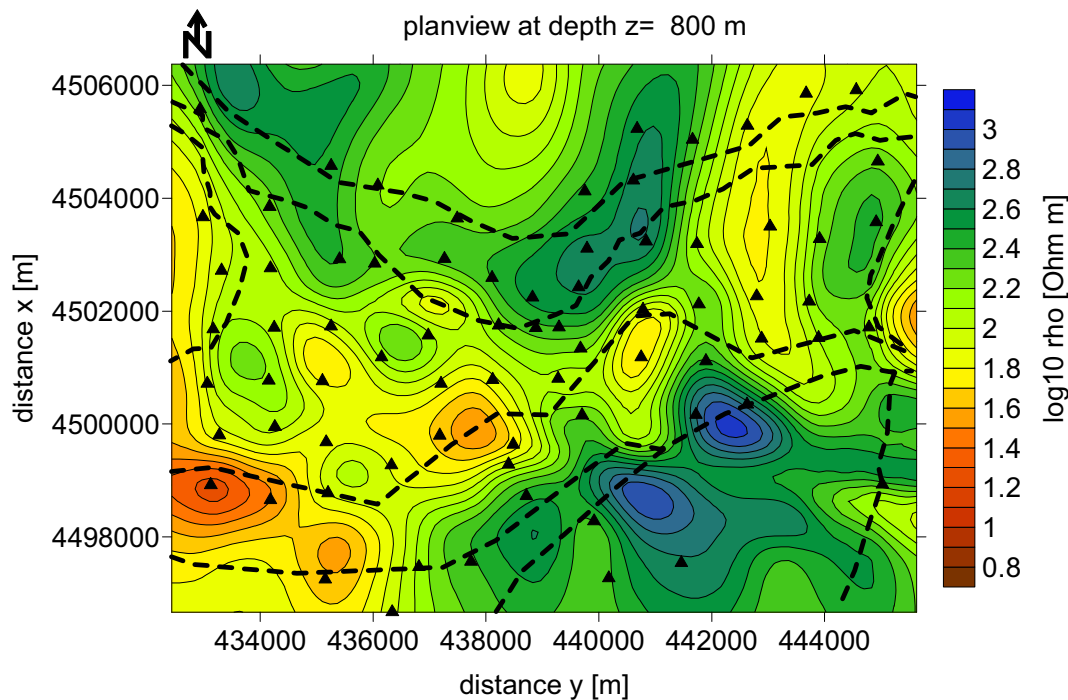


Figure 6: Plan view of 3D resistivity models at three different depths (triangles indicate MT site locations, hashed lines indicate geological boundaries).

Figure 6 shows representative examples of the 3D inversion results of our last run as plain view at three different depths. The resistivity of the lacustrine sediments (approx. $40 \Omega\text{m}$) in the valley contrasts clearly with the higher resistivities of the surrounding basement rocks allowing to map the structures at near surface ($z = 60 \text{ m}$) in very good accordance to the geological setting as displayed in Figure 2. In a larger depth range at $z = 250 \text{ m}$, the influence of a basement high is more pronounced towards the centre of the area. The sediments (approx. $40\text{--}60 \Omega\text{m}$) in the Eastern part of the area seem to vanish whereas in the Western part sediments are still visible. At a depth of $z = 800 \text{ m}$ only sediments in the South Western part of the valley are still recognisable, the basement high in the middle of the valley is now fully developed and the sediments towards the East are not pronounced any more.

The normalised absolute misfit of the real part of the impedance tensor elements Z_{xy} and Z_{yx} are shown in Figure 7. The data misfit is higher in the Western part of the area than compared to the Eastern part, which is in agreement to the previous 2D model study. Large misfits can be found where the MT site distribution and/or data quality is poor. It is a striking feature in Figure 6 that the misfit follows the topography or morphology of the survey area which could mean that static shift and/or topographic effects may play a more important role in the error distribution than previously considered. Hence the entire data set should be re-analysed more carefully with respect to galvanic distortion models.

3D MODEL VOLUME

The 3D inversion results at distinct depth values can be presented using volume rendering tools. Since our task is to map the top-of-basement by means of an iso-resistivity layer, we have to confine the range of resistivity values that can be associated with the top-of-basement rocks. Due to the changing hydrogeological conditions which leads to different degrees of weathering of the basement rocks, it is not possible to allocate a unique resistivity value to the top-of-basement. Nevertheless, we can use available information from boreholes and VES data (Euroseistest, 1995; Tournas, 2005) to constrain the resistivity range and the depth to the basement rocks at specific locations. Our first approach is displayed in Figure 8 to highlight the onset of the basement resistivities, indicated by the transition between yellow and green colors (approx. $75 \Omega\text{m}$). From these considerations a $90 \Omega\text{m}$ iso-surface can be deduced that matches sufficiently to basement information from boreholes (see Fig. 9). Studying the conductivity distribution in general with in the 3D

volume, we see that the sediments with the lowest resistivities can be found in a band along the Southern rim of the valley. Towards the North, the resistivities of the sediments increase gradually.

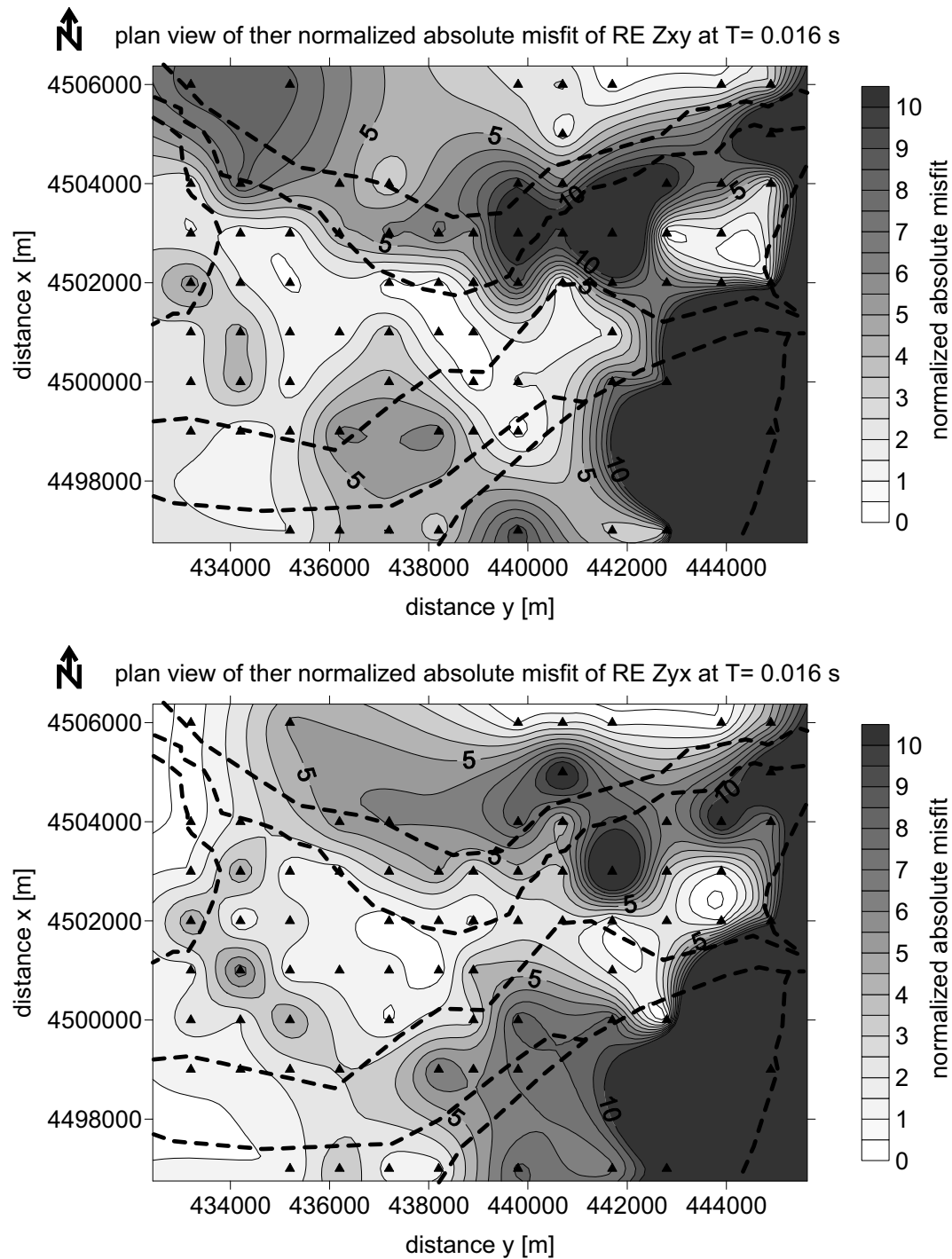


Figure 7: Plan view of the normalized absolute misfit at $T = 0.016$ s for the real part of the impedance tensor elements Z_{xy} (up) and Z_{yx} (below). Triangles indicate MT site locations, hashed lines indicate geological boundaries.

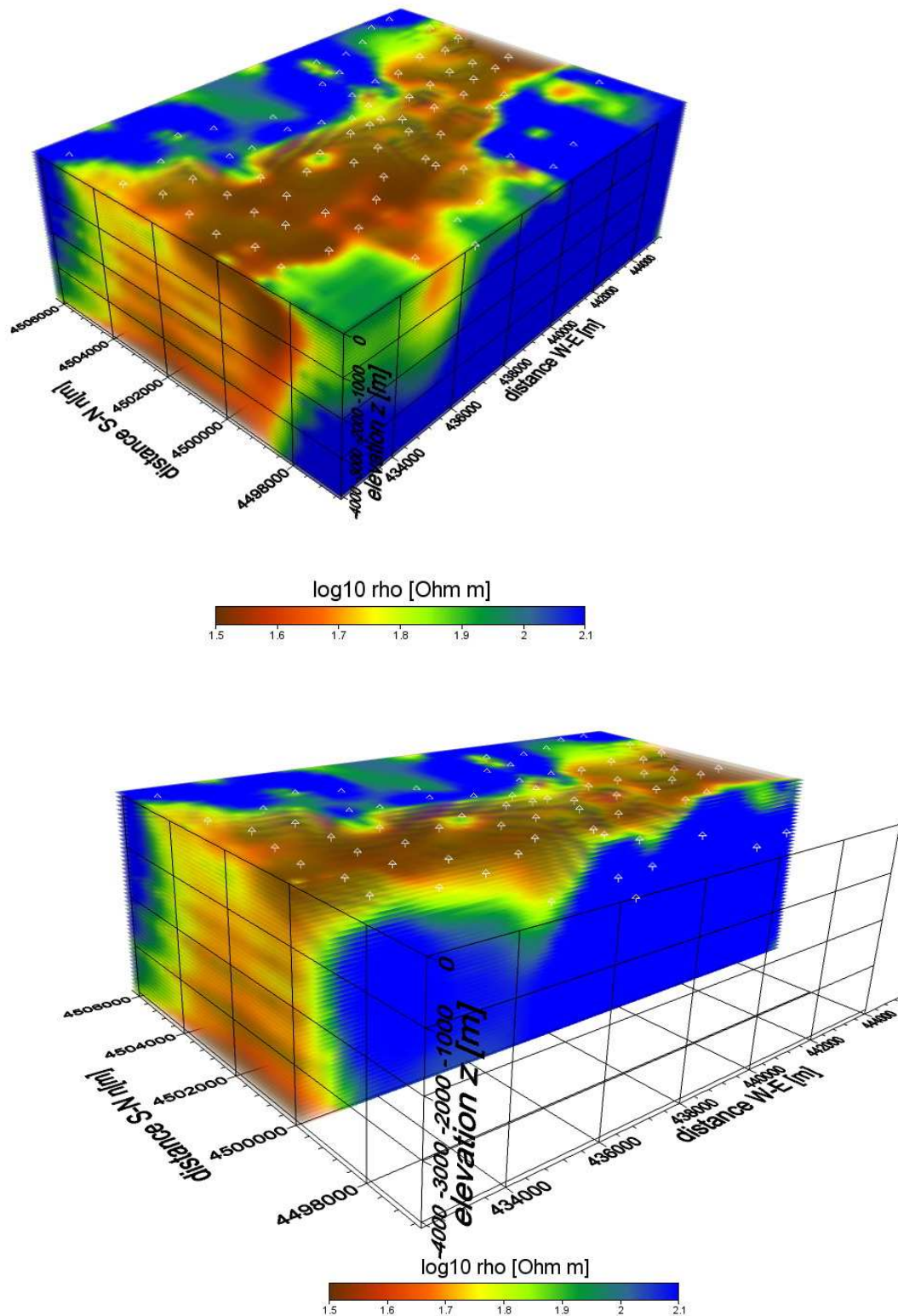


Figure 8: Two 3D bird views of the resistivity model in North-East direction. Triangles indicate MT sites.

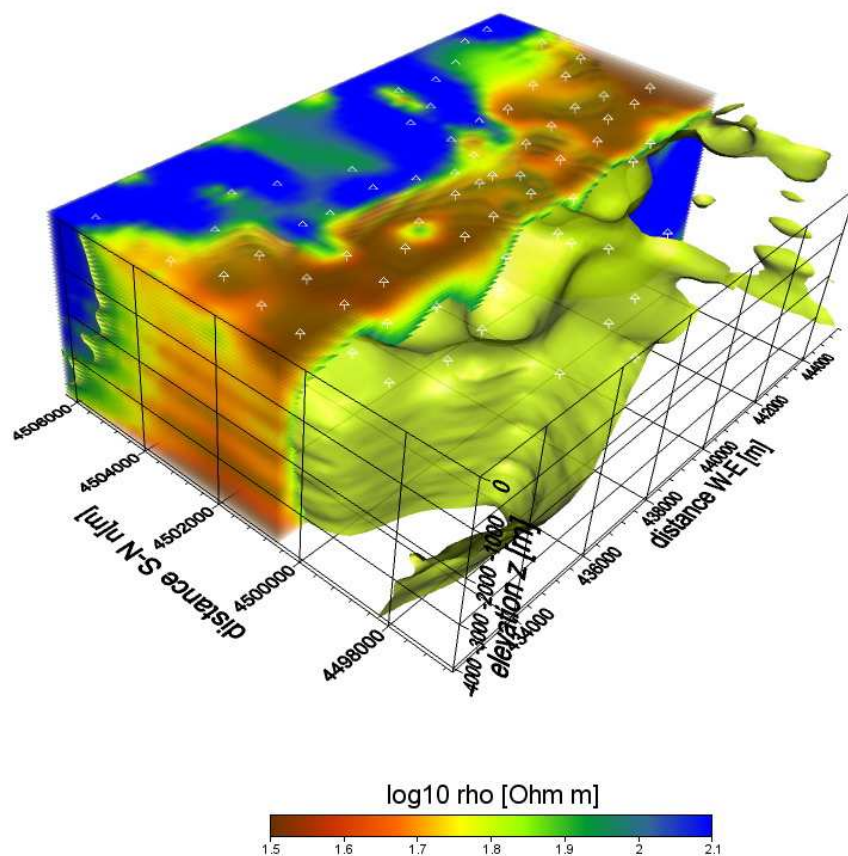


Figure 9: 3D view of the top of basement ($90 \Omega \text{ m}$ iso-surface, green color) in North-East direction. Triangles indicate MT sites.

CONCLUSIONS

MT array data were used to obtain new information about the sediment thickness and the slope of the top-of-basement in the South-Eastern part of the investigation area which is of particular importance for seismic wave propagation modeling. To support this finding in our future work new MT sites are going to be deployed to stretch the 2D sections A, B and C towards the South-East. In the next inversion attempt, we pay more attention to galvanic and topographic effects, as well as to confine the model results with available VES and RMT/CSTAMT information.

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MT measurements in the Cape Fold Belt, South Africa

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1. OVERVIEW

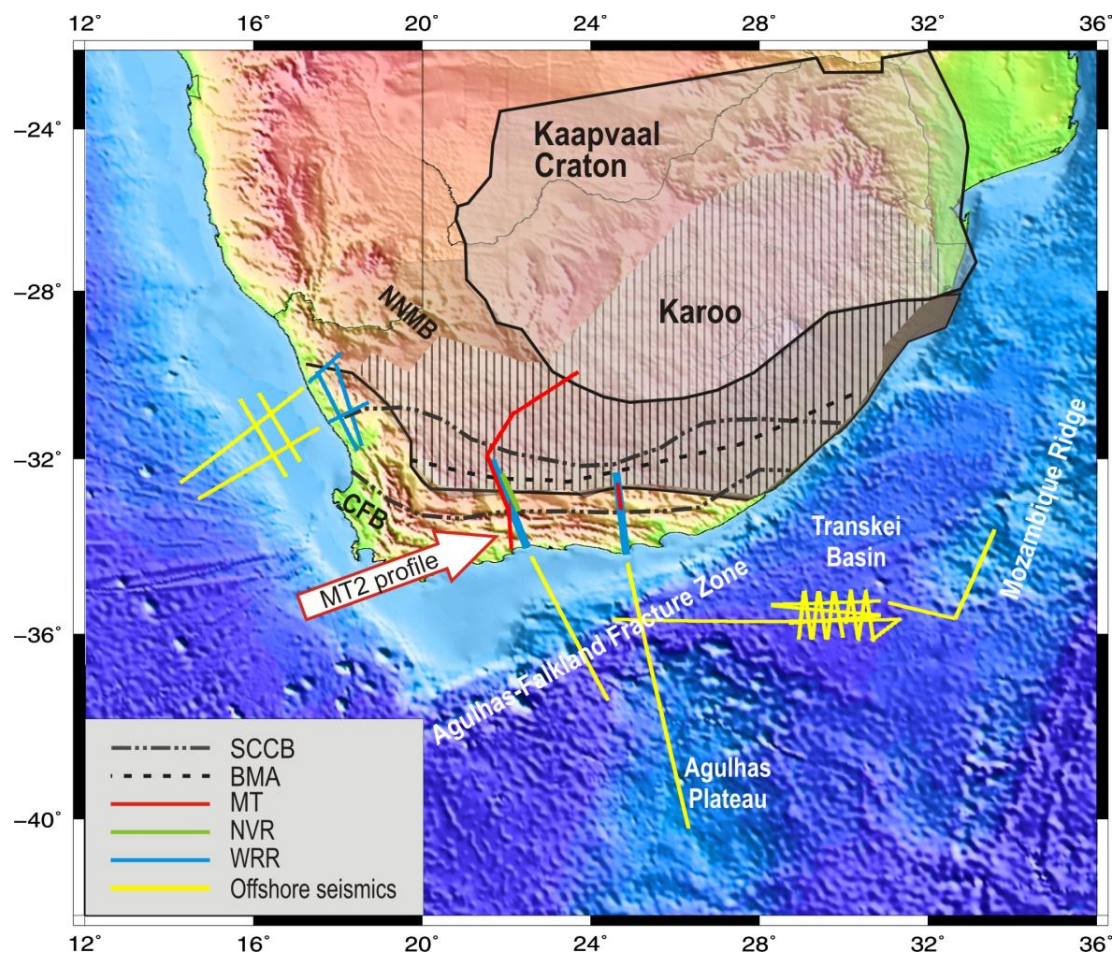


Figure 1: Simplified terrane map of Southern Africa with different on- and offshore geophysical profiles within the *Inkaba yeAfrica* project. The map also shows the Archean Kaapvaal Craton, the Mesoproterozoic Namaqua Natal Mobile Belt and the upper Paleozoic Cape Fold Belt. A large region is covered by the Paleozoic-Mesozoic sediments and igneous rocks of the Karoo Basin.

The southern part of the African continent consists of an assemblage of different continental masses. The Archean nucleus is the Kaapvaal craton. In the Proterozoic the Namaqua Natal Mobile Belt (NNMB) accreted in the south, followed by the collision of the Permo-Triassic Cape

Fold Belt (CFB). Within the *Inkaba yeAfrica* project (www.inkaba.org; de Wit & Horsfield, 2006), a number of geophysical on- and off-shore experiments were carried out along profiles across the southern margin of the African continent (Weckmann et al., 2007a; Weckmann et al., 2007b; Stankiewicz et al., 2007; Lindeque et al., 2007; Parsiegla et al., 2007). Focus of this paper is the MT2 profile which crosses the Cape Fold Belt (CBF). The magnetotelluric (MT) profile is located between the NNMB to the North, which is covered by the sediments of the Karoo Basin (shaded area in Figure 1) and the Indian Ocean to the South with the Agulhas Falkland Fracture Zone running approximately 150 km off the coast.

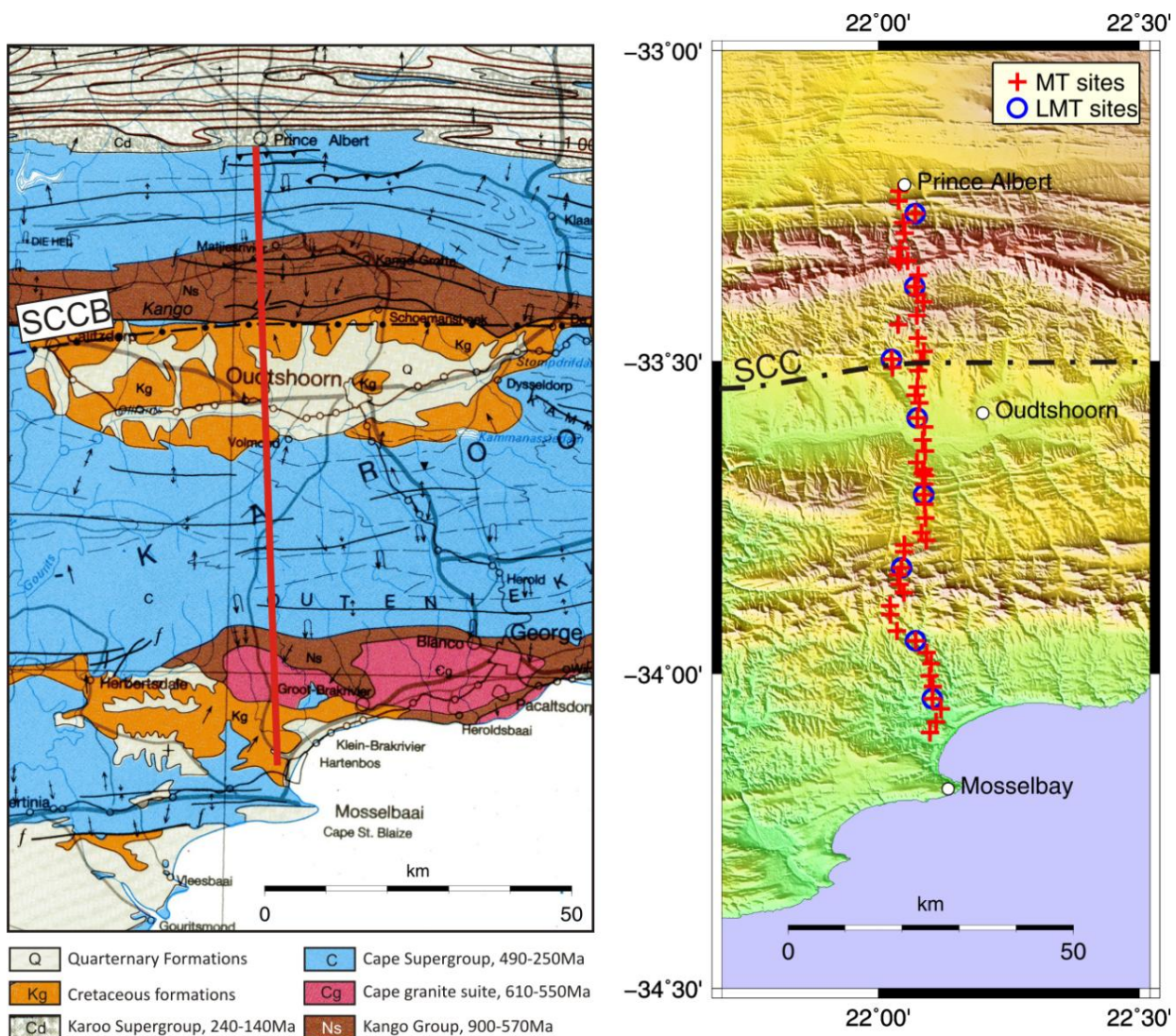


Figure 2: Left: geological map of the Cape Fold Belt along profile MT2. The red line indicates the location of the MT profile. Right: Detailed location map of broadband and LMT sites along the profile (MT2).

Whereas the northern part of the western traverse runs through the NNMB and the Karoo Basin (see Figure 1), profile MT2 (see Figure 2 right panel for details) is located in the CFB. This mobile belt mainly consists of Cape Supergroup rocks, which were formed during the Palaeozoic, when

deposition occurred in a basin created by inversion of a Pan African mobile belt flanking the southern margin of Africa (de Wit, 1992). Since then, the Cape Fold Belt has experienced several periods of re-activation. During Late Proterozoic / Early Cambrian eroded material from uplifts in the NNMB filled basins to the south, forming the Kango and Outshoorn inliers (Hälbich, 1993) (see Figure 2 left panel).

Open questions concern the kinematic history of the CFB inliers as well as the subsurface geometry of major tectonic and stratigraphic boundaries. A crucial point is if and to which extent the CFB may continue under the NNMB or vice versa.

2. MT DATA SET ACROSS THE CFB

54 MT sites recording electric and magnetic field variations in a period range from 0.001 s to 1000 s were deployed along a 105 km long profile from Prince Albert to Mosselbay in November 2005 (red crosses in Figure 2 right panel) crossing the Cape Fold Belt and its inliers. We acquired 5-component MT data using GPS synchronized S.P.A.M. MkIII (Ritter et al., 1998) and CASTLE broadband instruments together with non-polarizable Ag/AgCl telluric electrodes and Metronix MFS05/06 induction coil magnetometers. The average site spacing was approximately 2 km. Blue circles mark LMT stations, which extend the period range to 20000 s.

The MT data were processed with the EMERALD software package (Ritter et al., 1998) using both robust single site and remote reference techniques (Krings et al., this issue). Measurements were widely affected by extensive farming. To improve data quality, the frequency domain selection scheme after Weckmann et al. (2005) was applied.

Figure 3 shows MT and GDS data and the results from a distortion analysis after Becken & Burkhardt (2004) at selected sites. The MT data are shown in a NS/EW coordinate system for four exemplary sites. Site locations are marked with pink arrows in Figure 5.

Apparent resistivities in the Outeniekwaberger (site 137), the Rooiberger, and the Swartberg Mountains (site 108) are high, reaching values of up to $10^6 \Omega\text{m}$ whereas in the Kango Basin (sites 121 and 117) resistivities are very low and the curves are characterized by a steep descent to extremely low resistivity values ($< 1 \Omega\text{m}$) for periods longer than 1s (note the different scaling of the axes). In the northern profile section (north of site 130), nearly all sites show phases exceeding 90° for one (sites 121 and 108) or even both (site 117) polarizations at long periods ($> 1\text{s}$). Real parts of the induction vectors (in the Wiese convention) in Figure 3 show a common behavior for longer periods ($> 5\text{s}$) by pointing towards south. Only stations in close vicinity to the Indian Ocean show the “ocean effect” with northward oriented induction vectors. Shorter periods ($< 1\text{s}$) are mainly influenced by the resistive mountain chains and the conductive basins. Figure 4b shows induction vectors for four different frequencies along the entire profile MT2. Shaded areas (grey, pink and blue) indicate zones of enhanced conductivity which could be the cause for larger induction vectors and their reversals. Induction vectors for periods shorter than 1s mainly show the influence of the Kango Basin. The influence of the Indian Ocean in the South and the extremely conductive crust of the NNMB in the North can clearly be seen in the induction vectors for periods longer than 1s.

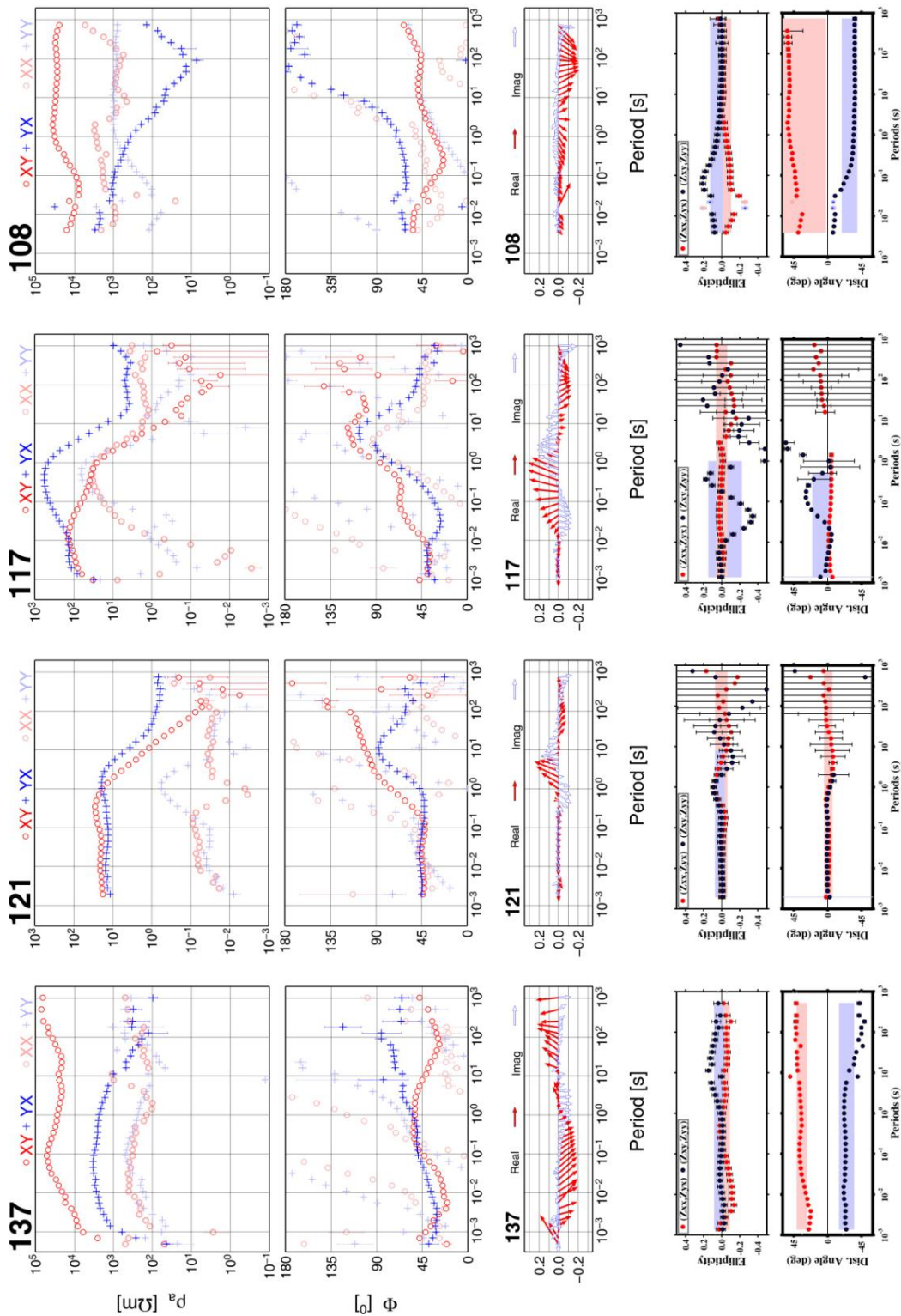


Figure 3: Off-diagonal and diagonal components of apparent resistivity, phase and induction vectors (Wiese convention) of four exemplary sites (in a geographic coordinate system: x = geographic north, y = geographic east). For site locations refer to Figure 5. Additionally, ellipticity parameters after Becken and Burkhard (2004) provide information on the directionality and dimensionality of the MT data.

The lower panels in Figure 3 show the impedance decomposition parameters for the four exemplary sites obtained from the analysis using Becken and Burkhardt (2004). The graph shows the ellipticities of the two independent current systems and the galvanic distortion angles. Shaded areas in light blue and red indicate the weighted means of respective quantities over period and their weighted r.m.s. deviations; the patches are centered at the mean values and have a width of two standard.

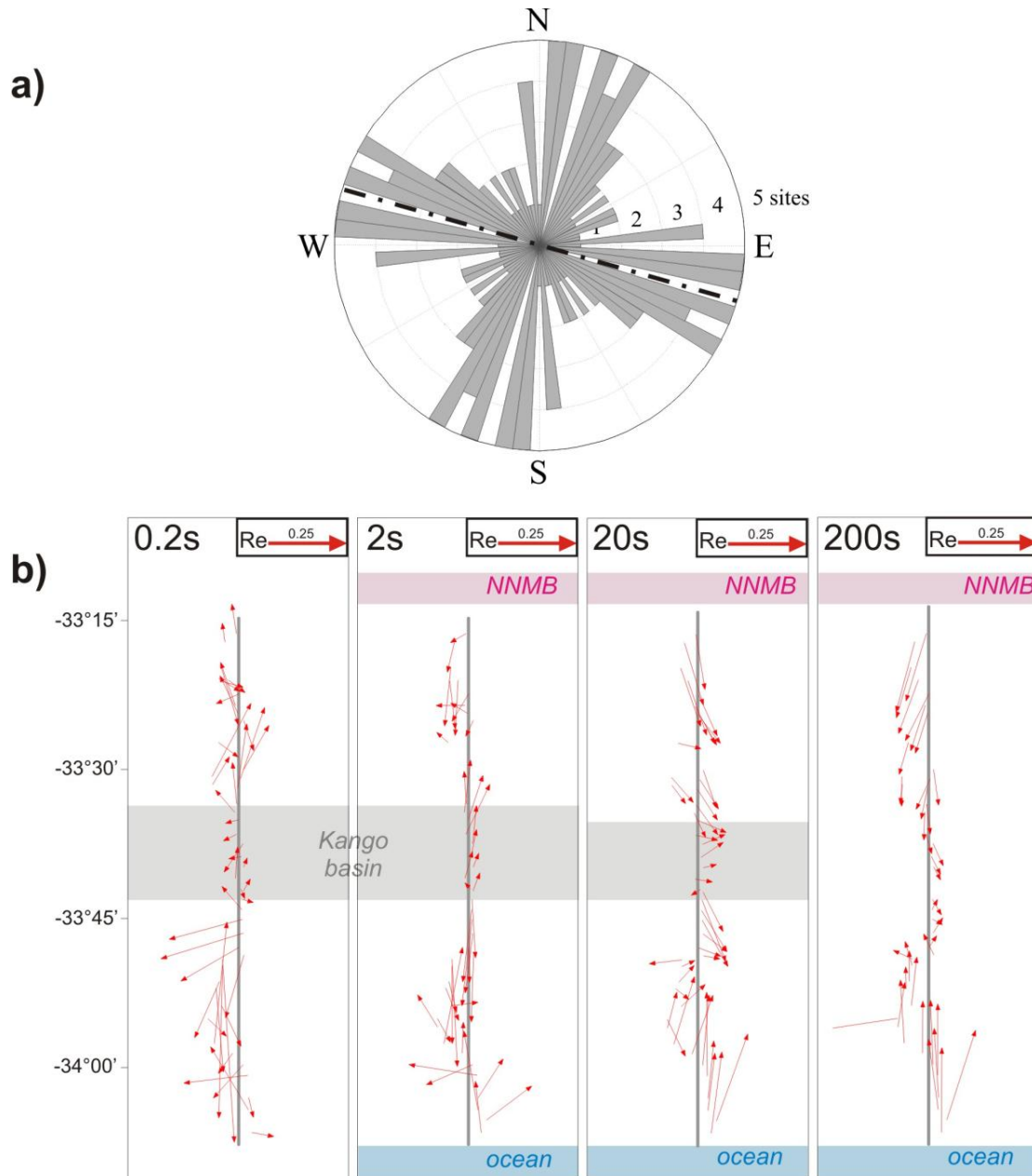


Figure 4: a) Regional strike angles shown in a rose diagram. The average amounts to -76° . b) Real Induction vectors in the Wiese convention for four different frequencies along our profile. Shaded areas (grey, pink and blue) indicate zones of enhanced conductivity which could be the cause for larger induction vectors and their reversals.

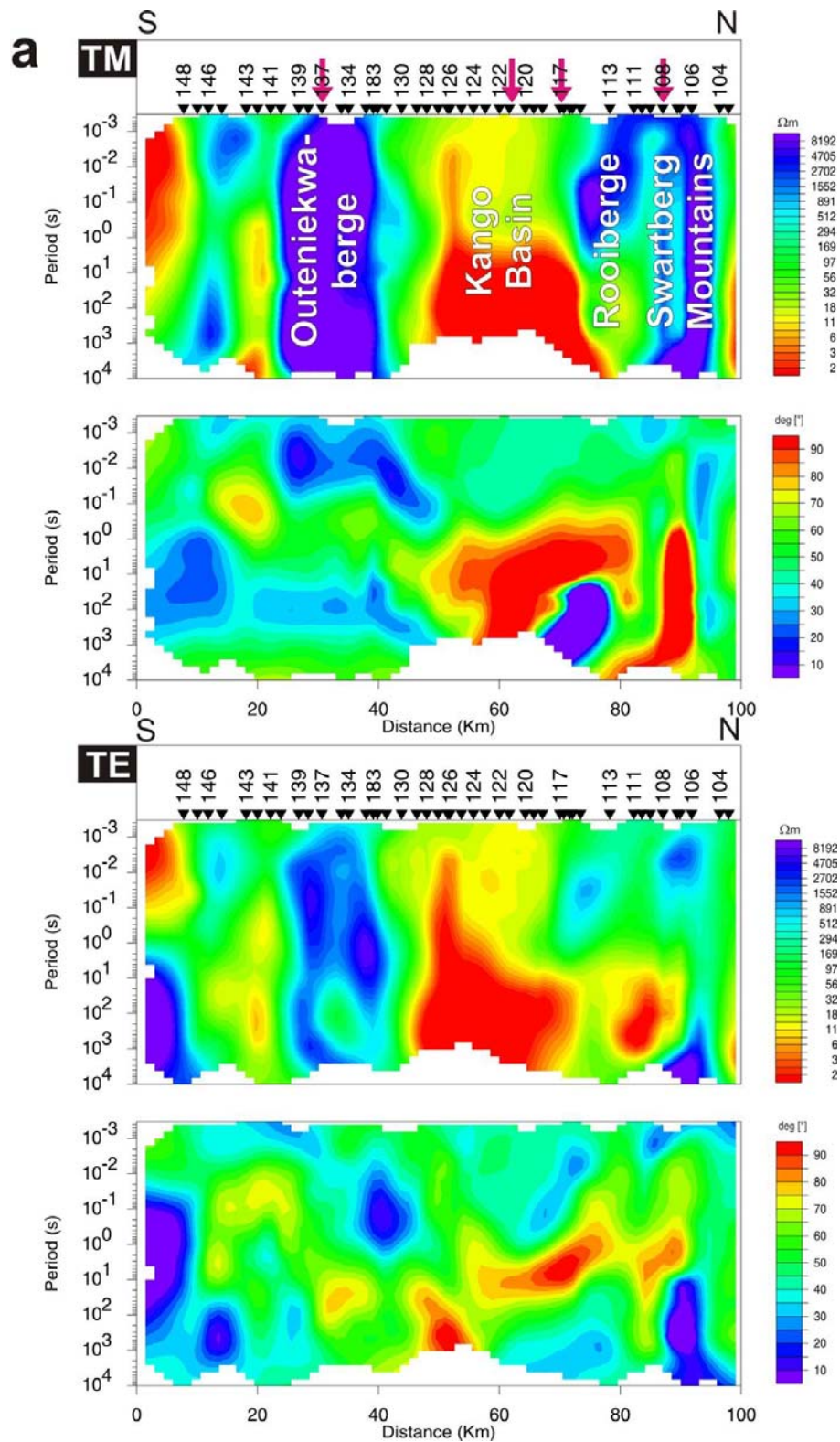


Figure 5: Pseudo-sections of measured apparent resistivity and phase of all sites along profile MT2. The upper panels show TM-polarization, the lower ones TE-polarization after rotation by -90° . Red arrows mark sites which are shown in detail in Figure 3.

A significantly large sub-set of sites (sites 137, 121, 117) shows ellipticities deviating from 0 for long periods ($>1s$), indicating 3D structures at depth / off-profile. In addition, several sites (e.g. site 108) show larger ellipticities for shorter periods ($< 1s$), which likely indicate local and shallow 3D structures. Distortion angles are stable for each site over the entire period range (sites 137, 121, 108), indicating frequency-independent distortion of the regional impedance tensor.. However, at some stations e.g. site 117 they vary strongly with period. This could be caused by the strong conductivity contrast between the mountain ranges and the sedimentary basins and different strike directions for these tectonic units.

Regional geo-electric strike angles are shown in a rose diagram plot in Figure 4a. The average strike direction, including data from all sites and over the entire period range, amounts to -76° . This direction corresponds roughly with the regionally east-west trending geologic structures of the CFB.

In a pseudo-section presentation different geological and tectonic features can already be attributed to zones of high and low conductivity. Figure 5 shows pseudo-sections for TM and TE polarizations after rotation by -90° . This direction was chosen as a compromise of geo-electric strike and the known strike direction of geologic and tectonic units. TE-mode data are now perpendicular, TM-mode parallel to the profile MT2. Along the profile, apparent resistivities range from $10^6 \Omega m$ in the Swartberg Mountains and the Outeniekwaberge to as low as $0.01 \Omega m$ at sites in the Kango Basin. For periods above 1s phases often exceed 90° at sites in the northern half of the profile.

In a preliminary interpretation approach we intend to assess which structures are consistent with 2D inversions. For the 2D inversion we only use MT data with phases staying in their assigned quadrant, with small ellipticities, and with consistent apparent resistivity-phase relationship (no significant deviation when applying the Sutarno phase criterion). Obviously, none of the observed 3D effects can be explained by a 2D model.

3. FIRST 2D INVERSION RESULTS

Figure 6 shows a first 2D inversion model along profile cut-off at 30 km depth. The 2D inversion model is the result of a minimum structure, non-linear conjugate gradient inversion algorithm after Rodi and Mackie (2001), which is implemented in WinGLink (www.geosystem.net). Prior to modelling the data were rotated by -90° . Inversion was started from a homogeneous half-space of $100 \Omega m$ with $\tau=10$ using TE and TM mode data. Preset error bounds of 5% for TM-polarization apparent resistivity and 200% for TE-polarization apparent resistivity and 0.6° for the phases of both polarizations were applied. Larger error floors were assigned to the apparent resistivity data, in particular to the TE-polarization to avoid problems with static shift effects and off-profile features.

In this is first 2D inversion approach the overall data fit (RMS =3.1) is quite high with consistently better fits (<2.5) in the northern half of the profile and larger misfits for sites in the Outenikwaberge area.

A large triangular shaped zone of low resistivities (<2 Ωm) is suggested for the Kango Basin area, extending further north under the Rooiberge. The bottom of the basin is not resolved (yet) and requires more modelling. High resistivities are depicted under the Swartberg Mountains down to a depth of 10 km. South of the Kango Basin the misfits are far above the average and indicate poorly resolved model features. Alternating resistive and conductive zones in the upper 5km between sites 126-135 indicate that electrical anisotropy could occur in this area. The conductive body between sites 135 and 142 at greater depth (below 10km) is also not constrained. The high conductivity in the southernmost part of the model corresponds to the Indian Ocean.

As a next step we will include topography and bathymetry into the models. With constraint inversions we will test if anomalous regions are required by the data and if we can resolve shape and structural details (e.g. electrical anisotropy).

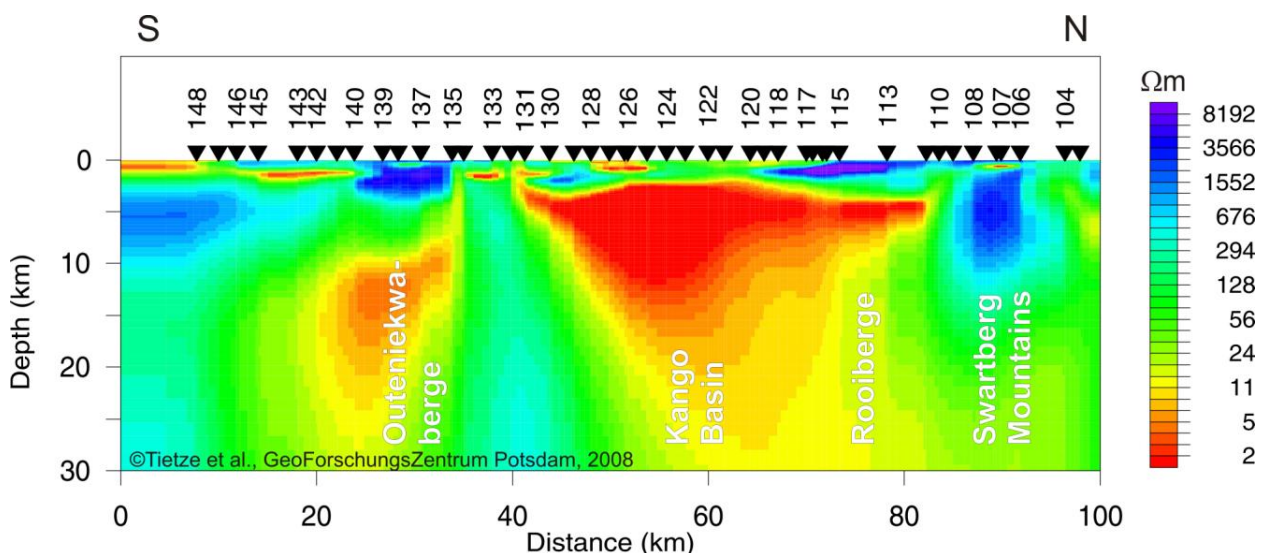


Figure 6: Electrical conductivity model along profile MT2 obtained from 2D inversion. The mountain ranges (Swartberg Mountains, Rooiberge and Outenikwaberge) correlate with zones of high resistivity and seem to have rather shallow roots (maximum 10km below Swartberg Mountains). The most prominent conductivity anomaly is the Kango Basin. Although its lower boundary is not resolved it could be deep reaching.

4. ACKNOWLEDGEMENT

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The electrical conductivity structure of the Dead Sea Basin obtained from MT measurements

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Abstract

The left-lateral Dead Sea transform (DST) is a major transform fault that separates the Arabian plate in the east from the African and Sinai plates in the west. It extends from Red Sea rift in the south to the Taurus collision zone in eastern Turkey, with a total length of more than 1000 km (Fig. 1). The total displacement along the fault is estimated to about 105 km since its development ~18 Ma ago with an average slip rate of 4mm/yr (Garfunkel et al, 1981).

During its evolution, the DST formed several deep sedimentary basins. These pull-apart basins formed the Gulf of Aqaba / Eilat, the lake of Tiberius and the Dead Sea basin (DSB). The DSB is probably the largest of these structures on earth.

Geophysical, geological and geochemical studies of the San Andreas Fault system in California showed clearly that the dynamics and thereby earthquake generation processes are closely linked with (over-pressurized) fluids (Zoback, 1987; Becken et al., 2007). In addition to meteoric water and formation water, fluids may originate from as deep as the earth's mantle. At the DST, the existence of several hot springs on both sides of the fault may indicate deep-reaching fault zones that provide pathways for fluids.

The main objectives for the MT experiment are: to determine position and depth extent of the major faults within the DS basin (e.g. Ritter et al., 2003), to establish if these fault zones are associated with deep reaching fluid channels (e.g. Becken et al., 2008) and to infer the thickness of sediments within DS basin and to image the internal structures of the DS basin.

Experiment design and Data processing

In the framework of the multi-disciplinary DESIRE (Dead Sea Integrated Research) project, several geophysical methods were applied to investigate the southern part of the Dead Sea basin. Besides MT, the methods used included active reflection and refraction seismic, aero-gravity, passive seismology, and thermo-mechanical modeling.

The MT profile in Figure 1 follows roughly the seismic profiles, crossing the DST at the place where the fault separates into an eastern and western border fault (EBF and WBF), where the basin fill is at its largest and where intruded salt domes are likely three-dimensional features.

The data were collected in October and November 2006. The measurements were carried out along two profiles. The main profile is oriented N70°E, approximately

perpendicular to the surface trace of the DST and has a length of ~ 110 km. A second, shorter profile (20 km) is oriented N20°E and runs parallel to the west coast of Al Lisan peninsula (Fig. 2). With the in-basin profile we intend to quantify the effect of the highly conductive Dead Sea brines ("ocean effect") and also to test if structural details within the sedimentary basin can be resolved.

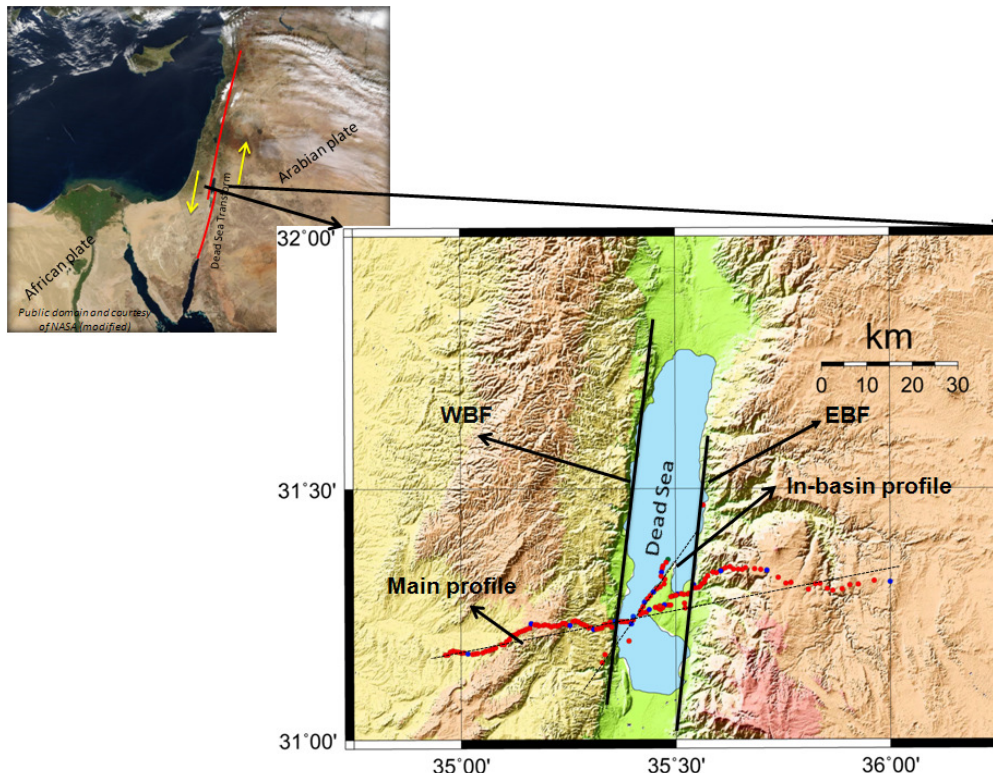


Fig. 1: Location map showing the locations of 148 MT sites. In the Dead Sea pull-apart basin the Dead Sea Transform separates into eastern (EBF) and western (WBF) fault branches

Data acquisition was accomplished with two independent teams, working at the same time in Jordan and Israel which allowed us to operate up to 30 sites recording simultaneously. This provides great flexibility to use remote reference processing with many combinations of sites.

In total, 148 MT stations were deployed along the profiles with denser site spacing within the Dead Sea basin and increased site spacing towards both ends of the profile. At each site, broad-band systems (red dots in Fig. 1, period range 0.001 s–1000 s) were used to measure time variations of the orthogonal magnetic (B_x , B_y , B_z) and electric (E_x , E_y) field components. Additionally, 20 sites with long-period systems (10 s – 10000 s) were distributed along both profiles (blue dots in Fig. 1).

The recorded time series were processed using the EMERALD package (Ritter et al., 1998; Weckmann et al., 2005) to compute the horizontal impedance tensor and vertical magnetic transfer functions. In addition, remote reference processing (Egbert, 1997) was useful to improve the data quality at particularly noisy sites.

The obtained impedances were analyzed to estimate the geo-electric strike direction. Based on the galvanic distortion analysis described by Becken and Burkhardt (2004) we discriminated the data into 3 subsets. Sites located to the west of the WBF and to the east of the EBF (purple and green profile segments, respectively, in Fig. 2) exhibit predominantly 2D conditions with a very stable strike direction of 10° - 15° E (90° ambiguous; cf. rose diagrams in Fig 2a and c, respectively). In the central segment of the profile, i. e. between the WBF and EBF, the geo-electric strike direction exhibits greater variability (see rose diagram in Fig. 2b) consistent with the dimensionality parameters (not shown) indicating moderate 3D conditions.

But this data analysis shows very clearly that the electrical resistivity distribution is strongly correlated with the fault structure and basin architecture. The geo-electric strike direction determined for different segments along the profile is very consistent with the orientation of the DST and the DS basin (10° - 15° , see rose diagrams in Fig. 2 a and c). However, data from the central part of the profile reveals a more complicated, three-dimensional nature of the internal structure of the DS basin with finite extent (Fig. 2c). Since the dominant strike direction in all parts of the profile is roughly north/south a 2D interpretation of the data is at least reasonable. The ultimate goal of data analysis is to compute 3D inversion models using the data from both the main profile and the in-basin profile.

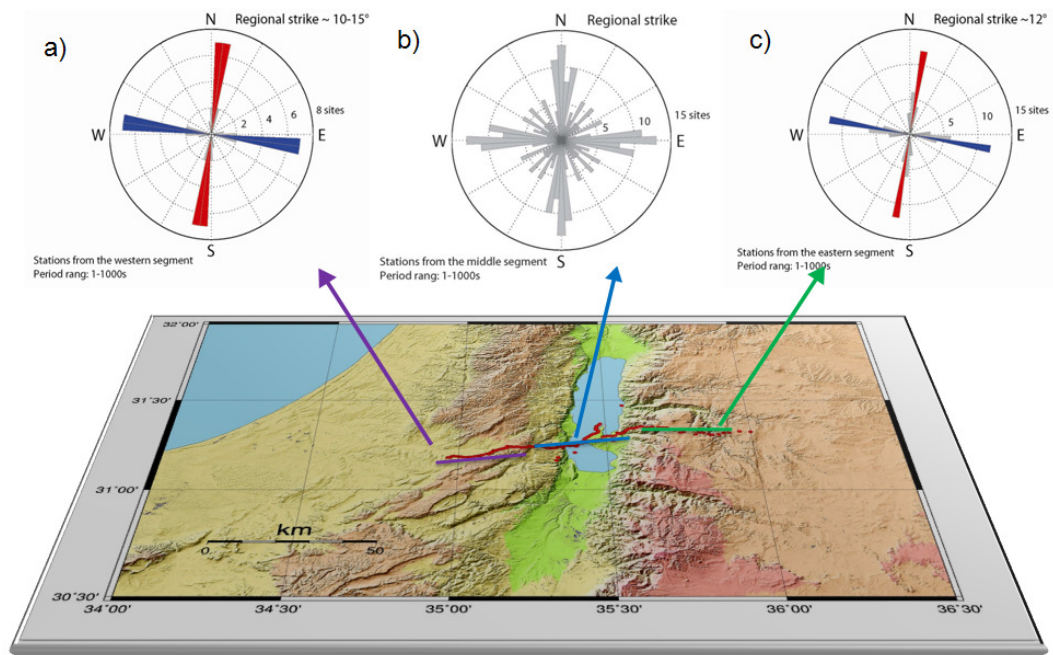


Fig. 2: Regional electrical strike direction analysis: a, b and c show rose diagrams for the western, central and eastern segments, respectively. The western and eastern segments (a and c) are consistent with the expected regional strike direction along the DS basin.

Preliminary 2D inversion results

For 2D inversion, we used the non-linear conjugate gradient algorithm described by Rodi and Mackie (2001) to invert the data of the main profile. The data were rotated into the regional strike (12°E) estimated for the eastern and the western profile segments, and assigned to the presumed E- and H-polarizations.

For the main profile, inversions using different model parameterizations, regularizations and a priori models were examined to test the robustness of the resulting resistivity model.

The model presented in Figure 3 was obtained from joint inversion of the E- and H-polarization apparent resistivity and phases and the vertical magnetic transfer function by using stations from the central part of the main profile. However, large error floors were imposed on the E-polarization apparent resistivity to reduce the effect of static shift in the data; emphasis was on fitting the phases. Furthermore, in this preliminary inversion study we confined the period range from 1000Hz to 10s to concentrate on the structures in the upper crust.

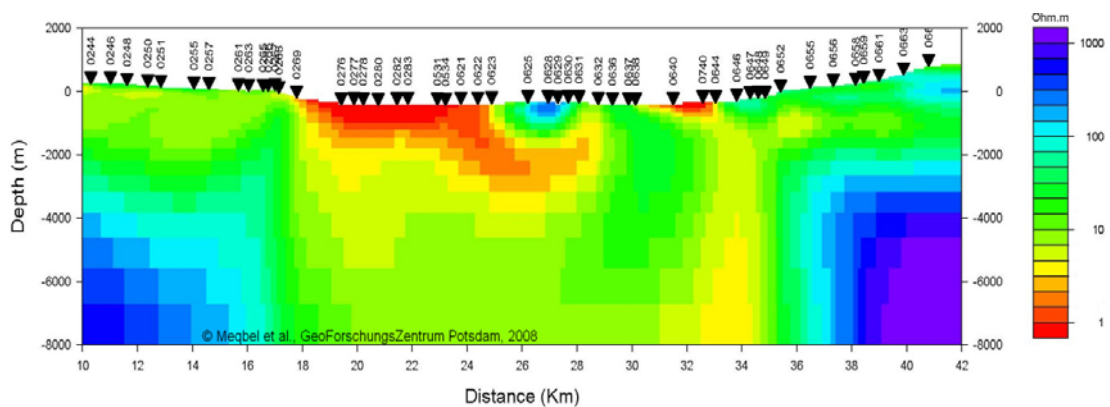


Figure 3: Resistivity model obtained for the central part of the main profile. The black triangles indicate the positions of stations. Red and yellow colours indicate zones of low electrical resistivity.

Our preliminary 2D resistivity section shows the following features (Fig. 3):

- Extremely low electrical resistivity ($< 0.5 \Omega\text{m}$) beneath the DS (red colours) extending to approximately 3km depth. This structure could indicate that the Dead Sea brines reach depths of several kilometres.
- Within this structure and directly beneath the Al Lisan Peninsula we observe a high resistivity body ($> 100 \Omega\text{m}$, green and blue colours) which seems to coincide with a known salt intrusion. From borehole information, a salt body (mainly halite) begins at 100m depth and extents to approximately 3000m depth (Powell, 1988).

- Very abrupt lateral changes from low to high resistivity ($> 1000 \Omega\text{m}$, dark blue and magenta colours) at depths of 3 to 4 km coincide with the surface traces of the EB and WB faults. These boundaries indicate the width of the Dead Sea basin at depth.
- Both fault zones appear to be associated with narrow, sub-vertical zones of low resistivity which may reach mid-crustal levels.

The deep resistive structures ($> 1000 \Omega\text{m}$) at depths of 3 (east) and 4 km (west) belong to the crystalline basement. The lower resistivity values beneath the central part of profile (Fig. 3) correspond to the sedimentary fill of the Dead Sea basin.

Outlook

Thorough resolution tests of the 2D inversion models will be important for the near future work. Furthermore, 3D modelling and inversion of the entire data set (including the in-basin profile) will help us to map the complicated electrical resistivity structure within the DS basin.

Our resistivity model will form an integral part of a joint interpretation of the many geophysical parameters obtained in the scope of the DESIRE project. Eventually, this will improve our understanding of large faults systems such as the DST.

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Magnetotelluric measurements in the vicinity of the Groß Schönebeck geothermal test site

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Introduction

The EU-funded I-GET (Integrated Geophysical Exploration Technologies for Deep Fractured Geothermal Systems) project is aimed at developing an innovative strategy for geophysical exploration. The basic idea is to integrate all available knowledge, from rock physics to seismic and magnetotelluric (MT) data processing and modelling, exploiting the full potential of electromagnetic and seismic exploration methods to detect permeable zones and fluid bearing fractures in the subsurface.

The Groß Schönebeck (Germany) deep sedimentary reservoir (Figure 2) is representative for large sedimentary basins which exist all over Europe. An existing borehole is currently used as an in situ geothermal laboratory. The Groß Schönebeck test site consist of a doublet made up of the wells GrSk 3/90 and GrSk 4/05 and is located NE of Berlin (Figure 1).

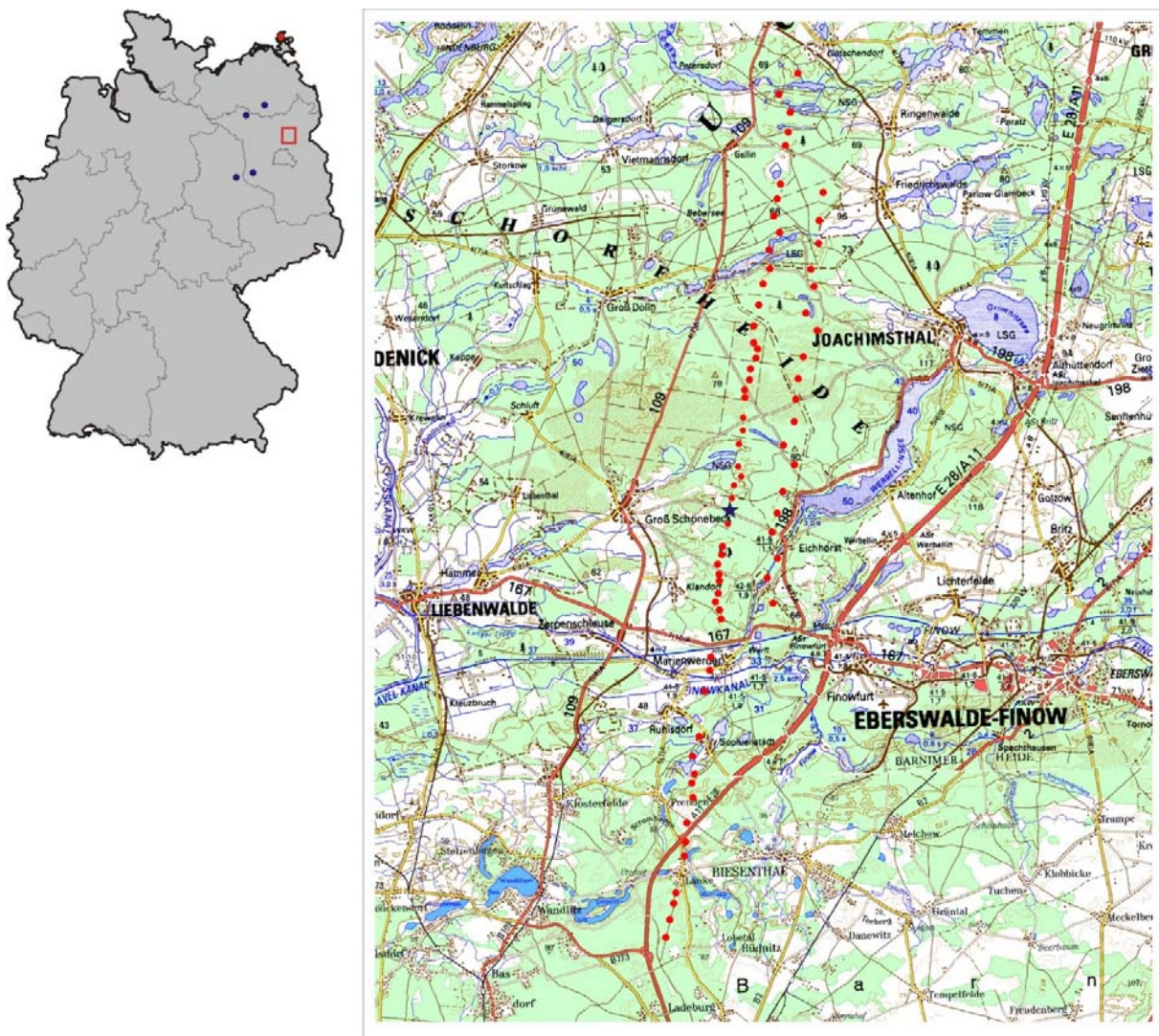


Figure 1. Location of the MT sites (red dots). The long profile located to the west (main profile) was acquired during June - July 2006. The short profile to the east (profile winter) was acquired in February - March 2007. The blue star indicates the location of the boreholes. The dots in the map of Germany indicate the locations of the remote reference sites (blue for the 2006 experiment, red for the 2007 experiment).

The geothermal reservoir is located in the Rotliegend strata of the NE German Basin (NEGB). Initial basin extension occurred between late Carboniferous and early Permian, with several NW-SE trending strike-slip faults dividing the basin in different areas. The movement along these faults is predominantly lateral, with very little vertical displacement. Due to wrench tectonics during the post orogenic period of the Variscan orogeny, weak zones in the NNE-SSW direction

were created; which lead to the development of a graben system (Baltrusch and Klärner, 1993). The extension phase was accompanied by the deposition of volcanic rocks which were subsequently covered by a siliciclastic sequence of alluvial fans, ephemeral stream, playa deposits and Aeolian sands (Rieke et al., 2001). Thick cyclic evaporites and carbonates were deposited during the Zechstein (Permian), overlain by thick strata of Mesozoic and Cenozoic sediments (Moeck et al., 2007).

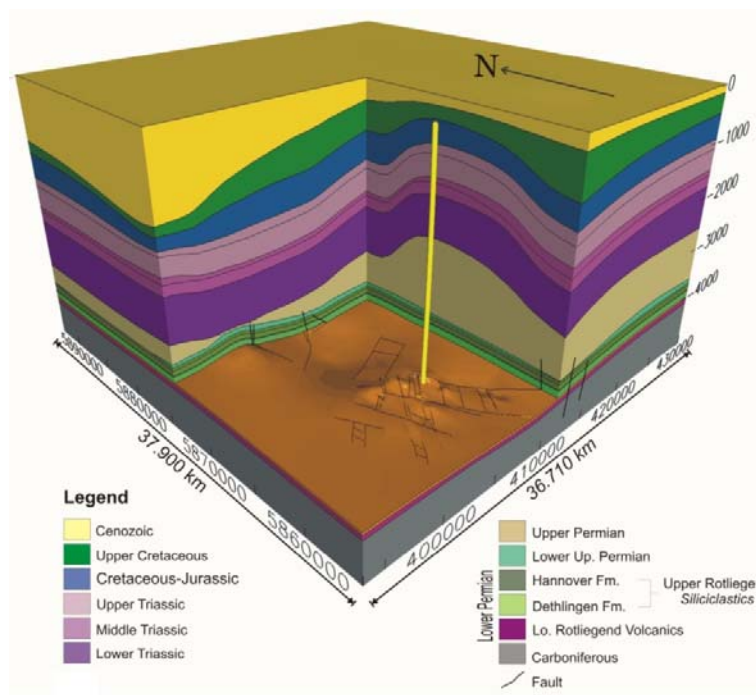


Figure 2. 3D model showing the geological environment of the Groß Schönebeck site. (From Moeck et al., 2007).

Data analysis

MT data was collected along a 40 km-long main profile centred around the well doublet and a shorter, approximately 20 km-long parallel profile located 5 km to the east (Figure 1). The main profile consists of 55 stations with a site spacing of 400 m in the central part (close to the borehole) of the profile, increasing to 800 m towards the profile ends. The parallel profile to the east consists of 18 stations with a site spacing of 1 km. The period range of the observations was 0,001 to 1000 s. At all sites, we recorded horizontal electric and magnetic field components and the vertical magnetic field.

Given the amount of electromagnetic noise present in the survey area, approximately 20 km north of the outskirts of Berlin, a long recording time was necessary to improve the statistical properties of the collected data. To further improve the data quality, additional MT stations were set-up to as remote reference sites for the data processing (Figure 1). The remote site delivering the highest quality was setup on the island of Rügen, approximately 350 km to the north of the profile. The geomagnetic and impedance tensor transfer functions were obtained using the robust processing algorithm described in Ritter et al. (1998), Weckmann et al. (2005), and with modifications by Krings (2007, see this issue) which were developed within the framework of the IGET project. For comparison the data were also processed using the remote reference code of Egbert (1997). Figure 4 show examples of the data.

The dimensionality of the data was analysed using the code of Becken and Burkhardt (2004). This technique examines the impedance tensor from the point of view of polarization states of the electric and magnetic fields. Analysing the ellipticities of polarization of the fields, the regional strike direction can be identified even if the data are affected by galvanic distortion. Figure 3 shows the rose diagrams of the strike analysis for both profiles. The multi-site regional strike direction for these data, according to the technique used is 20° for the main profile and 17° for the parallel profile. This coincides fairly well with the direction of maximum horizontal stress near the borehole, which was determined as $18.5^\circ \pm 3.7^\circ$ (Holl et al., 2005).

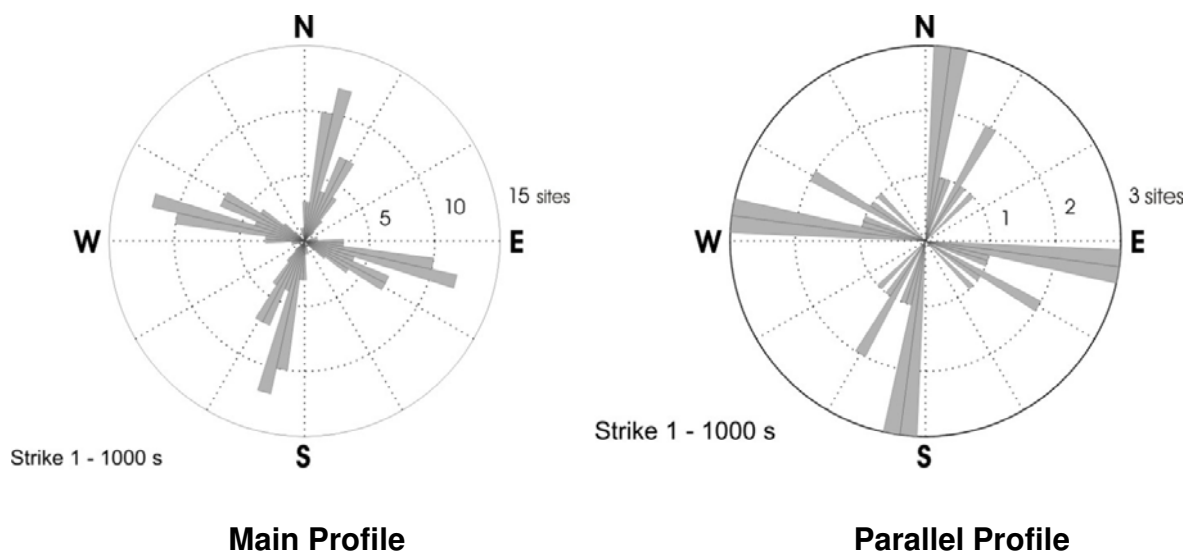
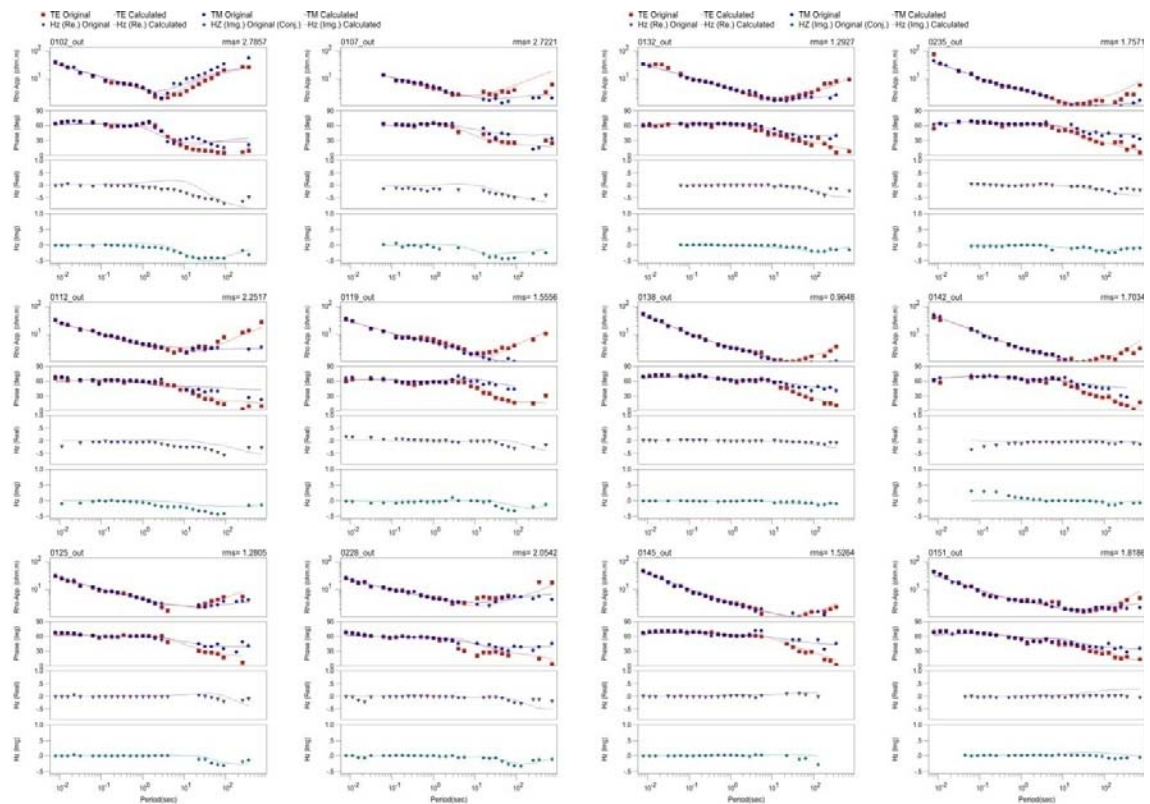


Figure 3. Regional strike direction of the sites of both profiles in rose diagram representation.

A)



B)

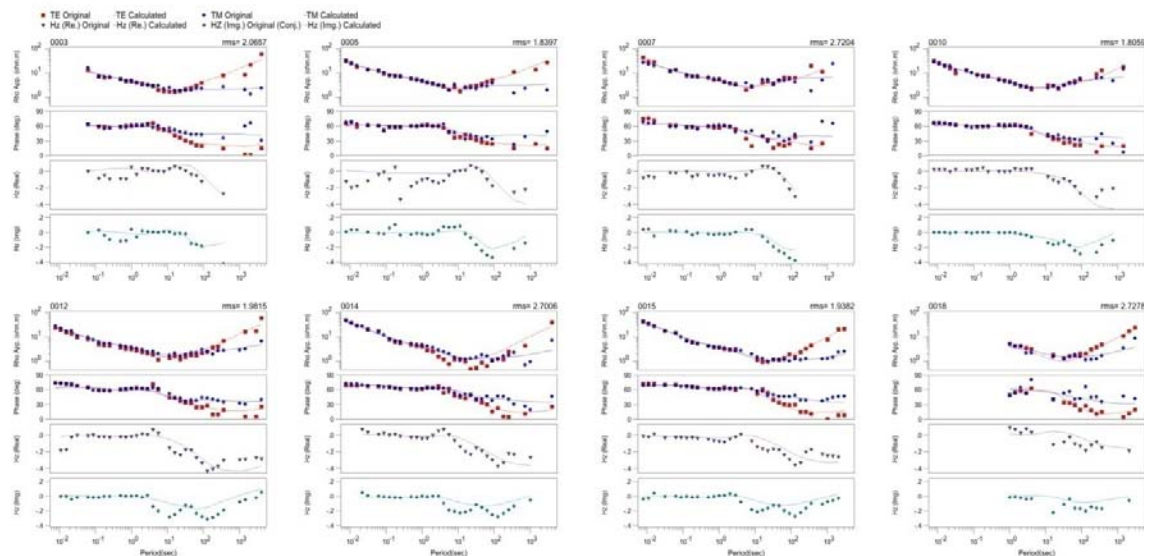


Figure 4. A) Data and model responses for some stations of the main profile. B) Data and model responses for some stations of the parallel profile.

Conductivity model

Prior to the inversion and according to the dimensionality analysis all data of the main profile were rotated to -70° and data of the parallel profile to -73° . The 2D inversion of the MT data was calculated using the algorithm of Rodi & Mackie (2001). For the inversion procedure, both TE and TM modes of apparent resistivity and phases and real and imaginary part of the geomagnetic transfer functions were used, in the frequency range from 1000 Hz to 1 mHz. Using an error floor of 10% for the apparent resistivities, 5° for the phases and 0.05 for the geomagnetic transfer functions an RMS misfit of 2.26 was obtained for the main profile and 2.28 for the parallel profile.

The resistivity model for the main profile (Figure 5) shows a shallow conductive layer extending from the surface down to depths of about 4 km, with an antiform-type shape below the central part of the profile. At a depth range of 4-5 km two conductive bodies are found, separated by a region of moderate conductivity. According to the seismic tomography (Bauer et al., 2006), which shows high velocity values for depths greater than 4 km, a resistive basement was introduced *a priori* in the resistivity model.

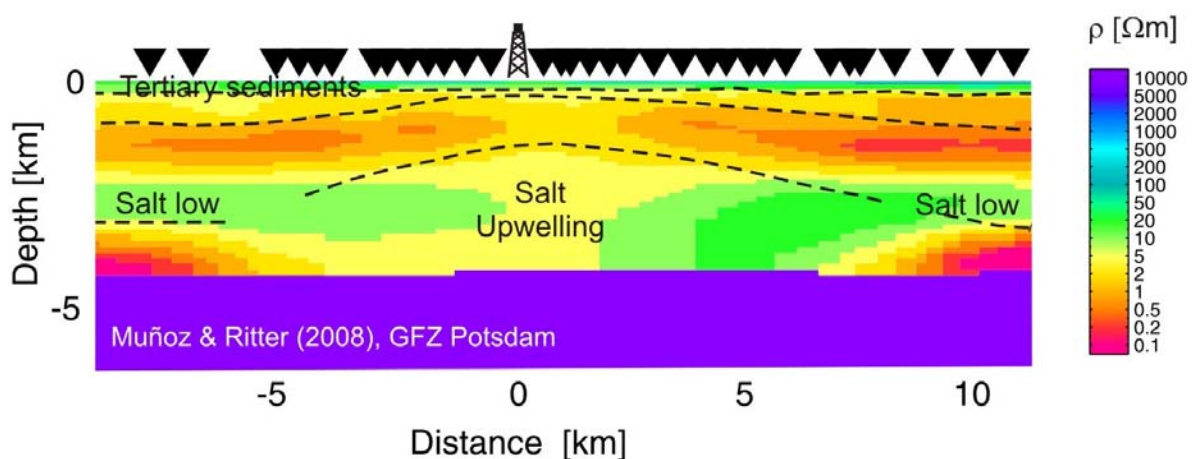


Figure 5. Interpreted MT section of the central part of the model obtained from inversion of the main profile.

At surface, the Tertiary sediments are imaged as a moderately conductive layer (20 – 50 Ωm). The antiform shape of the conductive layer coincides with the sedimentary sequences above the

Buntsandstein. Areas of higher conductivity image the transition to the Cretaceous, where the presence of more porous material could be responsible for the enhanced conductivity. The slightly higher resistivity at the hinge of the antiform can be explained by higher erosion (or lower deposition) rates of the porous material in this elevated area. The deep conductors at the left and right edges above the resistive basement coincide with the Rotliegend level. These conductive bodies occur below the areas where the Zechstein salt layer presents local lows, while we observe only moderate resistivities beneath the salt upwelling in the Rotliegend. The deep high conductivity bodies could be caused by fluids, possibly with salinities in excess of 260 g/l (Giese et al., 2001). With these salinities, a temperature in of 120 °C, and porosities around 15% (Holl et al., 2005) the modelled resistivities can be explained by virtue of Archie's law considering a fracture dominated porosity below the salt lows and a pore dominated porosity below the salt upwelling. Consequently, higher resistivity regions in the Rotliegend could be interpreted as a lower porosity (fracture density) or fluid conductivity. The main lithological unit of the Zechstein lows is anhydrite, which shows brittle behaviour under stress and is therefore perceptive for fracturing. In contrast, upwelling regions are caused by plastic deformation and up-doming of salt which is less susceptible for fracturing.

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Electrical characterization of Pathri-Rao watershed in Himalayan foothills region, Uttarakhand, India

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Abstract

Geoelectrical techniques have been used to define the geometry of aquifer system in Pathri-Rao watershed situated in the Piedmont zone of Himalayan foothill region, Uttarakhand, India. This has been done by integrating the results of dc resistivity and electromagnetic data recorded from the area. 2D resistivity imaging and time domain electromagnetic were carried out to define horizontal and vertical geometry of aquifer system and to infer the local groundwater flow condition. On the basis of resistivity values it has been found that shallow and deeper aquifers have different degree of interaction in the area. The study demonstrates the versatility of geoelectrical techniques.

Keywords: Electrical resistivity imaging, Piedmont zone, Himalayan foothill region, Aquifer system.

Introduction

The rate of withdrawal of groundwater in the piedmont area is increasing continuously due to the increasing pace of population growth, agricultural and industrial development. The over-exploitation of groundwater is manifested by the lowering of water table and creating regional imbalance associated with the problem of water scarcity for domestic, agricultural and industrial use. Hence, the detail identification of aquifer system is essential for the sustainable development of groundwater in the piedmont area of Himalayan foothill region.

The aquifer delineation based on the surface hydrogeological and geomorphologic features and do not provide reliable data of subsurface features of aquifer system. For a better understanding of aquifer system in a multi-aquifer system in the piedmont zone, geophysical methods can provide a better estimate of the depth,

thickness and lateral extent of the aquifer system. Recent development in the geoelectrical data acquisition and interpretation methodology provides electrical image of subsurface feature which could be directly used to infer the aquifer configuration. The geoelectrical image can be used to define the depth, thickness, horizontal extent and surface elevation of aquifer system. Thus by combining the data on the surface hydrogeological features with the subsurface information obtained from the geoelectrical investigations, one may define the subsurface features and details of aquifer geometry. Venkateswara Rao and Briz-Kishore (1991) have used the geophysical and hydrogeological methods for estimating the groundwater potential in arid and semi-arid areas of south India, where more than 80% of the land is underlain by crystalline rocks. Edet and Okereke (1997, 2002) used a similar approach for the Oban massif, Nigeria and calculated the groundwater potential in the area. Shahid and Nath (1999) have also used

the integration of remote sensing, and electrical sounding data for spatial hydrogeological modeling of a soft rock terrain in the Midnapur District, West Bengal, India. The objective of present study is to apply integrated geoelectrical techniques for the delineation of aquifer system and its geometrical configuration in the Pathri-Rao watershed of Himalayan foothill region, Uttarakhand, India.

Study area:

The study area, Pathri-Rao watershed, is shown in Fig. 1, located between Latitude $29^{\circ} 55' - 30^{\circ} 03'$ and Longitude $77^{\circ} 59' - 78^{\circ} 06'$ covering about 52 sq km area and falls in the Upper Piedmont zone of Himalayan foothills region, Uttarakhand, India. The Upper Piedmont zone is also referred as “*Bhabhar*” in northern India. The *Bhabhar* zone presents several difficulties in groundwater exploration and development due to the occurrence of thick deposits of poorly sorted sediments, a deep water table, and the associated drilling problems. Consequently, cost of the developing groundwater in the area is prohibitive and is just sufficient for domestic use. Therefore, delineation of aquifer system and its detail geometrical configuration has become an important problem for sustainable groundwater in the area.

Geology and Hydrogeology of the study area:

Pathri-Rao watershed is mainly characterized by quaternary deposit of Piedmont formation, formed by the sediments of Himalayan foothills and tertiary deposit of Siwaliks. The Siwaliks are exposed in the northern part of Pathri-Rao watershed. *Bhabhar* formation of Pathri Rao watershed is bounded by Siwaliks in the north and *Tarai* formation in the southern

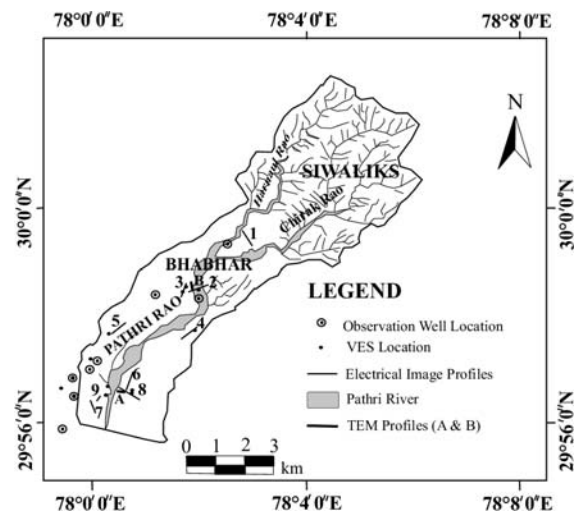


Fig. 1: Study area showing locations of investigated sites.

part. *Bhabhar* formation is part of Gangetic alluvium deposit, which forms a belt which extends in an elongated nature along the foothills region. It consists of unconsolidated coarse sand with boulder, fine to medium sand with pebble, boulder and clay, derived from the present day Siwaliks ranges. The depth to water level monitored in the observation wells located in area lies from 7 m to 31 m below the ground level (bgl).

Geoelectrical techniques:

Geoelectrical techniques are powerful tools and play a vital role in the delineation of aquifer configuration in complex geological environments. A planned geoelectrical investigation is capable of mapping aquifer system, clay layers, depth and thickness of aquifers and qualitative estimation of local ground water flow (Israil et al, 2006).

The popular electrical resistivity method of data collection requires the use of four electrodes, which are moved for each measurement. When the spacing between electrodes is maintained constant and the locations are moved across the ground

surface, a profile of apparent resistivity values across an area can be developed. When the spacing between electrodes is varied around a central location, a sounding is developed by spreading the electrodes further apart for each measurement. 1D inversion is used to model the geological features by geophysical parameters such as depth, thickness and resistivity.

Electrical image profiling using multi – electrodes resistivity imaging system, facilitates simultaneous recording of profiling and sounding data efficiently. Loke & Barker (Loke and Barker, 1996) have developed a rapid least squares inversion method for the data collected by multi – electrode resistivity system. Resistivity method is not successful in the area where galvanic contact of electrode to the ground is not possible or the surface is highly resistive. In such areas, inductive method (Electromagnetic) is more effective. Time Domain Electromagnetic (TEM) method does not require any galvanic contact with the ground for the current injection. Therefore, it can also be used in the highly resistive ground.

Thus the integration of Vertical Electrical Sounding (VES), Electrical Image Profile (EIP) and TEM data can be effectively used to determine the aquifer configuration in a complex geological environment.

Geoelectrical Data Acquisition and Interpretation:

The location of 11 VES, 9 EIP and 2 TEM profiles are shown in Fig. 1. VES data were collected using Schlumberger configuration with maximum electrode spacing of 900.0 m. As northern zone of the area is restricted and is inaccessible, therefore the survey was carried out only in the southern part. The electrode spacing is sufficient to provide information about

resistivity variation in near surface and deeper than 100 m, which is capable of delineating shallow and deeper aquifers. The least squares method is used to invert apparent resistivity data in terms of resistivity and thickness of subsurface layer. Interpreted 1D model consists of 3 to 5 layers with varying resistivity and thicknesses. Root mean square (rms) error obtained between the observed and modeled apparent resistivity lies below 6 % in all data sets. The resistivity of top layer in the northern part of the study area is very high. The resistivity decreases in deeper layer indicating the saturated condition in deeper zone. In southern area, the resistivity is relatively lower at all depths in comparison to the northern zone. For geological interpretation, the resistivity values are calibrated with known lithology from available borehole data. An example of lithological correlation of resistivity values obtained from the interpretation of measured apparent resistivity in the area is shown in Fig 2.

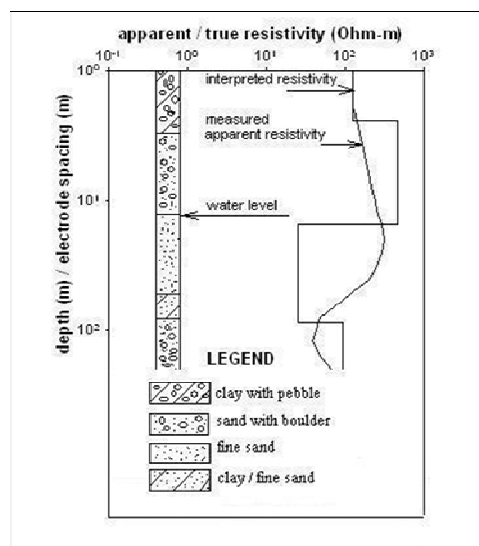


Fig. 2: Lithological correlation of resistivity data in *Bhabhar* zone.

With the objective to map the detailed lateral and vertical variation of resistivity in

the area, 9 Electrical Image Profiles (EIP) are systematically recorded to cover entire accessible region. These profiles were arranged serially from north to south. The exact location and orientation of these profiles are shown in Fig. 1. EIP-1, 2 and 8 are oriented nearly perpendicular to the Pathri-rao river whereas others are oriented almost along and on either side of the river bed. In 8 EIP lines, 72 electrodes are placed at 10 m intervals making a total length of each profile 710 m and the last profile (EIP - 9) is recorded at 5 m electrode spacing with a total profile length of 355 m. The data

were recorded using IRIS resistivity meter in a sequence generated using Schlumberger – Wenner configuration with 895 quadripoles (data points) in each sequence. Surface topographical elevations have been recorded using handheld GPS system along the profile line to define topographic elevation. The data were processed initially to eliminate spiky and error prone data. Subsequently, 2D inversion has been performed on EIP data set using RES2DINV (Loke, 1997) code, with smooth constrained least square method to delineate resistivity depth image along the profile line.

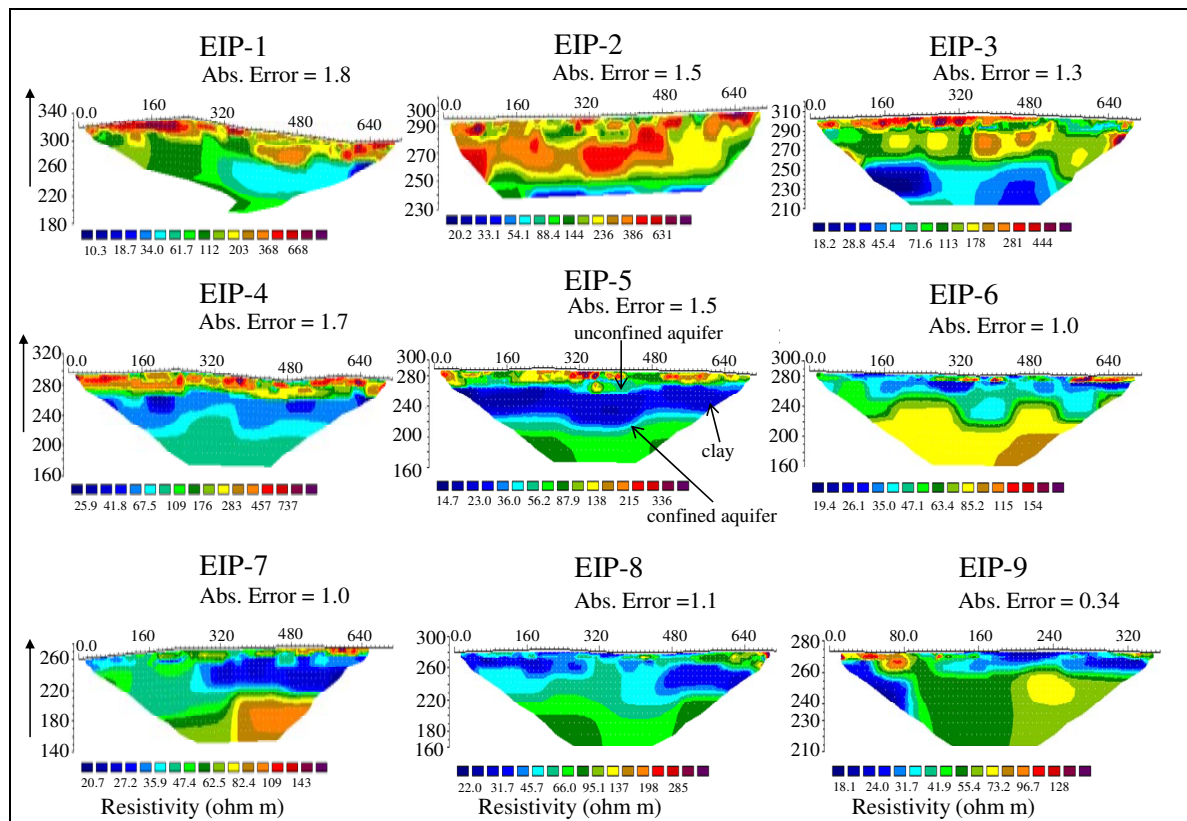


Fig.3: Interpreted resistivity depth sections along Electrical image profiles (EIP). Vertical scale represents the height above mean sea level (msl) and horizontal scale is the distance in meters along the profile line.

Fig 3 shows the 2D model representing resistivity-depth section along nine EIP's lines. The absolute root mean square error between the observed apparent resistivity data and the modelled one is less than 1.7 in all cases (Fig. 3). These geoelectrical sections present a systematic variation of electrical resistivity with depth along each profile line. In the northern area unconsolidated coarse materials and boulders transported from Siwalik Hills are characterized by very high resistivity (900 Ωm) for near surface unsaturated layer. At a depth of about 30 m, the resistivity is low (50 Ωm) indicating saturated sand below the river channel (Harnaul Rao). Further southward, EIP-2 indicates comparatively low resistivity (600 Ωm) of near surface formation. Underneath the top resistive layer a consistent aquifer zone dipping towards southwest is delineated, which is characterized by low resistivity range (50-100 Ωm). EIP-3 is also located near to EIP-2 oriented approximately along the river bed. A very low resistivity (18 Ωm) zone is present underneath the ends of the profile; these low resistivity zones are discontinuous in the middle of the profile (EIP-3). Further south to it, in EIP-4 and EIP-5, the low resistivity zone extends almost all along the entire profile line. EIP-5 represents a typical example of the existence of two aquifer system (shallow at about 10 m depth and deeper at nearly 30 m depth) separated by a thick clay layer. Further towards south, as indicated by EIP-6 to 9, the saturated zone is at shallower depths, which finally intersects the ground surface as manifested by the spring line (Israil et al, 2006). Thus the profiles oriented in different directions show the configuration of aquifer zone and its elevation along the profile line. The aquifer zones are normally dipping southward, indicating local groundwater flow direction from north to south in the area. The direction of groundwater flow delineated

from electrical images is consistent with the local hydrogeological setting. The middle part of study area is a typical example of two aquifers separated by a thick impermeable clay layer (EIP-5). Similar hydrogeological conditions are also revealed on the eastern side of the Pathri Rao River (Fig. 3).

In the study area, unconsolidated, and unsaturated coarse material transported from the Siwalik Hills, is represented by a high resistivity (900 Ωm), saturated sand with pebbles by a resistivity range of 30-100 Ωm and saturated clay by a resistivity range of 10-25 Ωm .

The time domain electromagnetic (TEM) data along two profiles (A & B) is shown in Fig. 1. Zonge GDP-32 NanoTEM system has been used to record data from 25 stations along two profiles (10 stations in profile A and 15 stations in profile B) with a station spacing of 20 m. The TEM technique does not require galvanic contact of electrode with the ground and hence it can be used in a very high resistivity ground also. Here our purpose is to improve the interpretation by comparing the dc and TEM results. The depth of investigation of TEM data is determined by the time at which the signal decays to noise level, the source strength, loop size and the earth's resistivity (Spies, 1989). In the present investigations, we have used 20x20 m² transmitter loop powered by 12 volts battery and 5x5 m² receiver loop placed in the centre of the transmitter. The data collected from the field are processed and interpreted to generate resistivity-depth image from the decay curve. For this purpose we used the STEMINV software package (MacInnes & Raymond, 2001), which is a smooth model inversion technique. The inversion results are presented as the isolines of resistivity values along the two profiles in Fig. 4(A & B). The resistivity section obtained from

TEM profile A & B can be compared with the EIP profiles EIP-8 & EIP-3 respectively. The interpreted TEM section along profile 'A' and EIP-8 both are showing the resistive layer at the top up to the depth of 10 m and below this resistive layer, there is a conductive layer which is approximately 20 m thick. Resistivity starts increasing again beneath this conductive layer. The interpreted section along TEM profile 'B' shows approximately 40 m thick resistive layer at the top and below this layer resistivity starts decreasing. Hence, the resistivity depth section obtained from TEM data at shallow depth broadly agrees with the corresponding resistivity depth section obtained by imaging and VES data.

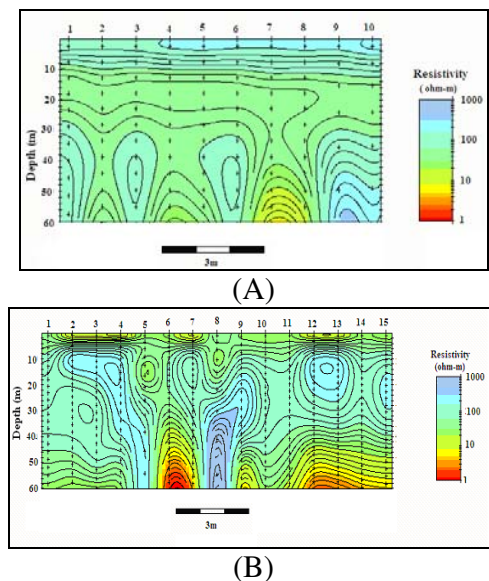


Fig. 4: Smoothed resistivity depth section obtained from Time domain Electromagnetic (TEM) data along two profiles (A & B).

Conclusion:

Aquifer geometry and groundwater flow direction in a complex geological environment of Himalayan foothills regions are defined on the basis of integrated VES, EIP and TEM geo-electrical techniques. The

study indicates high resistivity of unsaturated unconsolidated and porous coarse material of *Bhabhar* formation. Finer subsurface material in saturated condition towards the southern part is indicated by low resistivity. Two aquifers separated by a clay formation are inferred in the middle part of the area. The clay layer is discontinuous in some areas indicating interconnection between the two aquifers.

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Detection of subsurface salinity and conductive structures in Inche-boroon, Golestan, Iran, using magnetotelluric method

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1. Introduction

Magnetotelluric (MT) method is an important passive surface geophysical method which uses the Earth's natural EM fields to investigate the electrical resistivity of the subsurface. The depth of investigation of MT is much higher than other electromagnetic (EM) methods. The electrical conductivity of upper few kilometers of the Earth's upper crust is controlled by many parameters among them salinity of the subsurface structures. Very conductive Iodine structures are one of the subsurface salinity structures which can be formed at upper few hundred meters of the crust. Thus to study those kind of structures, magnetotelluric (MT) is the most effective method.

Gorgan plain is located in the northern Iran. In Feb-March 2006, an MT survey was conducted in the area. Data were collected along four east-west profiles (37 sites in total). Sites distances are 1 km and profiles distances are 1.5 km (Fig. 1 & 2).

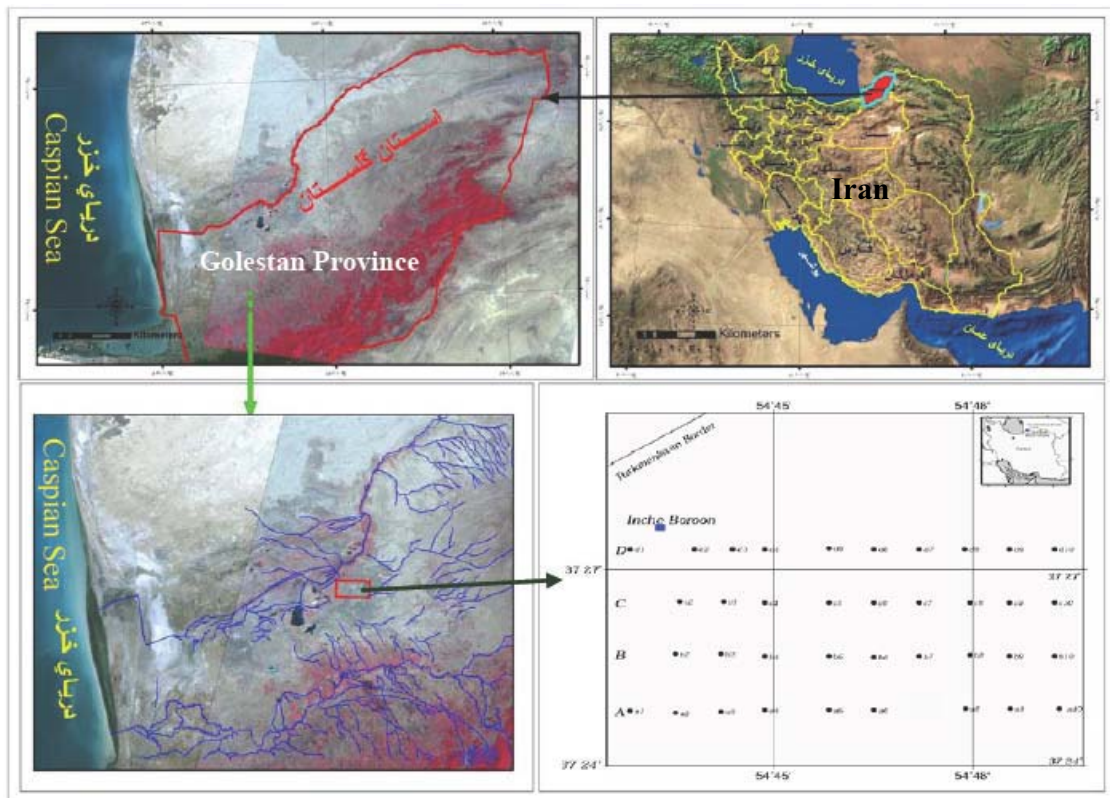


Figure 1. Geographic location of study area.

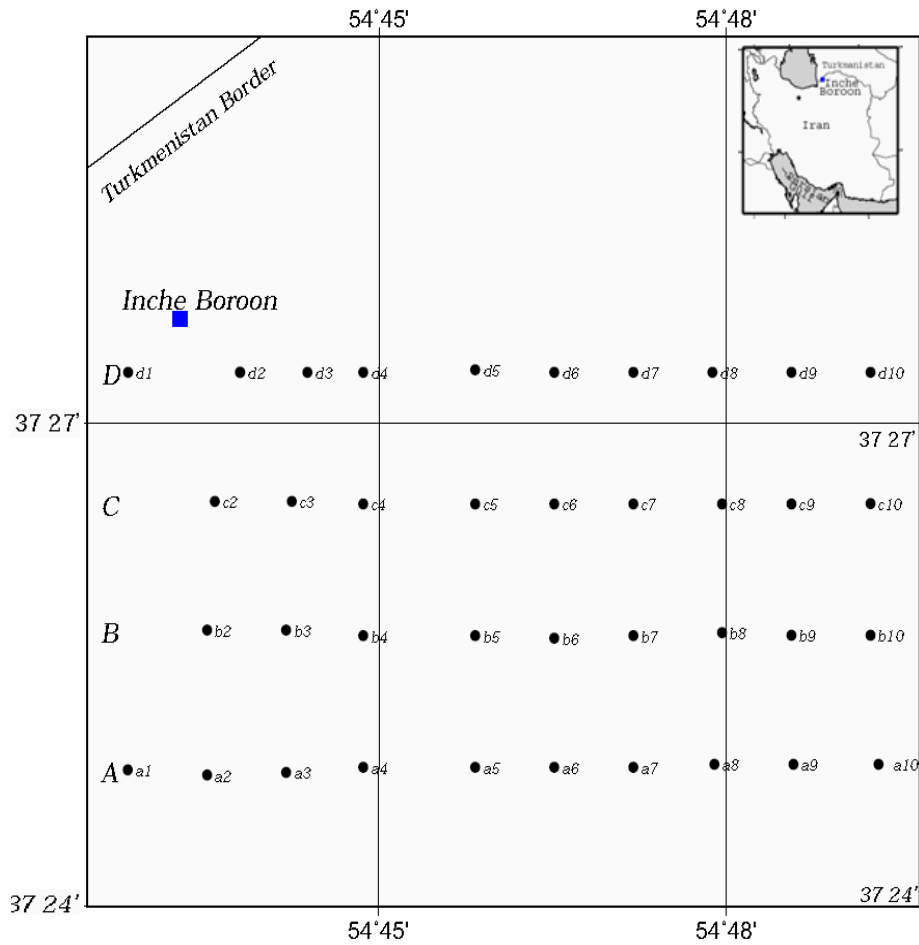


Figure 2. Location of profiles and sites

2. Magnetotelluric concepts

Measurements of the horizontal components of the natural electromagnetic field are used to construct the full complex impedance tensor, Z , as a function of frequency,

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

indicating the lateral and vertical variations of the subsurface electrical conductivity at a given measurement site. Apparent resistivity, ρ_a , and phase, φ , are the desired quantities calculated through the following relations,

$$\rho_{ai} = \frac{1}{\mu_0 \omega} |Z_i|^2 \quad i = xx, xy, yx, yy, DET$$

$$\varphi_i = \text{phase}(Z_i)$$

Where, μ_0 , is the permeability of free space, ω , is the angular frequency and , DET , denotes the determinant data. Time series measurements collected in various frequency ranges are transformed into frequency domain, and cross power spectra are computed to estimate the impedance tensor as a function of frequency.

The determinant of impedance tensor which is also called the effective impedance, Z_{DET} , (Pedersen and Engels, 2005), is defined as,

$$Z_{DET} = \sqrt{Z_{xx}Z_{yy} - Z_{xy}Z_{yx}}$$

Using the effective impedance, determinant apparent resistivities and phases are computed. The advantage of using the determinant data is that it provides a useful average of the impedance for all current directions. Furthermore, no mode identifications (transverse electric, TE mode: current in parallel with the strike; or transverse magnetic, TM-mode: current perpendicular to the strike) are required, static shift corrections are not made, and the dimensionality of the data is not considered, since the effective impedance is believed to represent an average that provides robust 1D and 2D models.

3. Dimensionality analysis

The Swift's (1967) skew, defined as, $S = |(Z_{xx} + Z_{yy}) / (Z_{xy} - Z_{yx})|$, indicates the dimensionality. Where Swift's skews are generally below 0.2, the structures can be regarded as undistorted 1D or 2D, otherwise, the structures are defined as distorted 1D and 2D structures or 3D structures. Dimensionality analysis of the MT data of the area shows that the assumption of undistorted 1D and 2D structures is correct. Figure 3 shows skew values for all sites measured across the profiles.

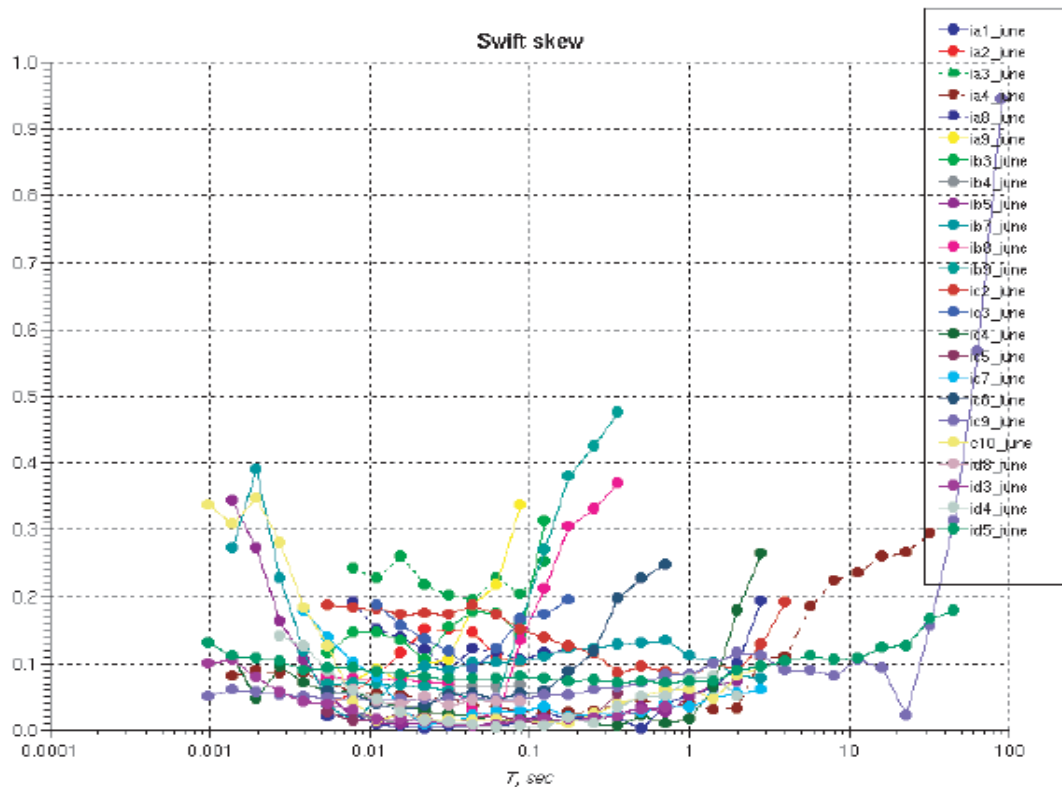
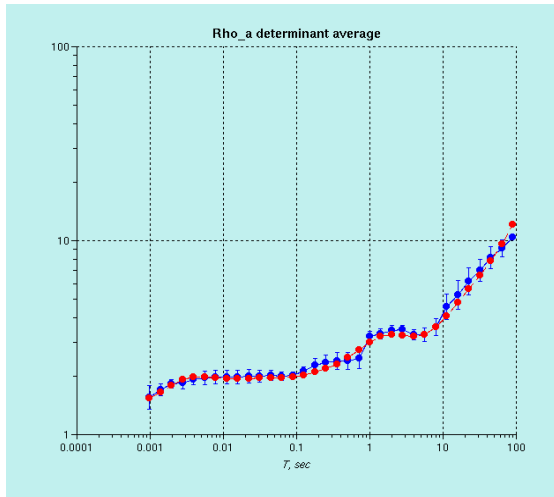


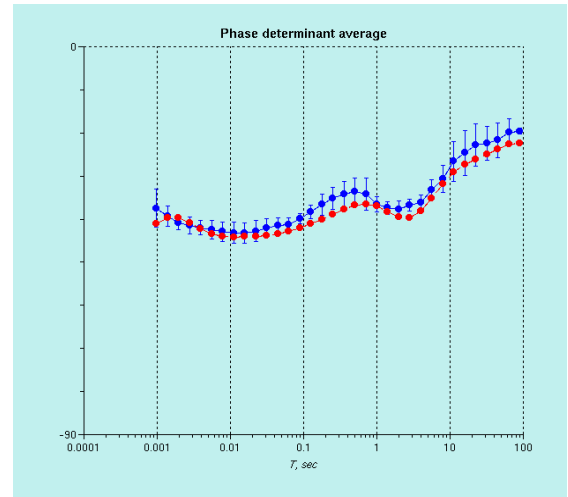
Figure 3. Swift's skew values for all sites measured across the profiles

4. Processing and 1D inversion of the MT data

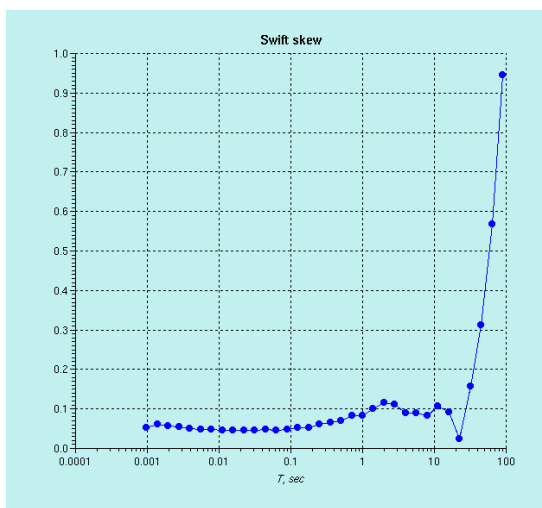
Processing of the data was done using Smirnov's (2003) approach. 1D inversion of the determinant (DET) data using a code from Pedersen (2004) was performed. The MT data in apparent resistivity and phase are given as inputs to inversion program. For instance, in figures 4a and 4b these data for site C9 are shown in blue. Red symbols are 1D model responses derived from data inversion. 1D model derived from data inversion of each site shows variation in Earth's layers conductivity in sites locations. 1D model of site C9 is shown in Fig. 4d. Also, to verify structure dimensionality assumption, Swift's skew values of site C9 are shown in Fig. 4c. By study of the resistivity sections (such as Fig. 4d) of the measured sites, it is concluded that although the area is conductive, two extremely conductive layers do exist between depths from 100m to 1300m. By separation of these two layers, isodepth maps and also isopach maps of them have been derived that are shown in figures from 5a to 5d. According to the isodepth maps, depths of the top of the first and second conductive layers from earth surface are estimated from 90m to 190m and from 250m to 650m, respectively. The thicknesses of the first and second layers are also distinguished from 5m to 30m and from 10m to 120m, respectively.



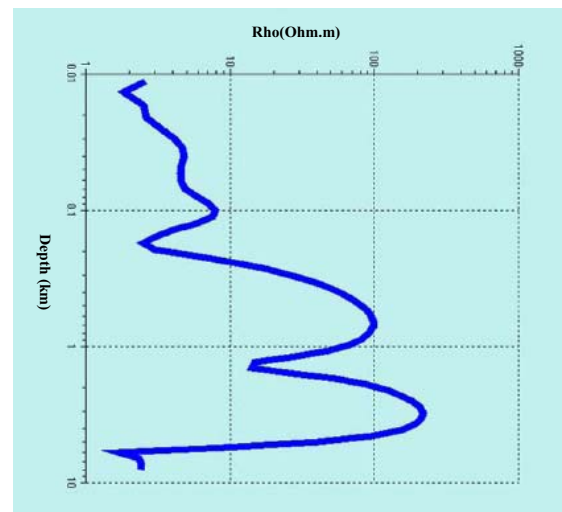
a



b



c



d

Figure 4. Data curve for site C9: a) Apparent resistivity, b) Phase, c) Swift's skew, d) 1D model

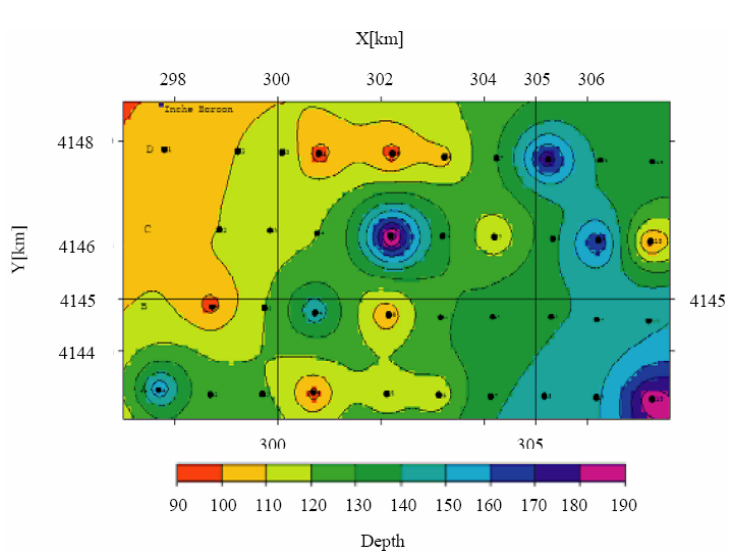


Figure 5a. Depth to the top of the first conductor

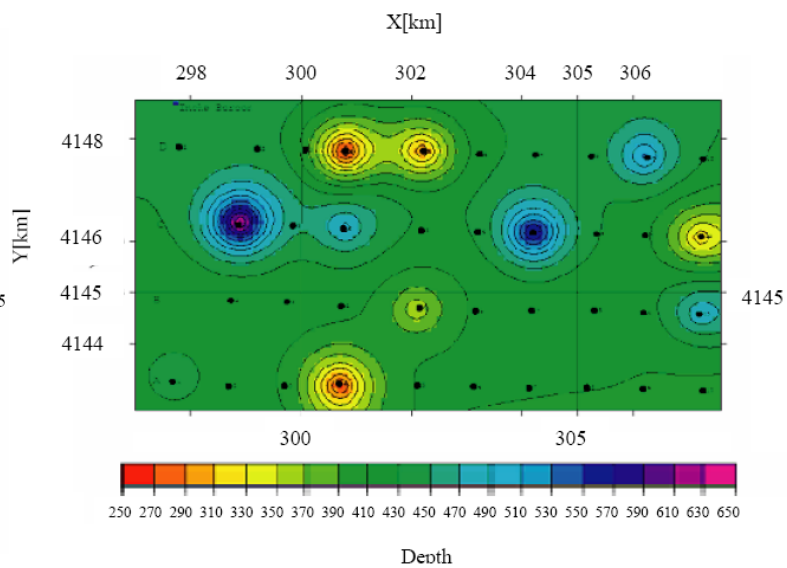


Figure 5b. Depth to the top of the second conductor

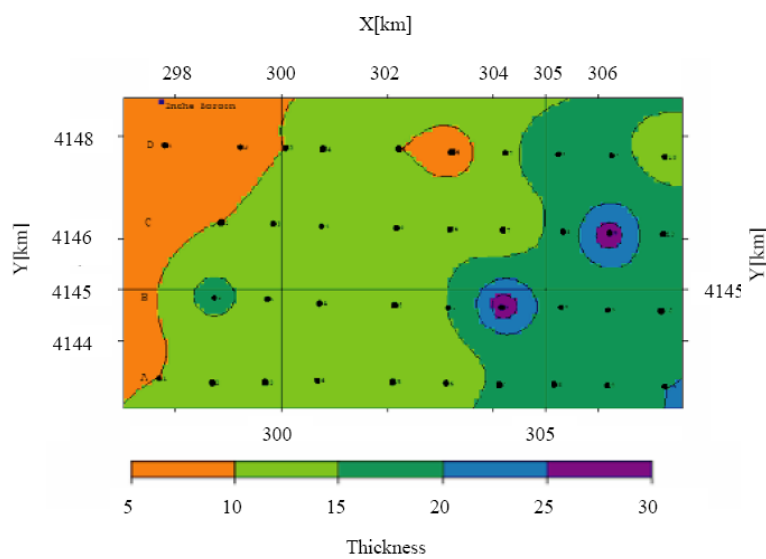


Figure 5c. Thickness of the first conductor

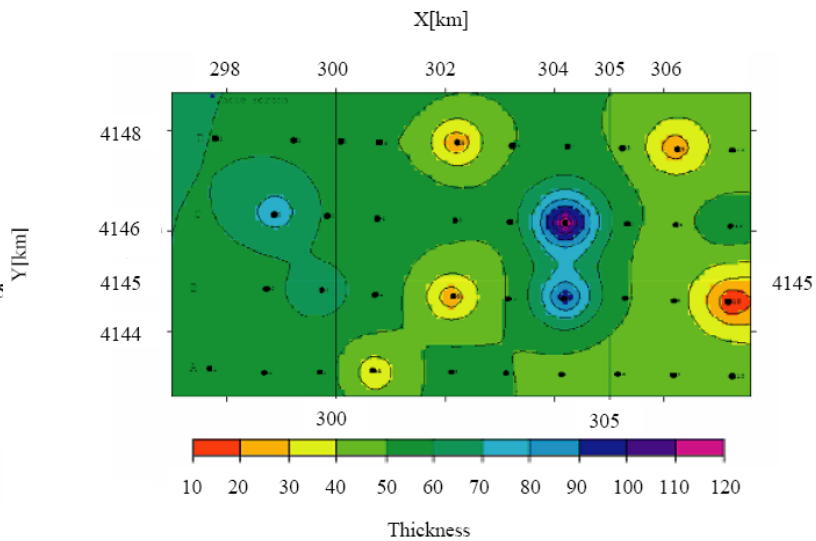


Figure 5d. Thickness of the second conductor

5. Comparison between borehole information and results of 1D inversion of the MT data

Subsurface electrical resistivity was verified by the information from a borehole in the vicinity of the area. According to the borehole log, there is a conductive layer consisting salt water at the depth of 670m to 840m. A schematic of the borehole log and the closeby resistivity section of the MT data are shown in Fig. 6.

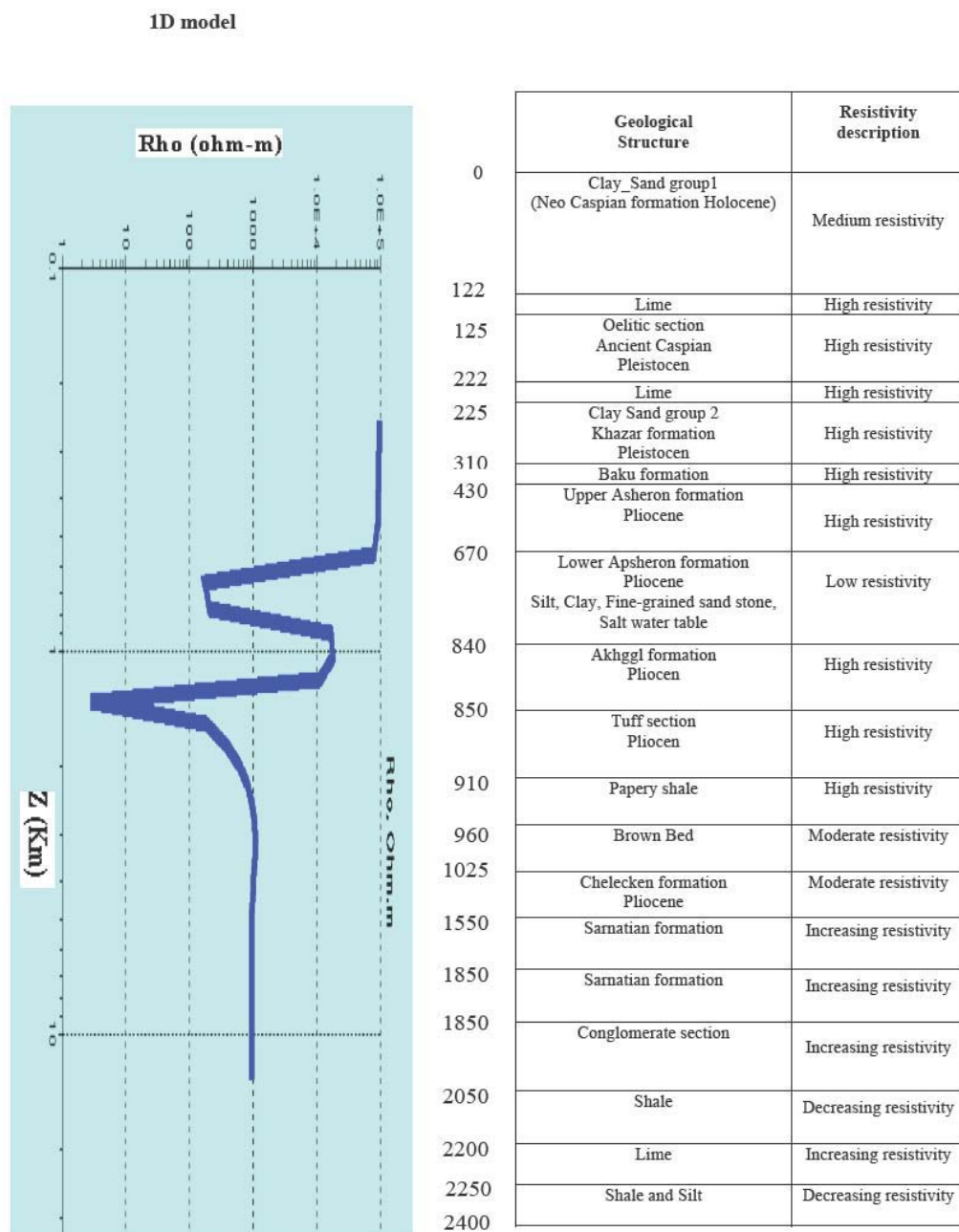


Figure 6. The well-log and the 1D model at the site close by.

6. 2D inversion of the MT date

A 2D inversion routine (Siripunvaraporn and Egbert, 2000) was used here to model the determinant data. We prepared initial models for 2D inversion of data based upon the results of 1D inversion. Observed, calculated and residuals of apparent resistivity and phase data along profiles are shown in Fig. 7. Final 2D resistivity models along four profiles are illustrated in Fig. 8.

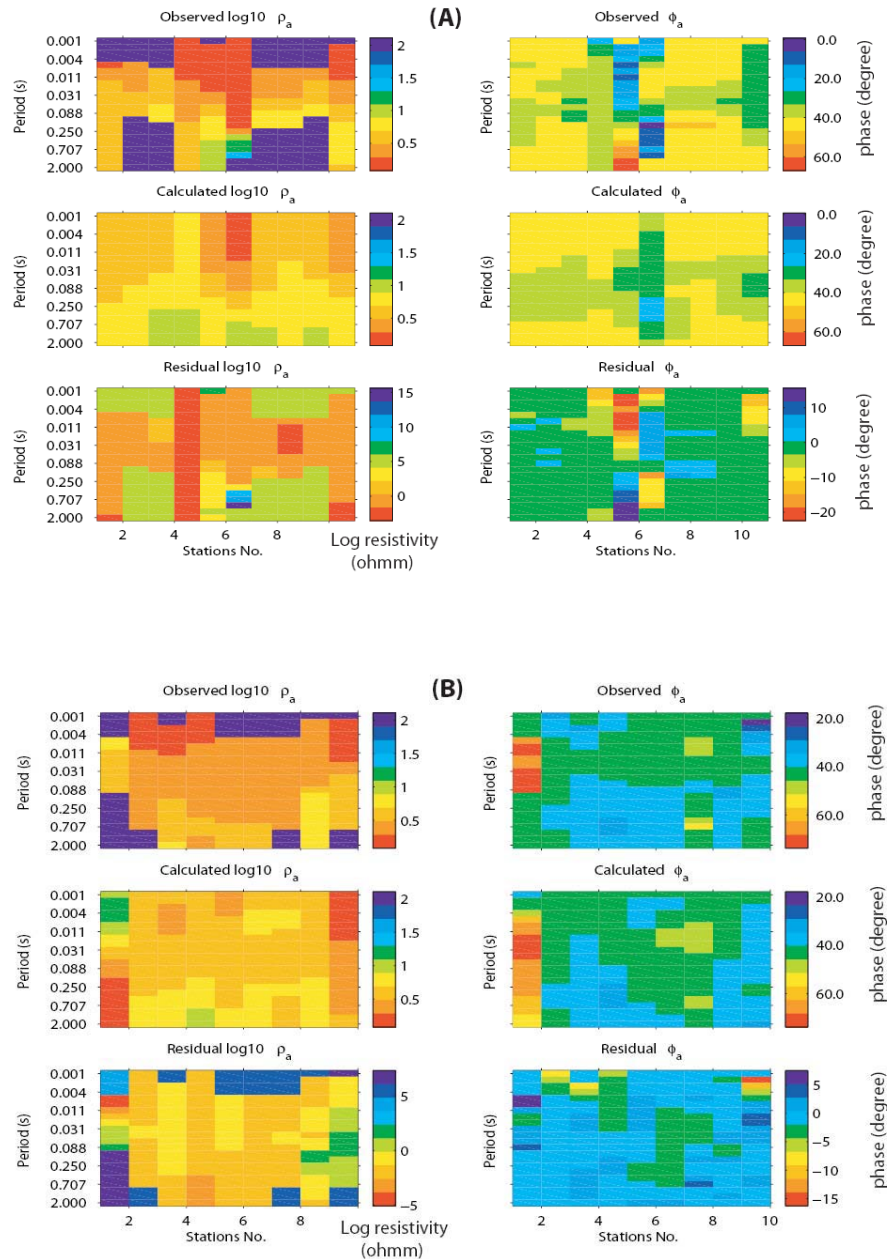


Figure 7. Observed, calculated and residuals of apparent resistivity and phase data along profiles A, B, C and D.

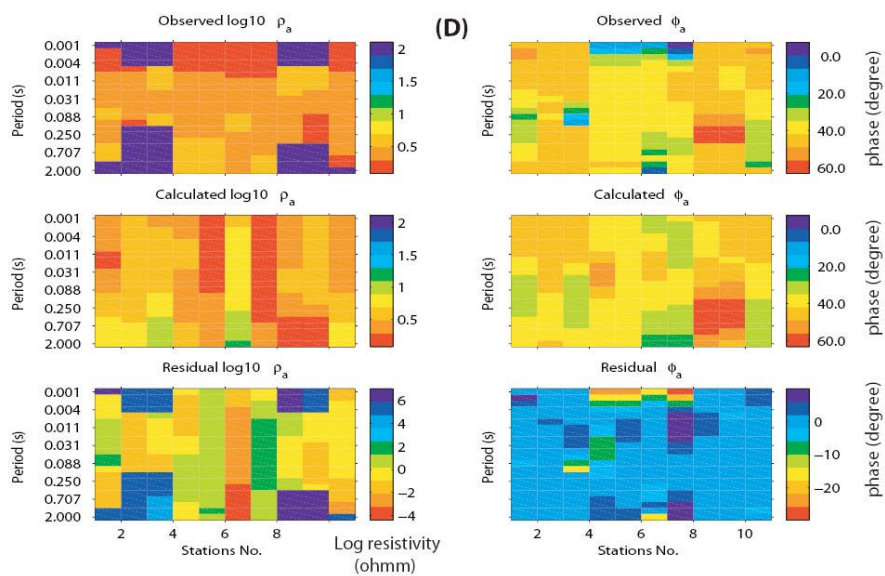
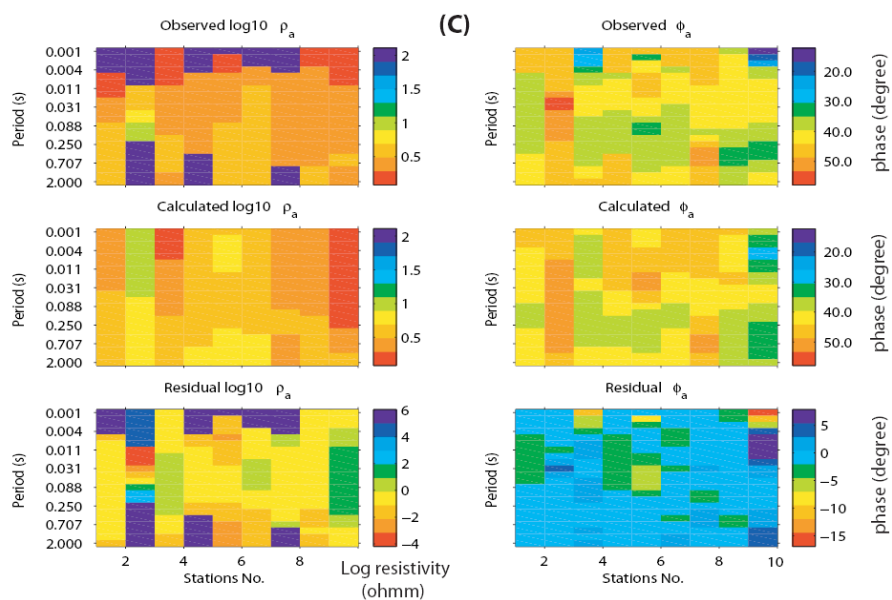


Figure 7. Continued.

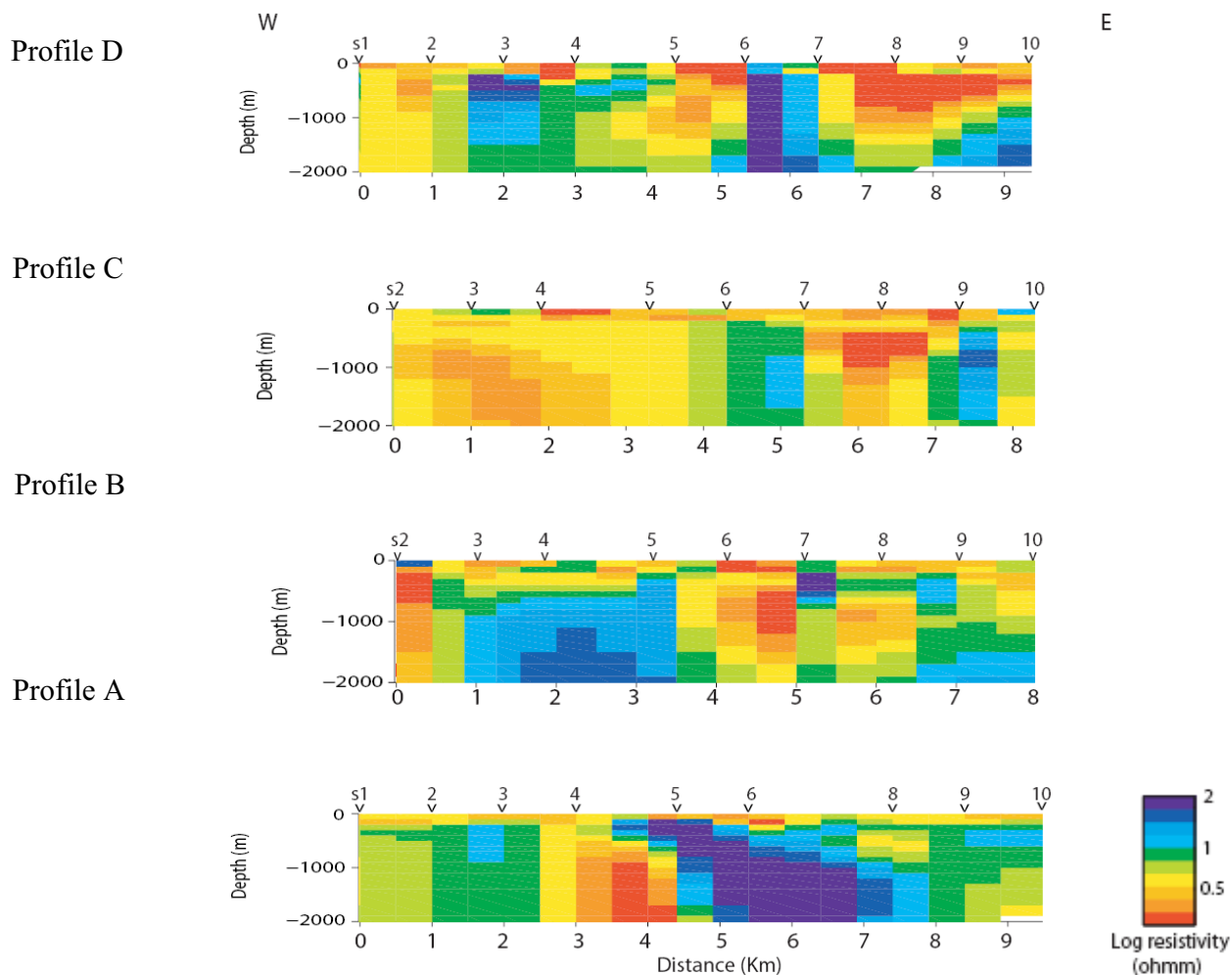


Figure 8. Final 2D resistivity models along profiles A, B, C and D.

7. Discussion and conclusions

Models of the MT data across the profiles, considerably express conductivity of the area that shows the MT advantage compared with other electromagnetic (EM) methods which aren't able to penetrate very deep in such a very conductive area. There is a good agreement between observed data and the model responses along profiles.

Derived resistivity models illustrate the presence of two conductors in the area. These layers are attributed to layers consisting salt water and are presumably reported as a reservoir for Iodine minerals.

As a final result, we suggest drilling an exploration well down to the depth of about 1000 meter at either site of c4 or c9.

8. Acknowledgement

The MTU2000 system from Uppsala University, Sweden, was used for the measurements at some sites which is acknowledged. The last version of the processing software from Maxim Smirnov is also gratefully appreciated. The Research Council of the University of Tehran is acknowledged for financial supports. We are grateful to Davood Moghadas, Alireza Babaei and Majid Zeinali for their assistance in harsh field conditions.

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1D and 2D Inversion of the Magnetotelluric Data for Brine Bearing Structures Investigation

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Abstract

A detailed standard Magnetotelluric (MT) study was conducted to recognize brine bearing layers in depths of less than 1200 m in northeast of Iran close to southeastern shore of Caspian sea. Long and medium period natural-field MT method has proved very useful for subsurface mapping purpose by determining the resistivity of the near surface structure.

MT data were analyzed and modeled using a 1D inversion scheme. Then corresponding data on eight profiles were inverted using 2D inversion schemes. To have the best possible interpretation all possible modes (TE-, TM-, TE+TM- and DET-data) were examined.

Down to 2 km, the resistivity model obtained from the MT data is consistent with the geological information from a 1200 m borehole in the area. Analysis of the MT data-set suggests signatures of salt water reservoirs in the area which are distinguished potentially positive to contain Iodine. Due to the very conductive nature of the sediments regardless of all difficulties in the interpretation stage because of the lack of a considerable resistivity contrast we could recognize the more conductive zones in the less conductive host as layers of saline water.

Key words: conductivity, Iodine, magnetotelluric, 1D and 2D inversion, resistivity.

1. Introduction

Conductive structures are ideal targets for Magnetotelluric method when located in a considerably resistive host. They produce strong variations in underground electrical resistivity. In cases where the electrical resistivity of the target is not substantially different from that of it would

be quite difficult to reach a promising result. Despite this limitation, we could get some useful results in our study.

Dashli-Boroon area is located in Golestan province in northeastern part of Iran right at the border with Turkmenistan. Geologically it is a part of Kopeh-Dagh sedimentary basin. Kopeh-Dagh has been formed by the last orogeny phase of Alpine and the erosion followed. Topography relief is very smooth and basically it is a flat plain consisting loesses occurring naturally between the Elburz mountain range and the desert of Turkmenistan. Quaternary sediments including clay and evaporates and particularly salt are impenetrable.

An MT survey was carried out using GMS05 (Metronix, Germany) and MTU2000 (Uppsala University, Sweden) systems in February 2007. MT data were collected at 60 sites in a network of 2 by 2 km meshes along eight EW profiles (Fig. 1).

For data processing a code from Smirnov (2003) was used. 1D and 2D inversions are conducted to resolve the conductive structures. 1D inversion of the determinant (DET) data using a code from Pedersen (2004) as well as the 2D inversion of TE-, TM-, TE+TM- and DET-mode data using a code from Siripunvaraporn and Egbert (2000) were performed.

A supplementary goal of this work is to evaluate the possibility of using surface MT measurements on the very conductive sediments to monitor the underground salt water bearing layers or bodies. Our concern which is followed in the current paper, only in the frame of one- and two- dimensional (1D and 2D) interpretation, is to emphasis on the characteristics of the extremely conductive structures which are supposed to bear Iodine in economic meanings. Based upon the MT results some sites were proposed for detail exploration by excavating deep exploration boreholes. As results the resistivity sections show a clear picture of the resistivity changes both laterally and with depth.

2. MT data acquisition

In February 2007, an MT survey was carried out at 60 sites in Dashli-Boroon area. All MT-sites are marked on the location map of Fig. 1. The magnetic declination in the area is less than 4° i.e. in the range of error of the array setup so that the geographic north is approximately the same as the magnetic north. The MT sites were projected onto eight EW lines for later 2D modeling. A regular grid mesh of 2 km was organized for the sites due to easy transportation in the plain. The study area has a topographic relief of about maximum ± 10 m which can be disregarded when using 2D inversion scheme compared with the 2 km depth of investigation.

In this study, data within a period range of 0.001-1000 s were analyzed. MT data were processed using a modern robust technique to obtain complete impedance tensor. The vertical magnetic components from various sites are not available due to an unknown source of noise which contaminated the data, therefore, we missed induction arrows which would be used to confirm the dimensionality analysis of the data. Overnight MT recording for minimum 12 hours per site have been arranged. Long time data collection was necessary to recover the proper signals from noise using statistical approaches. The problem with the unexpected noise could be dealt only by removing the noisy stacks manually since remote reference systems were not used for data acquisition.

3. MT data processing

MT data were processed as single sites using a robust routine from Smirnov (2003). The final results of processing for data from most sites were of a reasonable quality. For some sites very bad electric field data were gathered in either x or y direction most probably due to the currents directed from some power-lines in the area. Only one main component of the impedance was used for further analysis for such sites.

4. Dimensionality of the subsurface structures in the region

General morphology of the quaternary deposits shows no tendency to judge about the dimensionality. From the data itself Swift's skew (Swift, 1967) were estimated in order to analyze the dimensionality of the data. Swift's skew, defined as the ratio of the on- and off-diagonal impedance elements, approaches zero when the medium is 1D or 2D. In the case of our data, Swift's skews shown in Fig. 2 are generally less than 0.2, which shows a good indicator for almost 1D or 2D structures.

The regional strike of the survey area was calculated by applying a routine from Smirnov (2003). Relatively stable regional strike estimates defined a principle direction of about 90° from magnetic north at most of the sites at period range 10 to 100 s. This would correspond to both NS and EW strike directions. Since we do not have the tipper data due to the noisy vertical component of the magnetic field we cannot approve the either strike direction, therefore we would take the determinant data in the inversion stage.

In the following section, results of 1D inversion of the determinant data and 2D inversion of the determinant data, TE-, TM-mode data and joint inversion of TE- and TM-mode data are described.

5. Near-surface distortions in EM induction

The static shift of MT apparent resistivity sounding curves is a classic example of the galvanic effect. MT sounding curves are shifted upward when measuring directly over surficial resistive bodies and they are depressed over conductive patches. The physical principles governing electromagnetic (EM) distortions due to near-surface inhomogeneities have been understood for several decades and several methods appeared to correct these distortions. Two of these methods are: use of invariant response parameters like the determinant data (Pedersen and Engels, 2005) and curve shifting (Jiracek, 1990).

We did try to conduct DC-Electrical sounding in the region with no success due to extremely strong EM coupling which arose because of very highly conductive surface layer. That is why we used only the determinant data to proceed with the inversion.

6. Inversion

Depending on the dimensionality of the field structure defined by the geology, tectonics and MT data 1D, 2D and 3D modeling can be applied on the data. Models explain the data if their responses fit the measured data within their errors. Generally, the better the fit between measured and predicted data, the better the model resolution.

1D as well as 2D inversion of the determinant (DET) data were performed using a code from Pedersen (2004) and a code from Siripunvaraporn and Egbert (2000), respectively. The data were calculated as apparent resistivities and phases. To avoid of probable unrealistic small errors on the data for the 2D approximation an error floor of 5% on the apparent resistivity was defined.

MT data were collected in the period range 0.001 – 1000 which by taking into account that the average resistivity of the area is extremely low we consider a maximum depth penetration of about 2 km for our models.

6.1. One dimensional inversion

1D inversion of the determinant data (*DET*) was performed. The determinant provides a useful average of the impedance for all current directions. Furthermore it is unique and independent of the strike direction. Regardless of the true dimensionality 1D inversion of MT data and, in particular, inversion of rotationally invariant data like the determinants, provides an overview of the subsurface conductivity in a feasible sense. Based on the results of 1D inversion, a reasonable starting model and strategy can be constructed for higher order inversions of 2D or 3D. An examples of 1D modeling together with the data and calculated model responses is shown in Fig. 3.

6.2. Two dimensional inversion

Data along eight profiles were corrected were taken for 2D inversion purpose. Apparent resistivity and phase data exhibit fairly different characteristics in TE- and TM-mode and 2D modeling would therefore be expected to provide a more reasonable approximation of the true subsurface structure. Moreover, the results of the data analysis with respect to the dimensionality and the surface and deep geology of the area indicate that a 2D inversion of the MT data is required. 2D inversions of the determinant data (*DET*) profiles are performed in using the code of Siripunvaraporn and Egbert (2000).

Starting with a half-space or a priori model as the initial model, for all models, convergence to a possible minimum RMS misfit was achieved after limited numbers of iterations. Root mean squared (RMS) of data misfit normalized by data error is used to control the data-fit. The resulting models using the determinant data, data-fit and residuals along profiles E2 and E3 are shown in Figs. 4 and 5. The residuals are simply the arithmetic difference between the observed and calculated data. In some cases there is a large misfit which most probably is due to 3D structures, since outliers in the residuals are seen quite frequently.

A shallow conductor is identified as a horizontal layer in all resistivity sections at depth about 300 m. Sites 4 and 6 along the profile E2 and sites 3 and 6 along profile E3 show conductive bodies in larger depths. Data, model responses and residuals are plotted as a function of station in corresponding figures. For an easy comparison resistivity sections along all profiles are shown in Fig. 6.

7. Conclusions

1D and 2D modelling of the MT data at various sites and along profiles revealed remarkable signatures of the conductivity structures down to the depth of 2 km. Our MT investigation resolved the area very conductive which could be expected considering the common geology of the area. By depth we discovered even higher conductivities which somehow prove the idea of salt water bearing layers possibly containing Iodine minerals. Two very conductive layers are resolved in depth from 300 m to 400 m and from 800 m. The bottom of the second conductor could not be estimated accurately due to the problem of limitation on the penetration depth.

8. Acknowledgement

Professor Laust Pedersen from Uppsala University in Sweden is acknowledged for lending us the MTU2000 system which was used for the measurements at some sites. The last version of the processing software from Maxim Smirnov is also gratefully appreciated. The research council of the University of Tehran is acknowledged for financial supports.

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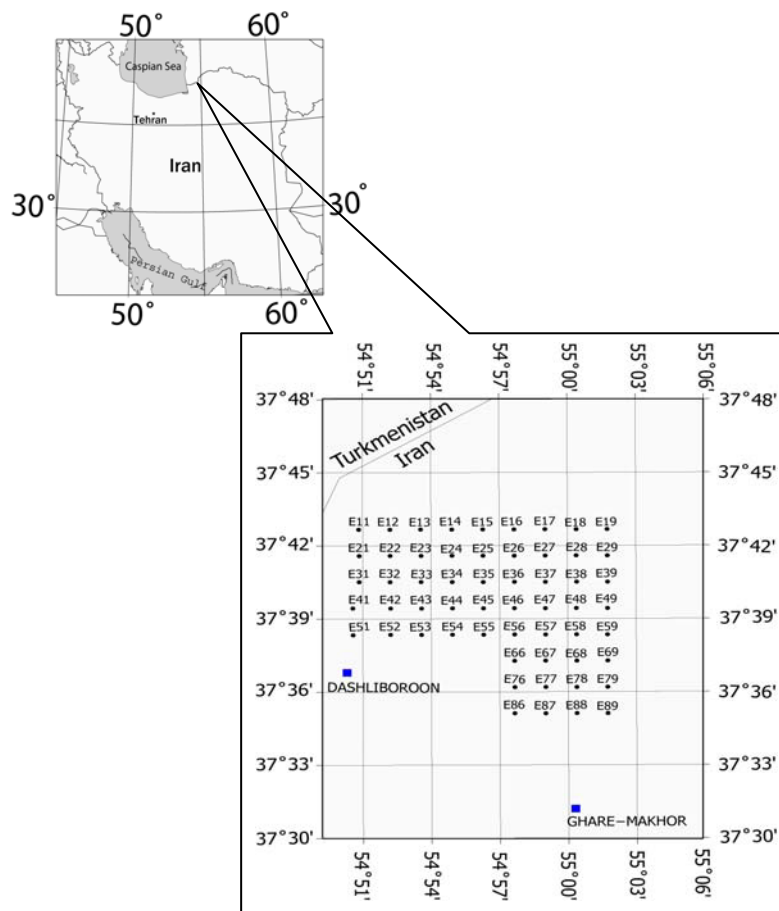
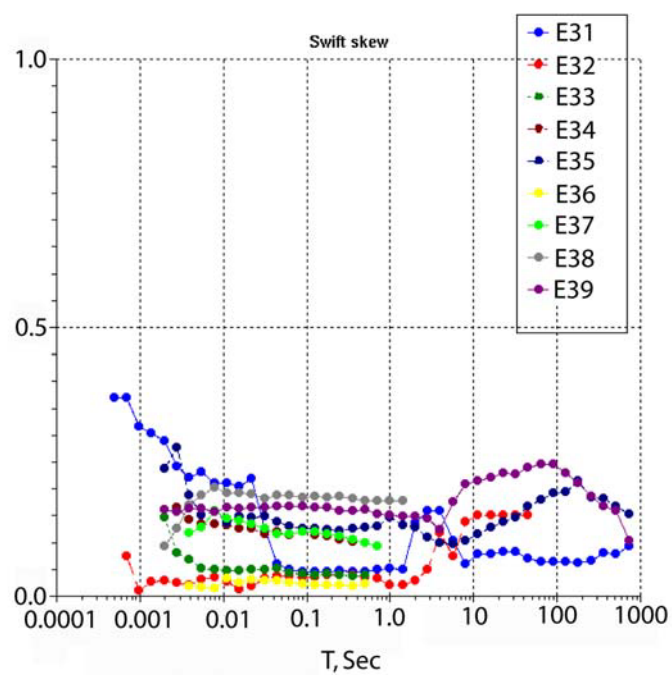
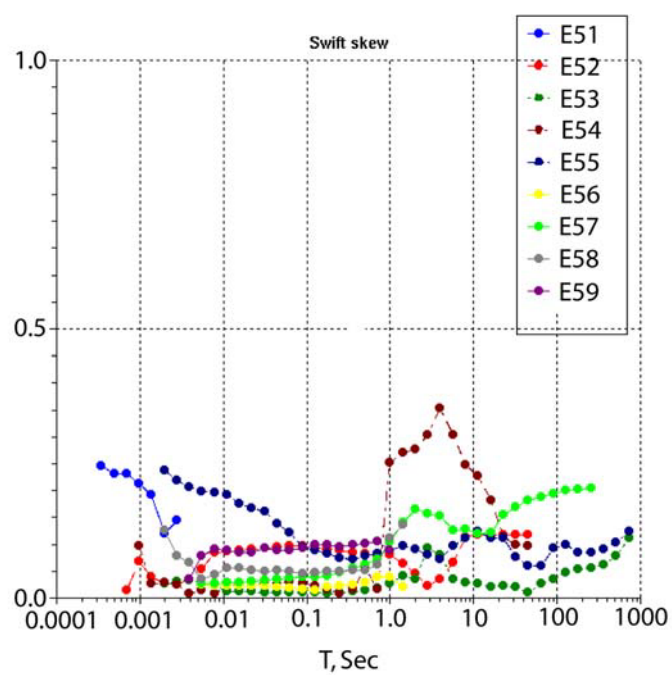


Fig. 1. A sketch of the MT-sites in Dashli-Boroon, north of Iran.



(a)



(b)

Fig. 2. Swift's skew values for all sites along profiles 3 (a) and 5 (b).

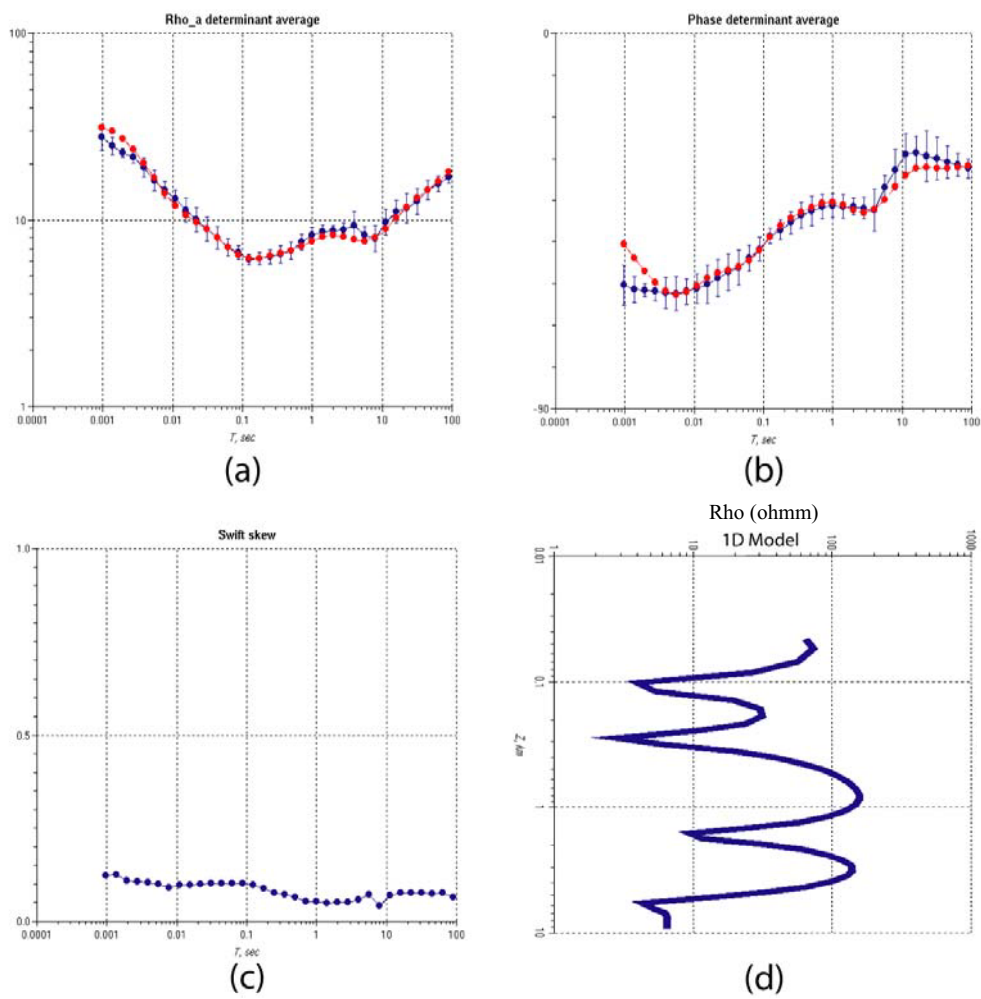


Fig. 3. Data-fit and the model for 1D inversion of the data from site E12, a) Apparent resistivity, b) Phase, c) Swift's skew, d) 1D model.

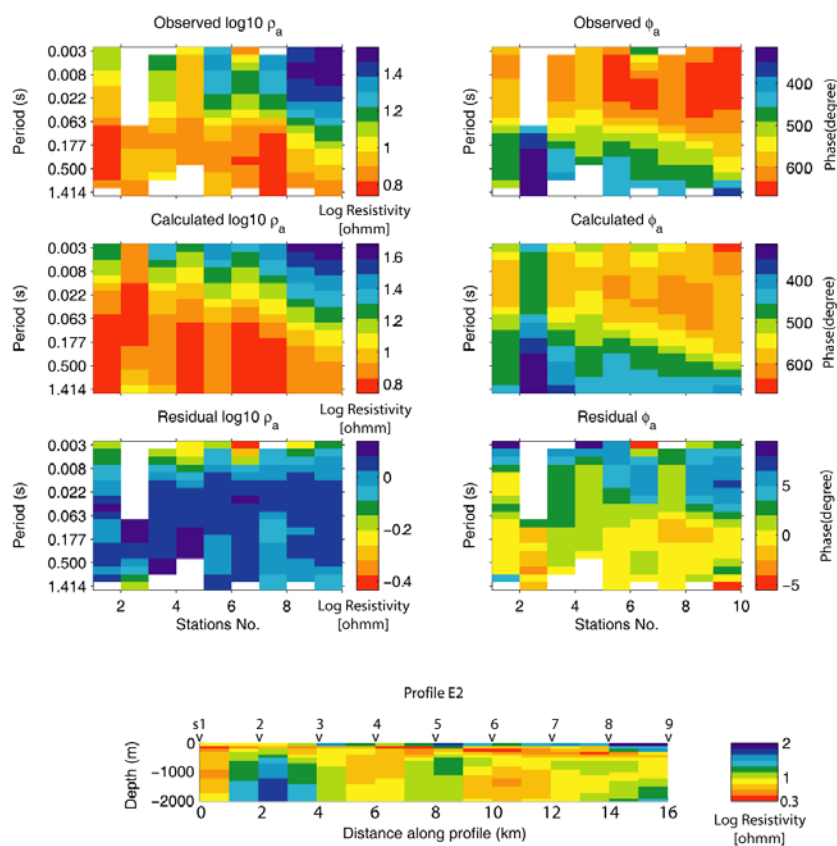


Fig. 4. Data, model responses, residuals and the model of 2D inversion for data along profile E2.

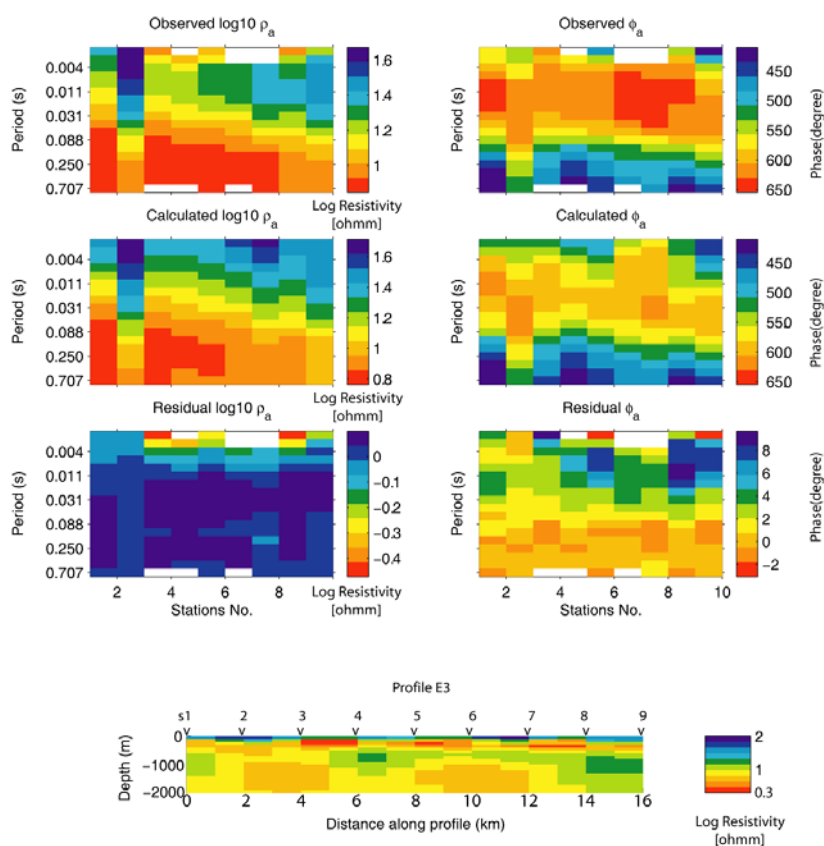


Fig. 5. Data, model responses, residuals and the model of 2D inversion for data along profile E3.

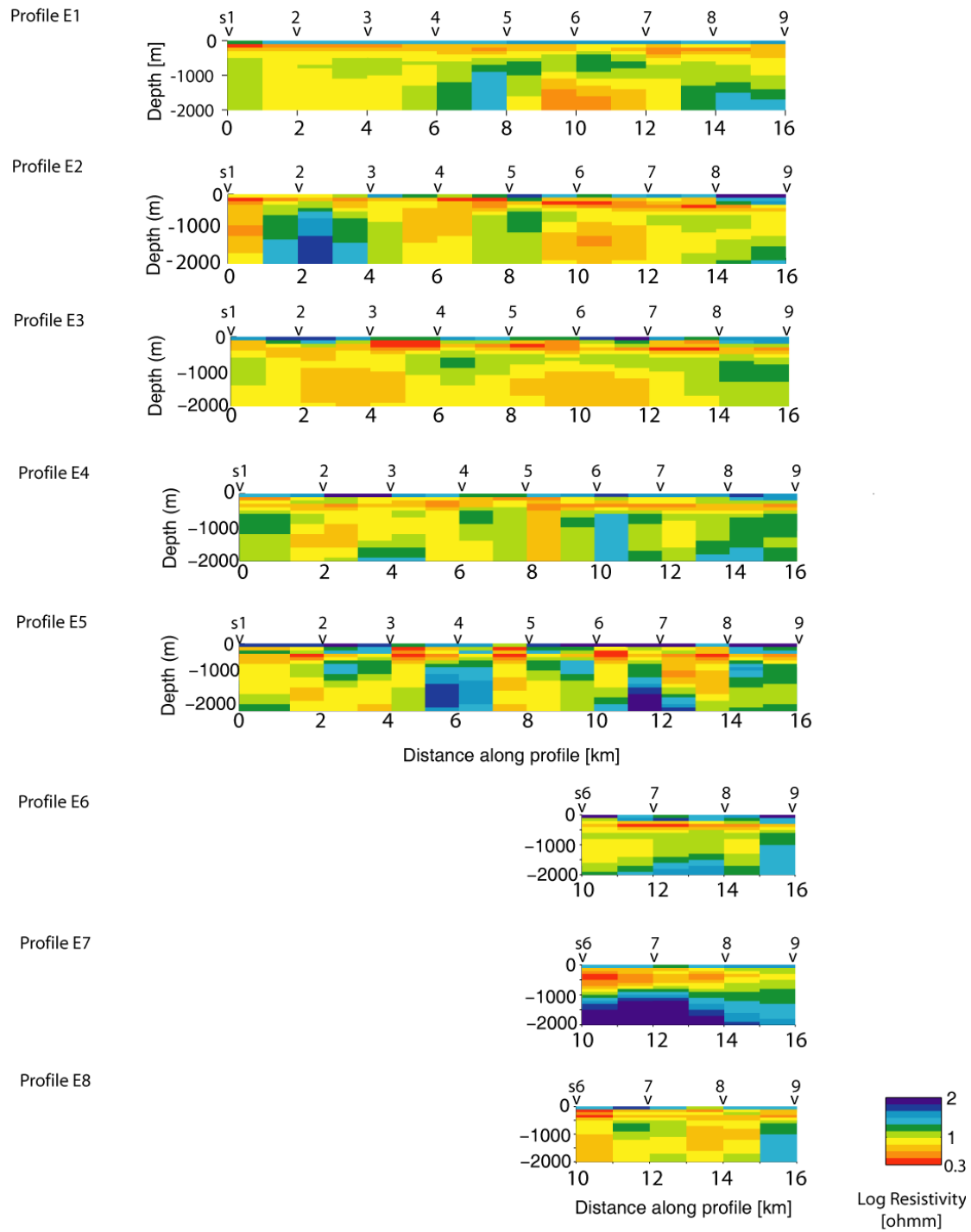


Fig. 6. 2D models along all 8 profiles for an easy comparison.

2D inversion of Magnetotelluric Data for Imaging the Subsurface Geological Structures Derived from Magnetotelluric Soundings

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Abstract

During year 2006 wide frequency range magnetotelluric measurements was carried out at the western part of Arak city in Iran to understand the crustal electrical conductivity of the region by putting emphasis on relocating the fault zones. The electric and magnetic field components were acquired from along a profile across the geological trend at 15 stations. A robust single site processing followed by the inversion and one dimensional as well as two dimensional modeling was performed. The inversion results revealed electrical conductivity structures in good correlation with geological features. As significant results, true locations of two major faults, Talkhab and Tabarteh Faults, were recognized in Arak area.

Keywords: Magnetotellurics, geoelectrical resistivity, Talkhab Fault, Arak, Iran.

1. Introduction

Because of the reflection and refraction of EM waves at vertical interfaces separating media of different electrical parameters, geoelectromagnetic methods have been developed and employed to recognize the fault zones in many regions. Due to its lateral resolution and also greater depth penetration MT method is one of the effective electromagnetic techniques to electrically image the subsurface structures.

Magnetotelluric (MT) transfer functions are calculated from measurements of horizontal electric and magnetic fields at the surface of the earth and could image the subsurface electrical resistivity in Arak area located in west central Iran (Fig. 1).

From the tectonostructure point of view the area of study is in the west of two NW-SE compressional strike-slip fault systems which are located in the border of the Central-Iran and Sanandaj-Sirjan zones. The faults are hidden under Quaternary alluviums. The seismicity of the area is controlled by these two parallel faults, especially Talkhab Fault which is the source of the slight seismicity of the region. Small magnitudes earthquakes on these long faults suggest an ambiguity in that; these faults are capable of producing great earthquakes. There are great numbers of earthquakes occurred in this area with ambiguous seismicity characteristics. The recurrence intervals of these faults are so long that can not be determined by instrumental seismicity. Therefore, identifying the characteristics of these faults must be dealt with first degree priority.

In this paper we present a case study of magnetotelluric survey conducted in 15 stations with site spacing varying from 5 to 10 kilometers to image the subsurface structures related to fault zones in Arak area.

2. Geological setting

The area of study is a part of Arak watershed located in the two Central-Iran and Sanandaj-Sirjan Zones. A simplified geological map of Arak area is shown in Fig. 1. The presence of folded mountains and pressure ridges are the main characteristics of this region. Two parallel faults named Talkhab and Tabarteh Faults pass through the region and divide it in to three blocks. These blocks are “Ashtian-Naragh” (ANB), “Haftad-Gholeh” (HGB) and “Sanandaj-Sirjan” Blocks (SSB). The Talkhab Fault separates ANB from HGB while Tabarteh Fault separates HGB from SSB. The amount of water discharge in HGB, SSB and ANB are different and decrease respectively. Talkhab and Tabarteh Faults control the seismicity of the region. Talkhab spring, travertine and the emanation of gas from some wells are the reasons indicating the activity of Talkhab Fault in Quaternary. Statistical analysis regarding the hypocenters of earthquakes shows that most of the events are located near Talkhab Fault. The oldest block in this region is SSB which involves crystallized limestones, slates from the Jurassic to cretaceous period that underwent faulting and

metamorphism without any volcanic activity. The HGB contains shale, Jurassic sandstones and cretaceous limestone with no metamorphism but severely folded and has a sequence of anticline and syncline without any volcanism.

3. Methodology

Magnetotelluric method is a passive electromagnetic technique that uses the natural, time varying electric and magnetic fields components measured at right angles at the surface of the earth to make inferences about the earth's electrical structure which, in turn, can be related to the geology tectonics and subsurface conditions. The depth of investigation of MT method is much higher than that of other electromagnetic methods. Measurements of the horizontal components of the natural electromagnetic field are used to construct the full complex impedance tensor, Z , as a function of frequency,

$$Z = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \quad (1)$$

The principal values of the tensor Z are two complex quantities which expressed in terms of rotational invariants, and, hence are independent of the direction of the axes (Berdichevsky and Dimitriev, 2002). For a 1D structure in which the resistivity of the earth varies only with depth, the diagonal elements of above matrix are equal to zero, whilst the off-diagonal components are equal in magnitude and have apposite signs. In a 2D case, wherein X or Y are aligned along the geoelectric strike, the diagonal element of Z would be zero again. We can define the geometric mean of this principal values as effective impedance which is a simplest approximation of the Tikhonov-Cagniard impedance,

$$Z_{DET} = Z_{eff} = \sqrt{Z_{xx}Z_{yy} - Z_{xy}Z_{yx}} \quad (2)$$

The advantage of using determinant data is that it provides a useful average of the impedance for all current directions. Furthermore, it is unique and independent of the strike direction. Using the effective impedance, determinant apparent resistivities and phases are computed and used for the inversion.

4. Data processing and analysis of the MT data

Time series measurements collected and cross power spectra computed to estimate the impedance tensor as a function of frequency. MT data were processed using a processing code from Smirnov (2003) aiming at a robust single site estimates of electromagnetic transfer function. As the area of measurements is populated and close to cultural noise sources, the recorded data were not with good quality which justify the low coherency between electric and magnetic channels. The parameter Swift's (1967) skew which is called a measure of asymmetry of a medium (Berdichevsky and Dimitriev, 2002) showed us dominant 1D and/or 2D structures in the area (Fig. 2). Swift's Skew values for the majority of sites are generally below 0.2. Some signatures of 3D effects are disturbing our modeling which could not be avoided due to lack of a 3D code at our disposal.

5. Inversion

5.1. One-dimensional inversion

One-dimensional (1D) inversion of the data carried out showing a good fit between the measured data and the responses of the models. Generally speaking the better the fit between measured and predicted data result in better resolution for model. We performed 1D inversion of the determinant data using a code from Pedersen (2004) for all sites. The resistivity model at site 2, located at the northeastern end of the profile, together with data and model responses is shown in Fig. 3. A transition from a less resistive formation (about 150 Ωm) at surface to a moderately resistive structure (about 500 Ωm) at 1500 m depth is resolved. By 10 km depth a consequence of conductive-resistive-conductive structure is provided by the 1D model.

5.2. Two-dimensional inversion

Regardless of the true dimensionality it is practicable to earn an overview of the subsurface resistivity with 1D inversion of MT data and actually, inversion of rotationally invariant data like the determinant data. Based on the results of the 1D inversion, a reasonable starting model and strategy can be constructed for two-dimensional (2D) inversion. Since the quality of the determinant data was acceptable,

we performed 2D inversions of determinant data using a code from Siripunvaraporn and Egbert (2000).

Due to relief topography compared with the depth of investigation, a flat topography is assumed. The final model, data, model responses and the residuals are shown in Fig. 4. Along with the profile at sites 1, 2, 3 and 4 a high resistive structure is resolved which corresponds to the red conglomerate (shown in Fig. 1 in red). A low resistive zone at the location of site 5 is a good signature of Quaternary clay formation. The data from site 6 is not involved in the inversion due to bad quality. Site 7 which is located on Talkhab fault shows a resistive thin layer at the surface which converts to a less resistive body towards the deeper parts of the ground. Site 8 is located on a resistive body which most probably is missed to be located on the geological map of Fig. 1. Site 9 at the border of the clay and limestone formations show a conductor from surface to the depth of a few kilometers. We measure this location as a probable hidden fault. Patches of limestone with clay at sites 10 and 11 show an intermediate resistivity structure. Site 12 located on Talkhab fault clearly shows a conductive zone spreading down to 3-4 km depth. At sites 13 to 15 again more resistive formations of garvel fans and travertine appeared.

6. Conclusions

The magnetotelluric technique is an influential method for recognizing fault zones. The case study presented here proved the efficiency of this method and the geological structures and formations are well recognized by interpretation of the MT data.

Highly resistive formations like conglomerate and limestone, a clay formation and two major fault zones were resolved along the profile. The 2D model significantly illustrate two highly conductive zones hidden under the Quaternary alluviums.

As significant results, in collaboration with geological information about the presence of Talkhab and Tabarteh faults the conductivity features can be attributed to the faults. Besides, a probable hidden fault is also recognizable.

7. Acknowledgment

We wish to place on record our thanks to professor Laust Pedersen from Uppsala University in Sweden for lending us the MTU2000 systems. The last version of the

processing software from Maxim Smirnov is also appreciated. We particularly acknowledge Arash Mansoori and Alireza Babaii for great helps both in field and processing works. We also would like to thank Dr. Mirzaie from Arak University for partial supports and providing facilities in the field. The Research Council of the University of Tehran is acknowledge for financial supports as well.

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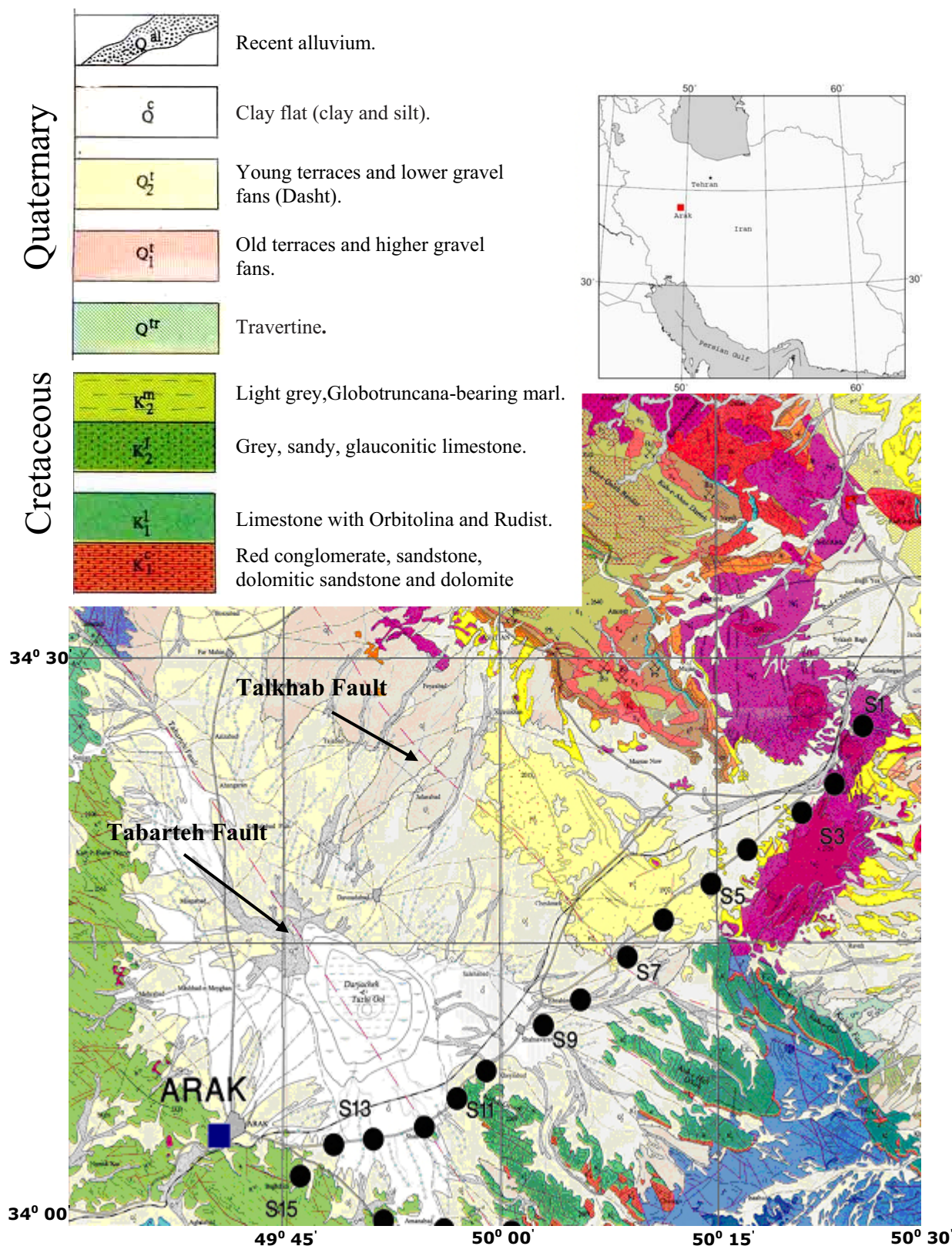
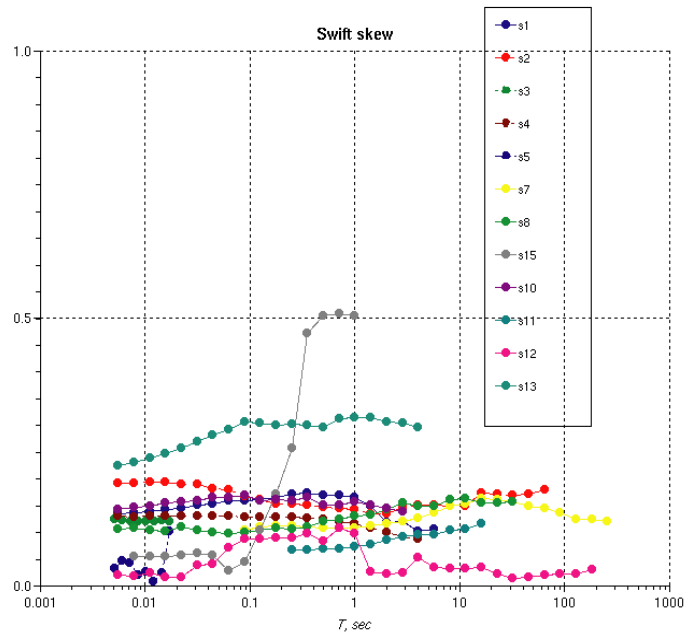
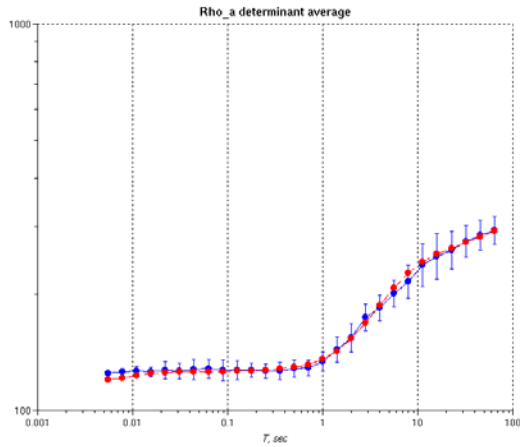


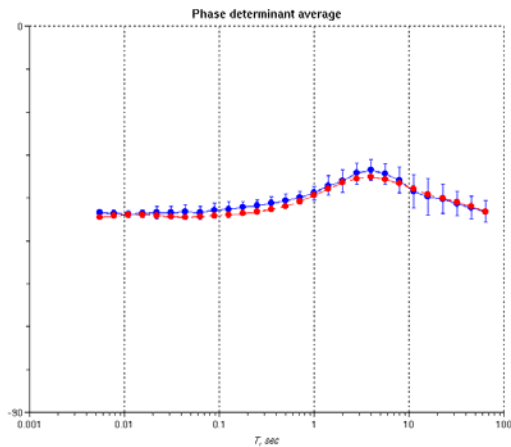
Fig.1. A simplified geological map of Arak and adjacent areas illustrating the major geologic and tectonic features. Location of the MT sites and Talkhab and Tabarteh faults are also shown on the map.



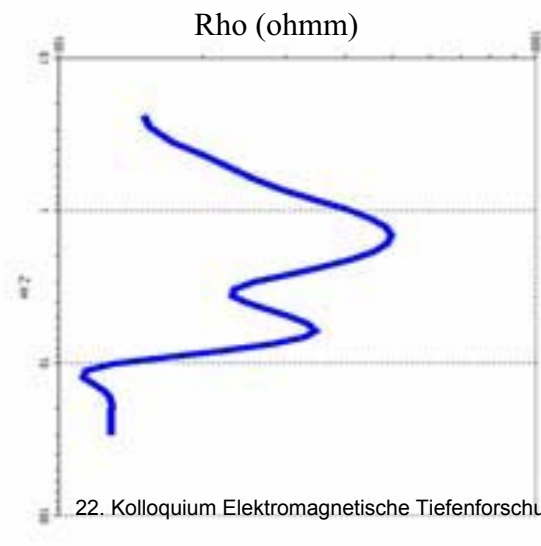
.Fig. 2. Swift's skew values for some sites along MT-profile



(a)



(b)



(c)

Fig.3. Data-fit and the 1D model of the data from site S2. (a) Apparent resistivity, (b) Phase and (c) 1D model.

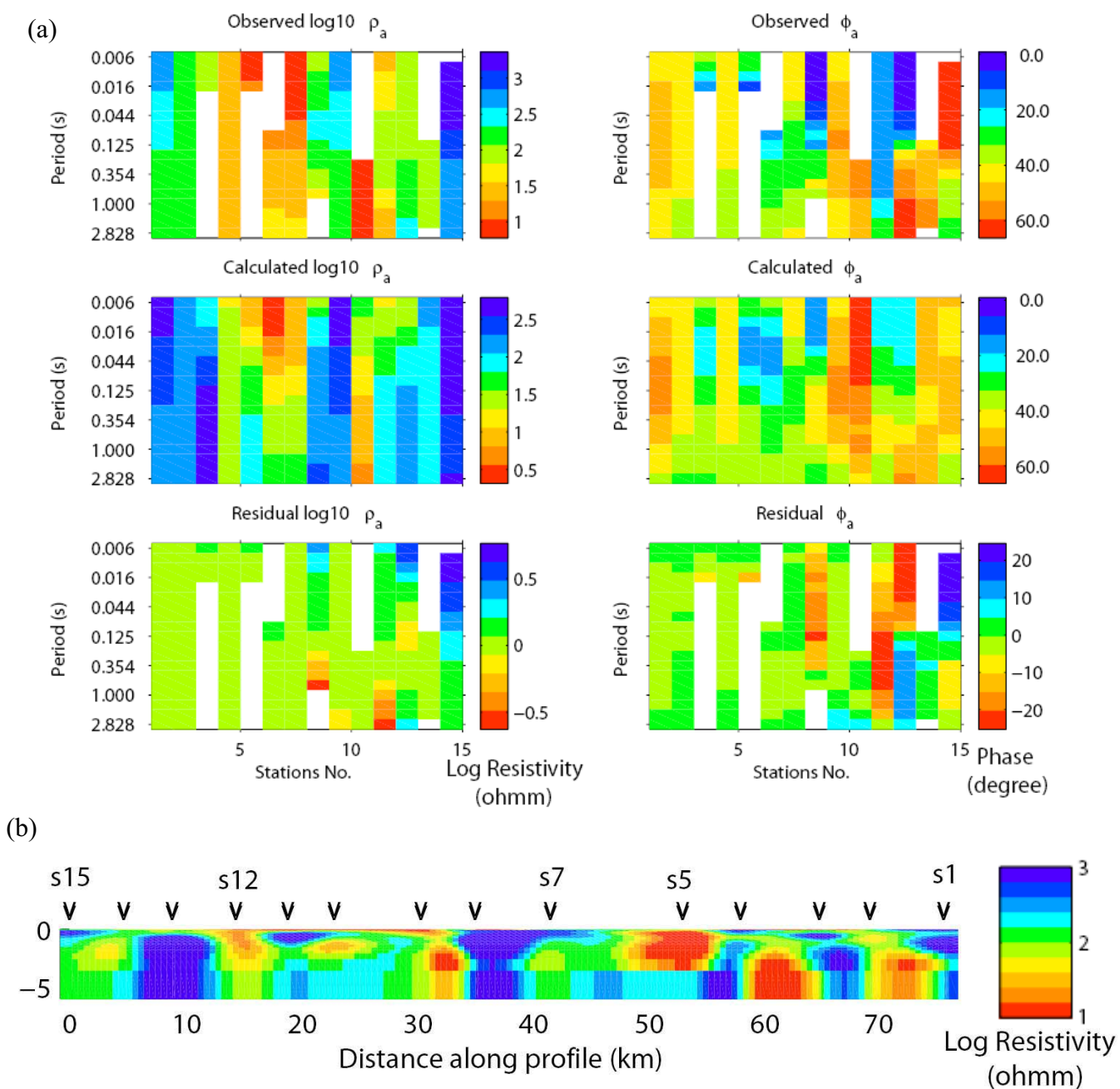


Fig 4. (a) Data, model responses and the residuals of 2D inversion of the determinant data along MT-profile. (b) Resistivity Depth Section of 2D modeling.

Inversion for Transient ElectroMagnetic (TEM) under inclusion of chebyshev series expansions and lateral constraints

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Abstract

In general TEM data are evaluated by inversion calculations. The found conductivity distributions are commonly ambiguous, especially concerning the noise of the measured data. The use of chebyshev polynomials for the description of the layer boundary and lateral constraints to connect conductivities within one layer can help solving these ambiguities through the assumption of an layered half space. The investigated methods use an 1D forward calculation, but the inversion algorithm is quasi 2D.

Introduction

Sedimentary soils are often provide an layered subsurface. These subsurfaces are mapped in profile orientated data and naturally invites a 2D interpretation. Unfortunately 2D forward calculations use huge amounts of computer resources and therefore inversion with this method is time intensive. In many cases where an layered subsurface is suspected, one can force the inversion method into this scheme. A possibility to achieve such results is a Lateral Constrained Inversion (LCI). Often a 1D forward solution with lateral constraints is sufficient to investigate an quasi-layered sedimentary environment (*Auken*

and *Christiansen* (2004)). An alternative, that also requires an layered subsurface, is an inversion with series expansion, where the layer boundaries can be described by chebyshev polynomials as basis functions (*Kis* (2002)). The model thicknesses in the model vector substitute with the coefficients of the used polynomials. Chebyshev polynomials have appreciable numerical advantages and are often used in geophysical applications, but the choice of the basis functions mainly depends on the geological model. This work compares the results for these two methods on examples for 1D and 2D synthetic data. The appropriate order of the polynomials and the importance of the profile length will be discussed.

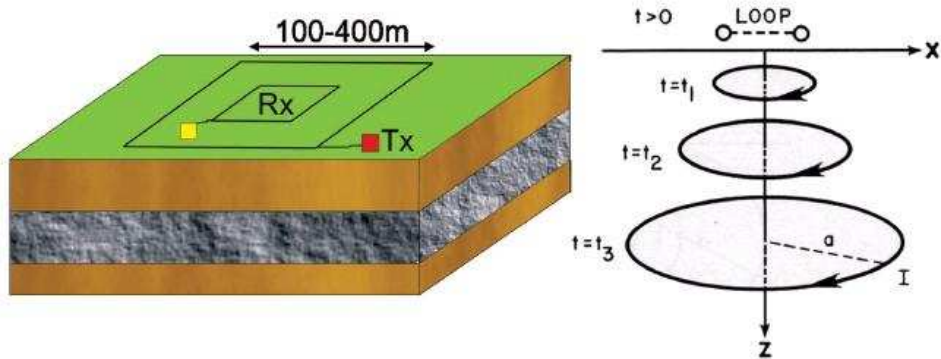


Figure 1: Field setup for Central Loop. *right*: Propagation of ring currents (smoking rings).

TEM

Transient ElectroMagnetic (TEM) is a geophysical method that is capable to derive the conductivity distribution of a subsurface with high resolution. The field setup consists of a transmitter, driven by an electric current that is suddenly switched off, and a receiver that measures the induced voltage of the subsurface currents, caused by the change of the magnetic field (*Helwig* (2003)). Theoretically the switch off process can be described by a Dirac Deltafunction, which contains, applying a Fourier Transformation, all frequencies. Unfortunately, practice relativates this fact. The received signal is called transient. Fig. 1 shows the central loop field setup, with a transmitter coil and a receiver coil in the center of the transmitter. Here the current system diffuses down and sideways, through smoking rings (*Nabighian and Macnae* (1991)). The setup is called Short Offset TEM (SHOTEM) and has an sheer inductive source. All results in this work refer to this setup. Another setup is

the Long Offset TEM (LOTEM), it is e.g. described by *Strack* (1992)

LCI

Geophysical inversion minimizes the misfit between measured and calculated data. The derived conductivity models are in general ambiguous within the data errors. A way to invert TEM data is the Marquardt-Levenberg method.

$$\delta \vec{m} = (\bar{J} \bar{J}^T + \lambda \bar{I})^{-1} \bar{J}^T \delta \vec{d}$$

$$\begin{aligned} \delta \vec{m} &= \vec{m} - \vec{m}_0 & \vec{m} & \text{model vector} \\ & & \vec{m}_0 & \text{initial model vector} \\ \delta \vec{d} &= \vec{d}_{cal} - \vec{d}_{obs} & \vec{d}_{cal} & \text{calculated data vector} \\ & & \vec{d}_{obs} & \text{observed data vector} \end{aligned}$$

Where the model vectors $\vec{m}, \vec{m}_0$ contain the resistivities ρ_i and the layer thicknesses D_i , and the data vectors $\vec{d}_{obs}, \vec{d}_{cal}$ contain the voltages. The damping factor λ stabilizes the inversion and determines the step length of one iteration. J is the Jaco-

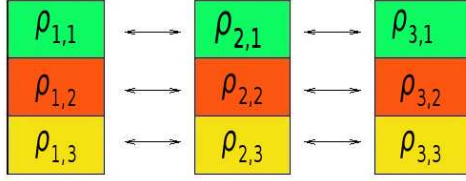


Figure 2: Three sites where the adjacent conductivities are connected

bian matrix and I the Identity matrix. In this work the LCI, which is developed by *Auken and Christiansen (2004)*, applies the Marquardt-Levenberg method to invert a profile, but uses a 1D forward calculation on each site. This is often called 1.5D Inversion. The main idea in LCI is the introduction of the roughening matrix R , that is capable to connect model parameters in the inversion scheme:

$$\bar{R}_{n \times m} = \begin{pmatrix} 1 & 0 & \dots & -1 & \dots & 0 \\ 0 & \dots & 1 & \dots & -1 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 1 & \dots & -1 \end{pmatrix}$$

In this manner all parameters can be connected, e.g. vertical parameters as well. With the background of an sedimentary quasi-layered subsurface, it is reasonable to apply lateral constraints for adjacent conductivities within one layer, shown in Fig. 2. The inversion problem can than be written:

$$\begin{pmatrix} \bar{J} \\ \bar{R} \\ \bar{I} \end{pmatrix} \delta \vec{m} = \begin{pmatrix} \delta \vec{d}_{obs} \\ \delta \vec{r}_r \\ \delta \vec{m}_{pri} \end{pmatrix} + \begin{pmatrix} \vec{\epsilon}_{obs} \\ \vec{\epsilon}_r \\ \vec{\epsilon}_{pri} \end{pmatrix}$$

where

$$\delta r_p = -\bar{R} \vec{m}_0$$

The first line describes the the inversion as normal with the observation errors ϵ_{obs} . The second line brings the lateral constraints into play, where ϵ_r determines the strenght of the constraint. The third line can be used for a priori information, with $\delta \vec{m}_{pri} = \vec{m}_{pri} - \vec{m}_0$. Where $\vec{m}_{pri}$ contains informations from e.g boreholes and $\vec{\epsilon}_{pri}$ sets the strenght for the a priori informations.

Inversion with series expansion

In the inversion with series expansion for chebyshev basis functions the Levenberg-Marquardt method is applied as well. It also requires a quasi-layered subsurface and inverts profiles in a quasi 2D manner, with 1D forward calculations on each site. But regularisation for the inversion is different. Instead of connecting adjacent conductivities, layer boundaries are described by polynomials (*Kis (2002)*), shown in Fig. 3, therefore the inversion scheme is merely capable to find solutions for an layered subsurface.

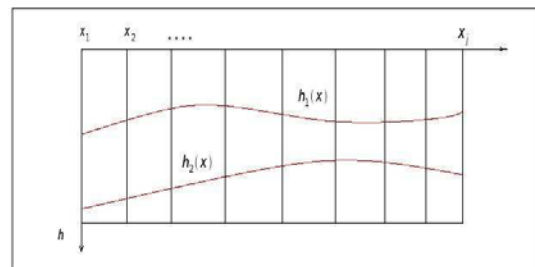


Figure 3: Boundaries are described by polynomials $h_j(x)$. The x_i indicates the measurement sites

The chebyshev polynomials are defined by (Bronstein et al. (1993)):

$$T_n(x) = \cos(n \cdot \arccos(x)) \quad \text{with } x \in [-1, 1]$$

or can be derived by the recursion formula.

$$\begin{aligned} T_n(x) &= 2xT_{n-1} - T_{n-2} \\ \text{for } T_0(x) &= 1 \\ \text{and } T_1(x) &= x \end{aligned}$$

The new model vector $\vec{m}$ contains the coefficients C_i for the chebyshev polynomials in the series expansion on each boundary:

$$\vec{m} = (\rho_{11}, \dots, \rho_{i1}, \dots, \rho_{ij}, \dots, \rho_{mn}, C_{11}, \dots, C_{1K}, \dots, C_{(m-1)0}, \dots, C_{(m-1)K})$$

where:

- $i = 1, \dots, m$ is the number of the layers and therefore $m - 1$ the number of the boundaries.
- $j = 1, \dots, n$ is the number of the sites in the profile.
- $k = 0, \dots, K$ is the order of the chebyshev polynomials and therefore $K + 1$ is the number of coefficients for each polynomial.

1D Syntehetic Data

To verify the LCI and the chebyshev inversions 1D TEM data were generated using the program Emuplus (Scholl (2005)). The distances between the sites are assumed to be equidistant. The data errors are 5% gaussian distributed and increase strongly for

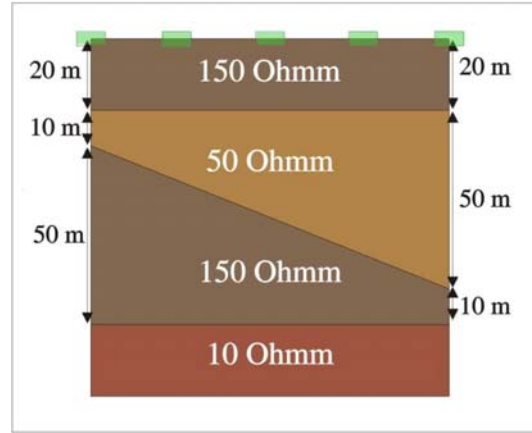


Figure 4: Four layers with five sites (green) and a dipping layer.

late times, as in field measurements. The misfit is calculated by χ , where:

$$\chi = \sqrt{\frac{1}{N} \sum_{i=1}^N \frac{(d_i^{obs} - d_i^{cal})^2}{\epsilon_i^{obs}}}$$

Fig. 5 shows the result for an inversion without constraints. It shows, that the resistivities ρ_i , especially in layer 3 are hardly represented. The depth on sites 3,4,5 are too low. But within the data errors the model is fitted well. Fig. 6 shows the result after applying lateral constraints. Although the resistivities ρ_i are still too low, the course of the dipping layer is represented much better. The initial model were $\rho_0 = (180, 80, 100, 20)\Omega m$ and $D_0 = (30, 30, 30)m$ in both cases and the inversion was stopped, when $\chi \leq 1$. Like in this example the LCI was always capable to improve the results of 1D data compared to an inversion without constraints.

To apply the chebyshev inversions it is important find out which order of the polynomials is appropriate for inversion prob-

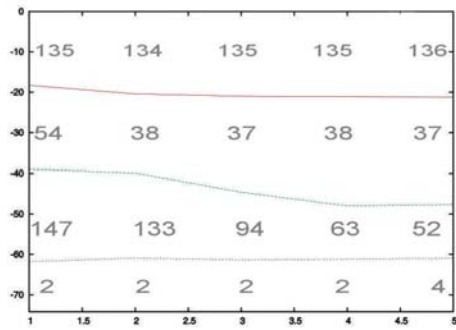


Figure 5: LCI without constraints, resistivity values ρ_i in Ωm . $\chi = 1.00$; 5 iterations.

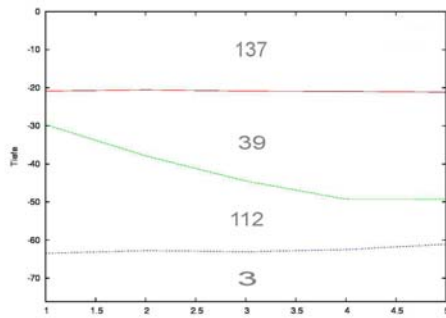


Figure 6: LCI with constraints of adjacent resistivities, resistivity values ρ_i in Ωm . $\chi = 0.93$; 6 iterations.

lems like this. In Fig. 7 and 8 a fit is shown for polynomials of the order 5 and 12. The synthetic data has no errors. Comparing the results, the course of the nonlinear boundary is already fitted well by the polynomial of order 5. In Fig. 8 appears an extrem misfit at the borders. This happens, because the higher the order of the polynomials, the more unknown the inversion has to determine, compared to the length of the measurement lines. It turns out, that orders from 4 to 7 are acceptable, where higher orders do not improve the results. Inverting the model from Fig. 4 with a series expansion

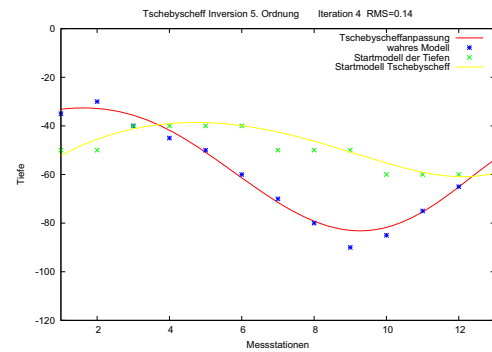


Figure 7: Chebyshev fit with polynomials of order 5 (red) for a 2 layer boundary (blue), initial depth (green) and initial polynomial (yellow).

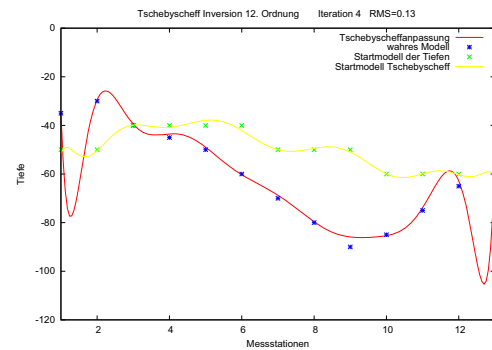


Figure 8: Chebyshev fit with polynomials of order 12 (red) for a 2 layer boundary (blue), initial depth (green) and initial polynomial (yellow).

sion of order 5 does not succeed, because the misfit creeps without appreciable progress over the iterations and does not converge. A reason is, that the chebyshev inversion has more parameters to determine (18 coefficients and 20 resistivities) than the LCI (15 thicknesses and 20 resistivities) for the same data information. Unfortunately the parameter dependencies for the chebyshev inversion are also more difficult to solve. An approach for this problem is to enhance the

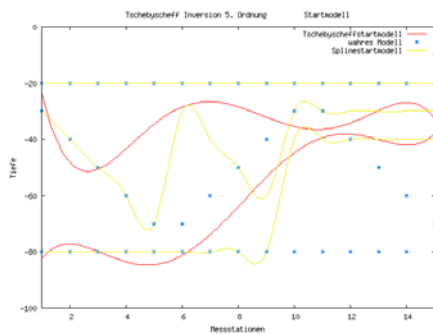


Figure 9: Chebyshev fit with polinomials of order 5 (red) reference model (blue), initial depth (yellow) described by a cubic spline.

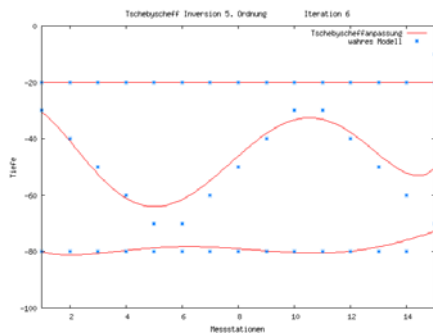


Figure 10: Chebyshev fit with polinomials of order 5 (red) reference model (blue). 6 iterations.

line of measurements or decrease the order of the polynomials. Fig. 9 and 10 show a result for a four layer model with fifteen sites, and no data error. In the initial model the resistivities are kept the same as in the reference model, only the depth are varied (Fig. 9). This shows, that the chebyshev inversion needs much more data information to achieve satisfying results.

2D synthetic data

The data for 2D models was generated with the program `sldmem3t`, that was written by

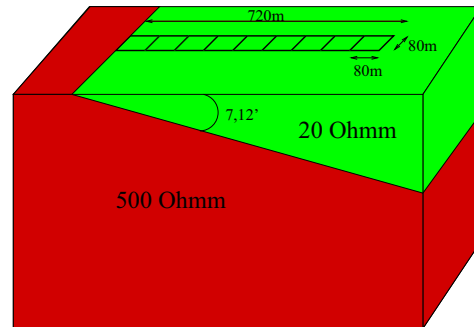


Figure 11: 2D Model with dipping layer and 9 central loop sites.

Druskin and Knizhnerman (1988) and base on finite differences. To test the ability of the inversions to compensate the slightly different transients for a 2D case with the regularisations based on a 1D forward calculation the data for a two layer subsurface with dipping layer of $7,2^\circ$ was calculated (Fig. 11). The data was inverted without constraints (Fig. 13) and with constraints on adjacent resistivities (Fig. 14). The initial model is shown in Fig. 12. The inversion was stoped, when χ could not improve anymore. The data misfit is 1% gaussian distributed, increasing for late times. The inversion without constrains is not capable to resolve the reference model on site 6 and 7 (iteration 6; $\chi = 20.2$), more iterations does not improve this result and the resistivity and depth values become even worse, whereas the inversion with lateral constraints retrieve the values of the reference model well (iteration 5; $\chi = 1.87$). Comparing this two inversions the lateral constraints have an clear advantage here, because the outliers on site 6 and 7 can be balanced by the constraints. In Fig. 15 the results of the chebyshev inversion can be seen. They are almost as good, as the LCI

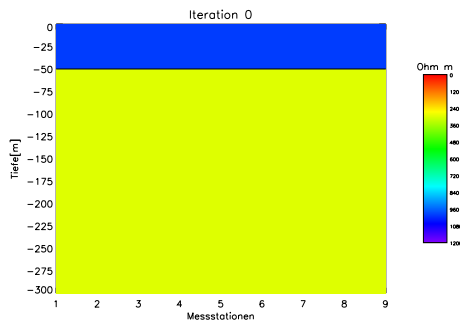


Figure 12: Initial model

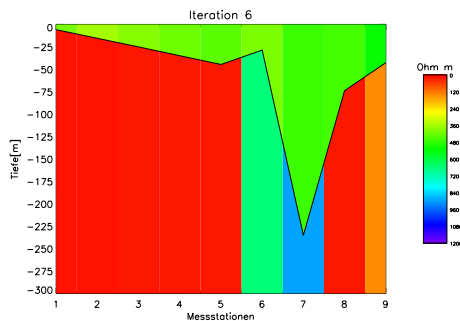


Figure 13: Inversion without constraints, $\chi = 20.2$

results (iteration 9; $\chi = 1.72$), but the resistivities on site 1 are slightly too low and the chebyshev inversion needed 9 iteration for this result.

Conclusion

Assuming a quasi-layered subsurface, both methods are capable of achieving results from 1D and 2D synthetic data. LCI has proved as a reliable tool, that has short calculation times. The results are in general preferable to an inversion of a profile without constraints.

The Chebyshev inversion needs large profiles for a convergence on the misfit. The numerical effort is more expensive and

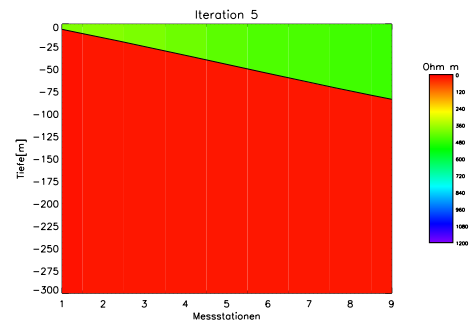


Figure 14: Inversion with lateral constraints, $\chi = 1.87$

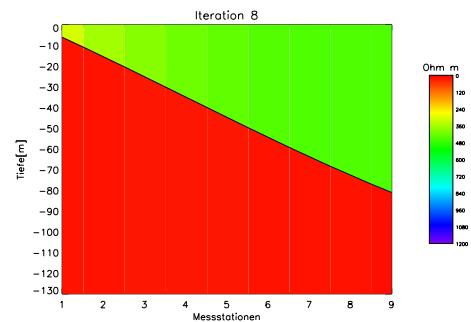


Figure 15: Chebyshev inversion, $\chi = 1.72$

therefore the calculation time is larger than for the LCI, but both methods need just fractions of time compared to a full 2D inversion. The advantages of the chebyshev inversion should further be tested for long measurement lines.

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Ein Vergleich von Widerstandsmessungen mit einem Multielektrodensystem und dem OhmMapper™

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Einleitung

Die Bestimmung von spezifischen elektrischen Widerständen bzw. Leitfähigkeiten im Untergrund erfolgt nach wie vor überwiegend mit Strom- und Messelektroden, die in den Boden gesteckt werden (galvanische Ankopplung). Insbesondere bei Oberflächen mit sehr hohen spezifischen Widerständen (z. B. trockene Sande) ist die Ankopplung der Elektroden jedoch mit erheblichen Schwierigkeiten verbunden oder – wie beispielsweise auf versiegelten Flächen – nahezu unmöglich. Dieses Problem ergab sich auch bei der Untersuchung von Permafrostböden und führte dazu, dass vor knapp 40 Jahren intensiv damit begonnen wurde, geoelektrische Messsysteme ohne galvanische Ankopplung an den Untergrund zu entwickeln (TIMOFEEV 1973, 1974). Hierfür bot sich die Übertragung des elektrischen Feldes auf kapazitivem Weg an im Gegensatz zur galvanischen Ankopplung wie in der klassischen Gleichstrom-geoelektrik. Dieses Prinzip macht sich neben wenigen anderen Herstellern auch die Fa. Geometrics (USA) beim OhmMapper™ zunutze, dessen erste Version vor etwa 10 Jahren auf dem Markt erschien.

In Anbetracht der schnellen Datengewinnung und der Möglichkeit zu Messungen auf versiegelten Flächen (urbane Gebiete) erscheint der OhmMapper™ als eine interessante Ergänzung zu herkömmlichen Gleichstromgeoelektrik-Systemen (DC).

Der OhmMapper™

In der einfachsten Ausführung besteht der OhmMapper™ aus einem Sender mit zwei Koaxialkabeln zu beiden Seiten und einem Empfänger, der ebenfalls mit zwei Koaxialkabeln versehen ist (Abb. 1). Sowohl im Sender- als auch im Empfängergehäuse befinden sich neben der entsprechenden Elektronik je zwei Batterien für die Stromversorgung. Die Verbindung zwischen Sender- und Empfängerkabel stellt ein nichtleitendes Zugseil variabler Länge dar. Vom Sender wird ein elektrisches Feld erzeugt, das sich im Untergrund in Abhängigkeit von der Leitfähigkeit ausbreitet und vom Empfänger gemessen wird. Die Daten werden über ein Kabel, das vom Empfängerkabel galvanisch getrennt ist, zur Messkonsole übertragen und gespeichert. Die Speicherkapazität der Konsole beträgt über 2 GB.

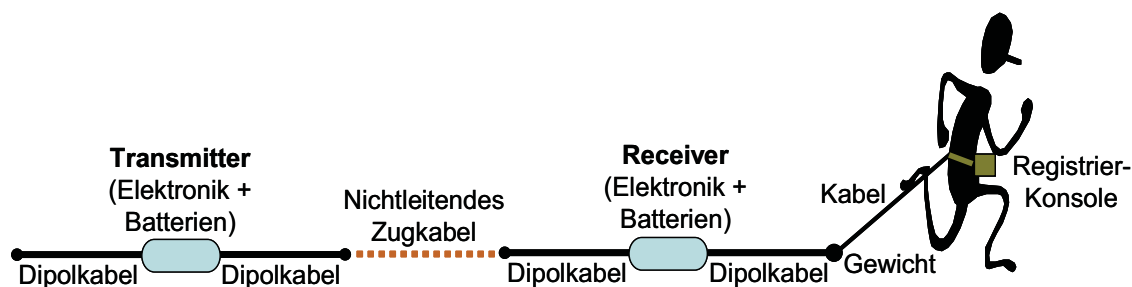


Abb. 1: Aufbau des OhmMappers™

Der vom Sender erzeugte Wechselstrom von 16,5 kHz wird an die Abschirmung der beiden Koaxialkabel angelegt (Abb. 2a). Die Abschirmung wirkt als Platte eines Kondensators und die Isolierung der Kabel als Dielektrikum, in dem Ladungsverschiebungen stattfinden. Der Erdboden stellt die andere Kondensatorplatte dar. Auf der Empfängerseite wird umgekehrt die Spannung im Untergrund kapazitiv aufgenommen und an der Abschirmung gemessen. Der auf der Senderseite eingespeiste Strom variiert zwischen 0,125 mA und 16 mA und wird automatisch den Untergrundbedingungen angepasst und dem Empfänger über ein aufmoduliertes 4-Hz-Signal mitgeteilt. Die Vorgänge auf Sender- und Empfängerseite sind in Abb. 2b in Form von Schaltbildern verdeutlicht.

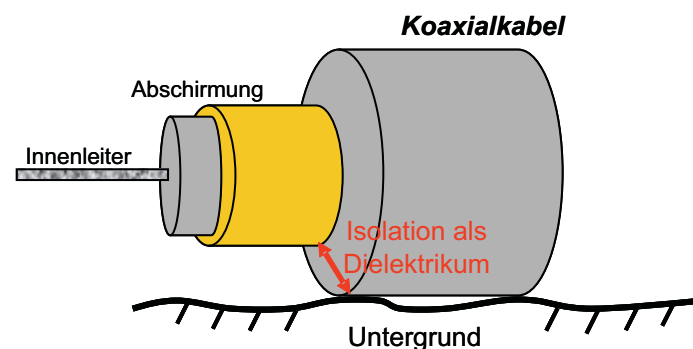


Abb. 2a: Prinzip der kapazitiven Ankopplung beim OhmMapper™ (nach Geometrics)

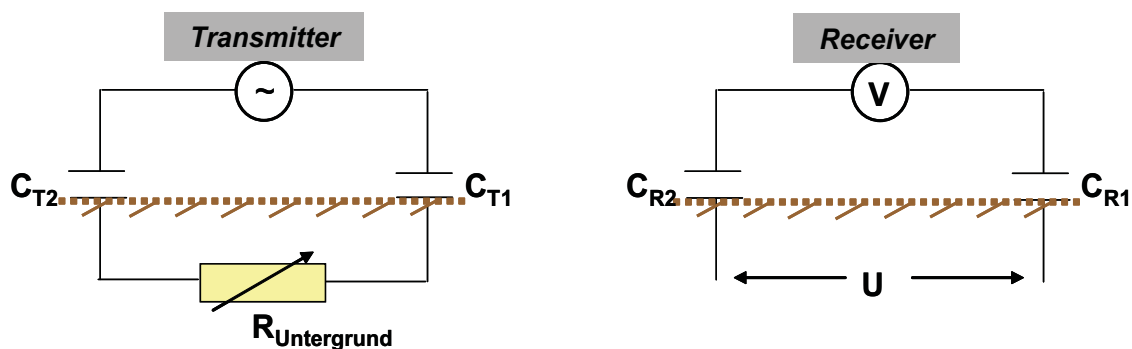


Abb. 2b: Schematische Darstellung in Schaltbildern

Die Veränderung des Abstandes zwischen Sender und Empfänger durch unterschiedliche Seillängen ermöglicht die Erfassung unterschiedlicher Tiefen. Die Erkundungstiefe wird mit maximal ca. 20 m angegeben und entspricht damit dem Bereich, der i. a. auch mit dem Georadar erfasst werden kann. Die Tiefe ist darüber hinaus auch von den Kabellängen abhängig.

Standardmäßig sind die Koaxialkabel in Längen von 2,5 m und 5 m erhältlich und sorgen über die gesamte Länge für Ankopplung, so dass sie auch als Linien-Elektroden zu bezeichnen sind. Die Kapazität der 2,5 m langen Kabel wurde zu 205 bis 210 pF und die der 5 m langen zu 505 bis 510 pF ermittelt.

Der OhmMapper™ kann nach Aussage des Herstellers spezifische Widerstände im Bereich zwischen 1 Ohmm und 100.000 Ohmm erfassen. Die Signale können mit einer Zyklusrate von maximal 0,5 sec (2 Hz) aufgenommen werden.

Für den Betrieb von Sender und Empfänger werden jeweils 12 V DC und für die Konsole 28 V DC benötigt.

Die neueste Geräteausführung ermöglicht den gleichzeitigen Einsatz von maximal fünf Empfängern.

Datenqualität

Der OhmMapper™ wurde in sechs unterschiedlichen Gebieten getestet. Das Hauptaugenmerk lag hier auf der Reproduzierbarkeit der gemessenen Daten und dem Vergleich mit Werten, wie sie die klassische Geoelektrik liefert. Für die DC-Messungen wurde eine Resecs-Apparatur eingesetzt.

Das OhmMapper™-System wurde überall von einem Fahrzeug gezogen, wodurch die Daten im Abstand von ca. 1,5 m (laterale Auflösung) registriert wurden mit einer Zyklusrate von 1 Hz.

Zur Unterscheidung von DC-Messungen wird im Folgenden für die Messungen mit dem OhmMapper™ auch die Bezeichnung CR für „capacitive resistivity“ (KURAS 2002) verwendet.

a) Reproduzierbarkeit

Zur Überprüfung der Reproduzierbarkeit wurden in drei Gebieten Testserien durchgeführt, bei denen das Profil jeweils in beide Richtungen mit demselben Sender-Empfänger-Abstand gemessen wurde. Abb. 3 zeigt die Pseudosektionen der Rohdaten des scheinbaren spezifischen Widerstandes aus dem Gebiet *Schillerslage*.

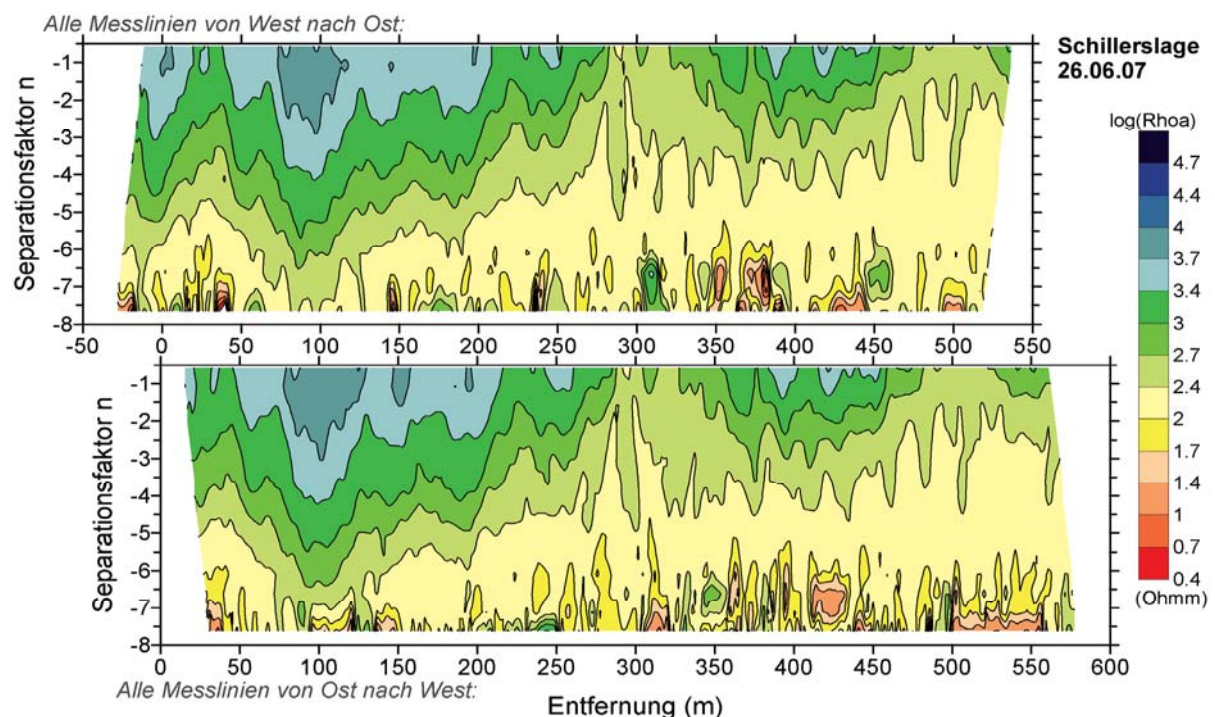


Abb. 3: Reproduzierbarkeitstest mit 5m-Dipolen in entgegengesetzter Richtung

Beide Darstellungen haben die Form eines Parallelogramms.

Bis zur Pseudotiefe $n=5$ ist eine sehr zufriedenstellende Übereinstimmung gegeben, insbesondere vor dem Hintergrund, dass ein bewegtes System nicht exakt dieselbe Spur trifft. Mit zunehmender Tiefe ist dann der Einfluss gestörter Signale erkennbar, die zu einem sehr diffusen Bild führen.

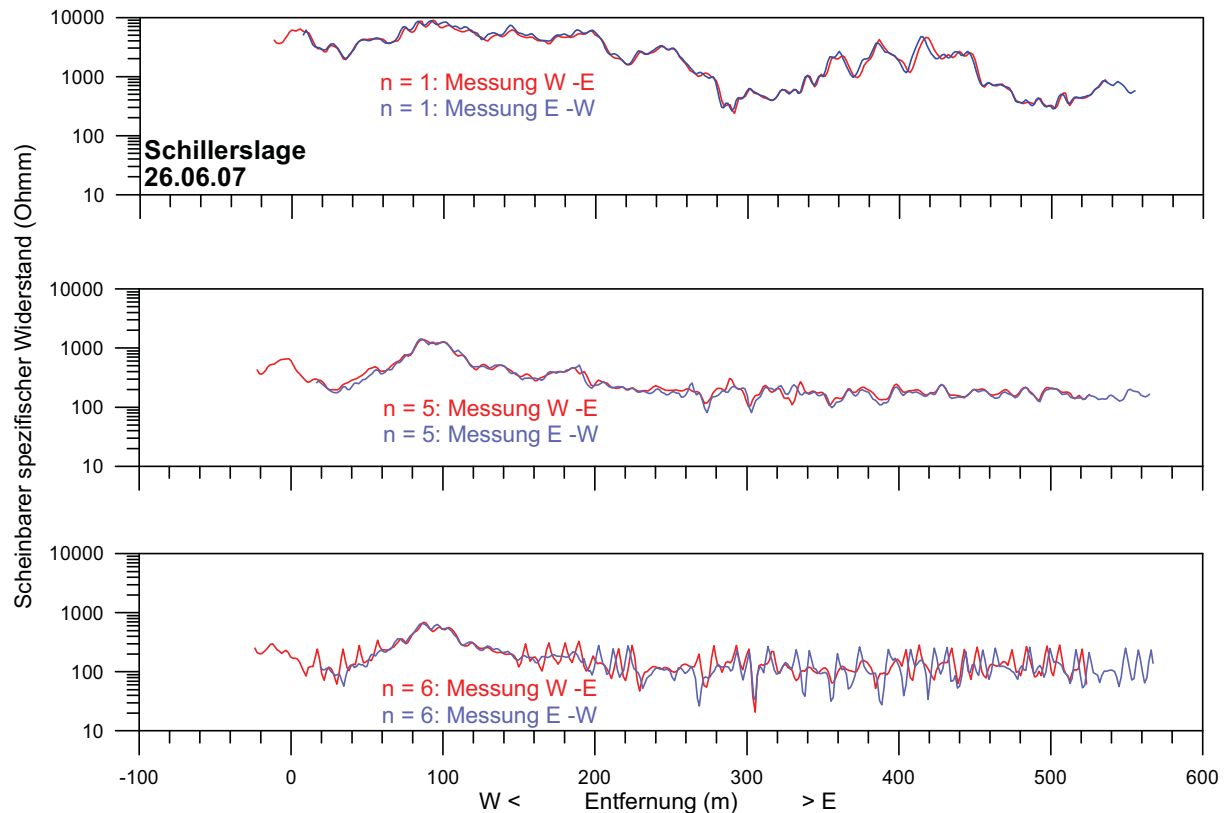


Abb. 4: Scheinbare spezifische Widerstände mit 5m-Dipolen für drei unterschiedliche Sender-Empfänger-Abstände

In Abb. 4 sind die scheinbaren spezifischen Widerstände des Profils für drei unterschiedliche Sender-Empfänger-Abstände ($n=1$, 5 und 6) dargestellt, was den Seillängen 5 m, 25 m und 30 m entspricht. Die Ergebnisse zeigen neben der guten Reproduzierbarkeit im oberen Bereich auch, dass Strukturen, die nur durch wenige Werte in Erscheinung treten, gut reproduzierbar sind. Der scheinbare spezifische Widerstand nimmt mit zunehmender Tiefe ab und relativ plötzlich, bei $n=6$, ist die Reproduzierbarkeit im Bereich der hochohmigen Struktur zwischen 60 m und 150 m zwar noch gut, verschlechtert sich aber in östlicher Richtung erheblich. Der Grund liegt in der Zunahme von gestörten Signalen, deren Ursache in einem zunehmend schlechter werdenden Signal-Noise-Verhältnis zu sehen ist. Auch in anderen Messgebieten wurde eine Verschlechterung beobachtet, die mit zunehmender Leitfähigkeit im Untergrund korreliert. Der Verstärkung kleiner Signale ist durch den Batteriebetrieb eine Obergrenze gesetzt, da die Stromeinspeisung maximal bis 16 mA möglich ist.

Auch die Ergebnisse aus den beiden anderen Messgebieten lassen eine gute Reproduzierbarkeit erkennen.

b) Vergleich mit DC-Messungen

Die Vergleichsmessungen mit der Resecs-Apparatur konnten wegen des Zeitaufwandes nicht überall auf den gesamten Profillängen durchgeführt werden. Für einen optimalen Vergleich wurden nach Möglichkeit die drei Sonden-Elektroden-Kombinationen Wenner-Alpha, Wenner-Beta und Dipol-Dipol verwendet.

Hier werden die Ergebnisse aus zwei Gebieten vorgestellt, die sich im spezifischen Widerstand signifikant unterscheiden. Im Raum *Marwede* sind in den Sanden der Südheide tief reichend sehr hohe spezifische Widerstände von über 10.000 Ohmm bekannt. Bei *Hämelerwald* reichen mächtige Tone der Unterkreide mit ihren niedrigen Widerständen bis dicht unter die Oberfläche. Daher müsste sich dieses Gebiet für den Einsatz des OhmMappers™ als äußerst ungünstig erweisen.

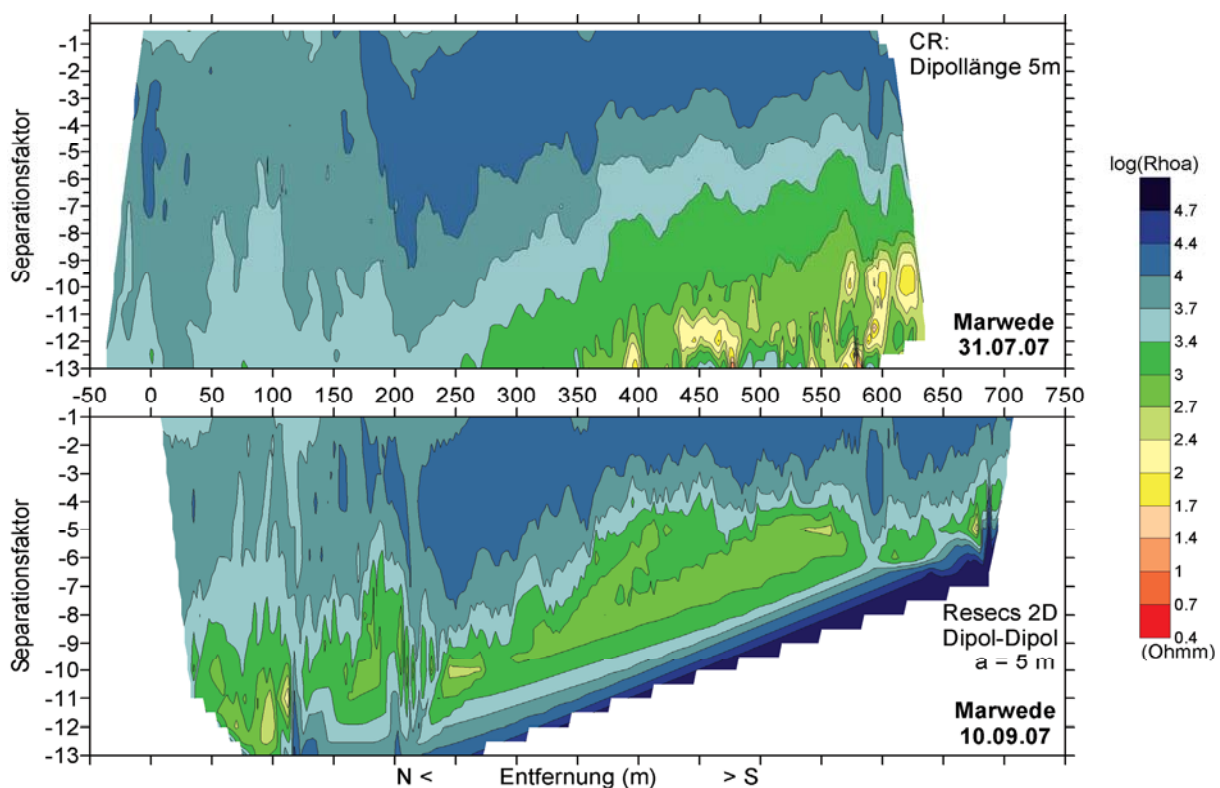


Abb. 5: Pseudosektionen der scheinbaren spezifischen Widerstände für CR- und DC-Messungen

Abb. 5 zeigt die Pseudosektionen aus dem Gebiet *Marwede*. Für die DC-Messungen wurde ein Dipolabstand von 5 m gewählt. Im unteren Teil der Abbildung sind die Ergebnisse der Dipol-Dipol-Messungen dargestellt und im oberen Teil die CR-Messungen mit Dipollängen von 5 m. Der zeitliche Abstand der Messungen betrug ca. sechs Wochen. Beide Ergebnisse zeigen im nördlichen Profilabschnitt tief reichend hohe scheinbare spezifische Widerstände um 5.000 Ohmm. In südlicher Richtung treten Werte über 10.000 Ohmm auf. Gleichzeitig verlieren die hohen Widerstände aber an Mächtigkeit, so dass im südlichen Bereich des Profils deutlich niedrigere Widerstände um 1.000 Ohmm schon in geringerer Tiefe anzutreffen sind. Beide Verfahren zeigen zwar eine sehr gute Übereinstimmung in der Tendenz, allerdings

differieren sie bezüglich der Mächtigkeiten. Die DC-Pseudosektion liefert geringere Mächtigkeiten.

Für dieses Profil sind in Abb. 6 die Einzelwerte des scheinbaren spezifischen Widerstandes für zwei Pseudotiefen ($n=1$ oben, $n=4$ unten) gegenüber gestellt. Für die Pseudotiefe $n=1$ liegen die Werte der CR-Messungen über denen der DC-Messungen. Dagegen ist in der Pseudotiefe $n=4$ eine wesentlich bessere Übereinstimmung gegeben. Die Abweichungen in Oberflächennähe sind mit größter Wahrscheinlichkeit auf unterschiedliche Durchfeuchtung infolge der zeitlichen Differenz der Messungen zurückzuführen.

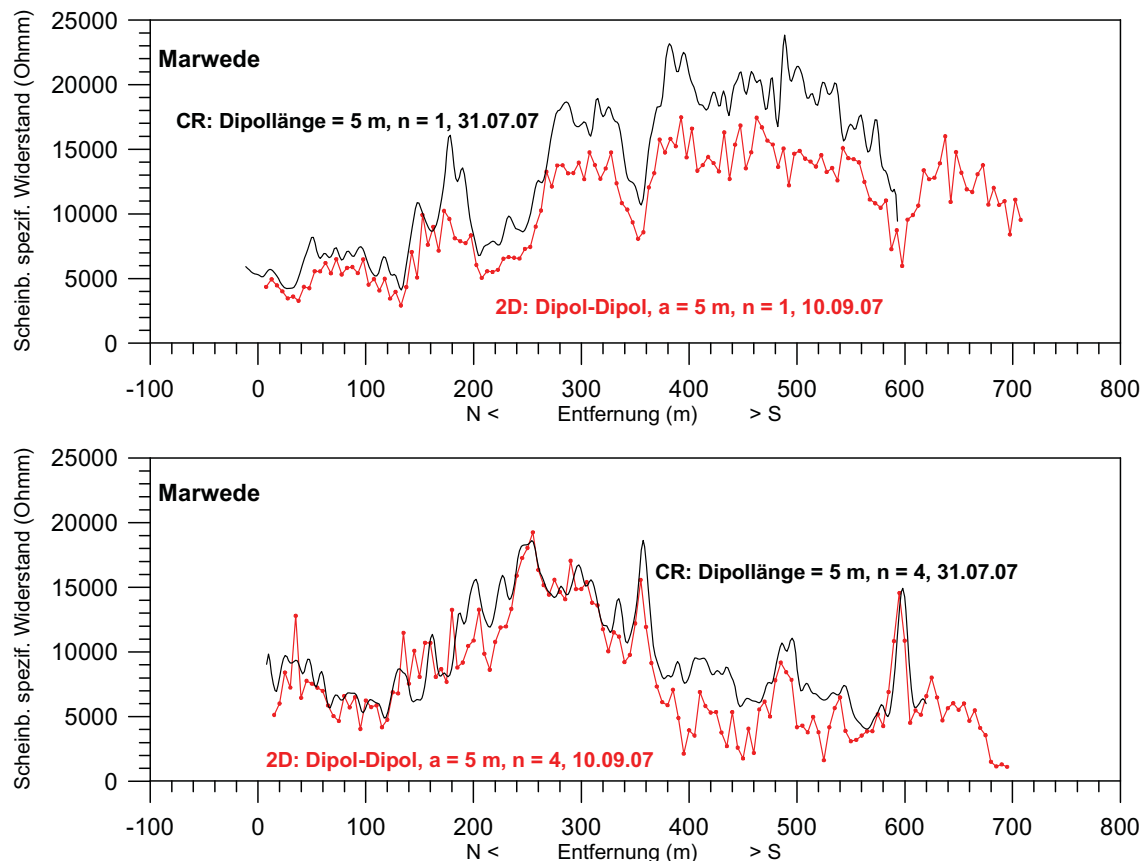


Abb. 6: Scheinbare spezifische Widerstände der CR- und DC-Messungen für die Tiefen $n=1$ und $n=4$

Bei den Messungen im stark leitfähigen Gebiet *Hämelerwald* (Pseudosektionen in Abb. 7) sind erwartungsgemäß die Grenzen des OhmMappers™ sehr schnell deutlich geworden. Bereits mit einer Seillänge von 10 m ($n=2$) konnten auf dem 900 m langen Profil nur noch an sehr wenigen Stellen Signale empfangen werden. In Materialien mit niedrigen spezifischen Widerständen kommen nur sehr kleine Signale am Empfänger an, da der Sender den eingespeisten Strom nur begrenzt erhöhen kann, wie bereits oben erwähnt. Auch mit Dipollängen von 10 m konnte hier nur eine unwesentlich größere Eindringtiefe erreicht werden.

Die Kreidetone mit spezifischen Widerständen um 10 Ohmm und weniger werden von einer geringmächtigen Deckschicht überlagert, die streckenweise scheinbare spezifische Widerstände um 100 Ohmm aufweist. Diese Bereiche sind in den DC-Ergebnissen, von denen hier die Pseudosektionen der Dipol-Dipol- und der Wenner-Alpha-Messungen dargestellt sind (Abb. 7 Mitte und unten), ebenfalls gut zu erken-

nen. Der Elektrodenabstand betrug 2 m. Aus der Wenner-Alpha-Darstellung ist ersichtlich, dass die Tone in beachtlicher Mächtigkeit vorhanden sind.

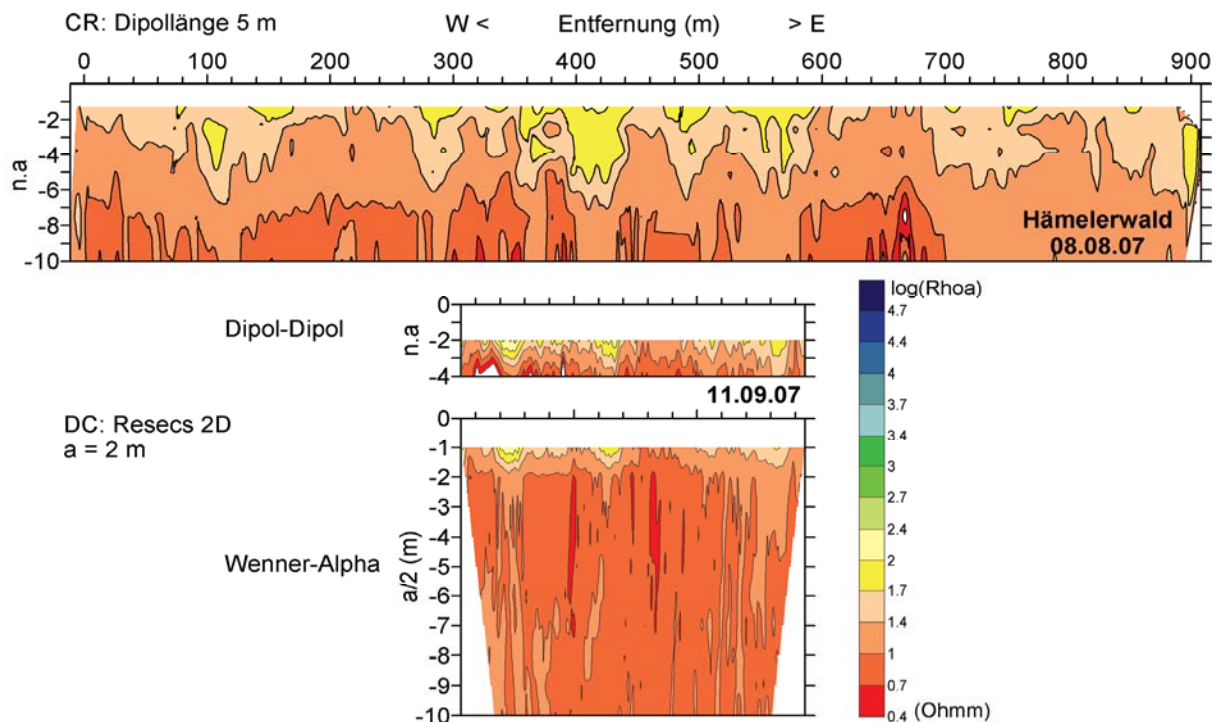


Abb. 7: Pseudosektionen der scheinbaren spezifischen Widerstände für einen gut leitenden Untergrund

Erkundungstiefe

Bezüglich der Erkundungstiefe h werden vom Hersteller lediglich die folgenden Angaben gemacht: Für $n=1$ beträgt $h=0,416 \cdot \text{Dipollänge}$ und für $n=3$ entspricht h der vollen Dipollänge, wobei n der Separationsfaktor ist (Sender-Empfänger-Abstand/Dipollänge). Darüber hinaus existieren keine weiteren Angaben.

Da kein linearer Zusammenhang zwischen der Tiefe und dem n -Faktor besteht, ist eine Grobabschätzung wie in der DC-Methode hier nicht möglich. Nur über eine Inversion der Daten sind Angaben zur Erkundungstiefe erhältlich. Hierfür wurde das Programm DC2dInvRes (GÜNTHER 2007) benutzt. Es ist, wie die kommerziellen Programme auch, speziell für gleichstromgeoelektrische Messungen entwickelt worden. In Abb. 8 sind die invertierten Daten für beide Verfahren aus dem Messgebiet *Marwede* dargestellt. Bei den CR-Messungen wurden Dipole von 5 m Länge eingesetzt. Für die DC-Messungen ist hier das Ergebnis der Dipol-Dipol-Messungen mit einem Elektrodenabstand a von 5 m gezeigt (Abb. 8 unten). Bei letzteren wurde eine Tiefe von ca. 12,5 m erreicht, während die CR-Messungen bei ca. 22,5 m enden. Zum besseren Vergleich ist die Untergrenze der DC-Ergebnisse im oberen Tiefenschnitt gestrichelt markiert. In beiden Darstellungen ist die deutliche Mächtigkeitsabnahme des hochohmigen Schichtpaketes in südlicher Richtung zu erkennen. In den CR-Ergebnissen ist dieses Paket über die gesamte Profillänge hinweg mächtiger ausgeprägt. Widerstände um 1000 Ohmm werden in den DC-Ergebnissen in geringerer Tiefe angetroffen. Die untersten Meter sind in beiden

Abbildungen von Artefakten geprägt. In den CR-Messungen treten sie noch erheblich stärker hervor. Dabei ist der Einfluss der unkorrigierten Daten nicht zu vernachlässigen. Für den obersten Bereich zeigen die DC-Ergebnisse eine bessere Auflösung.

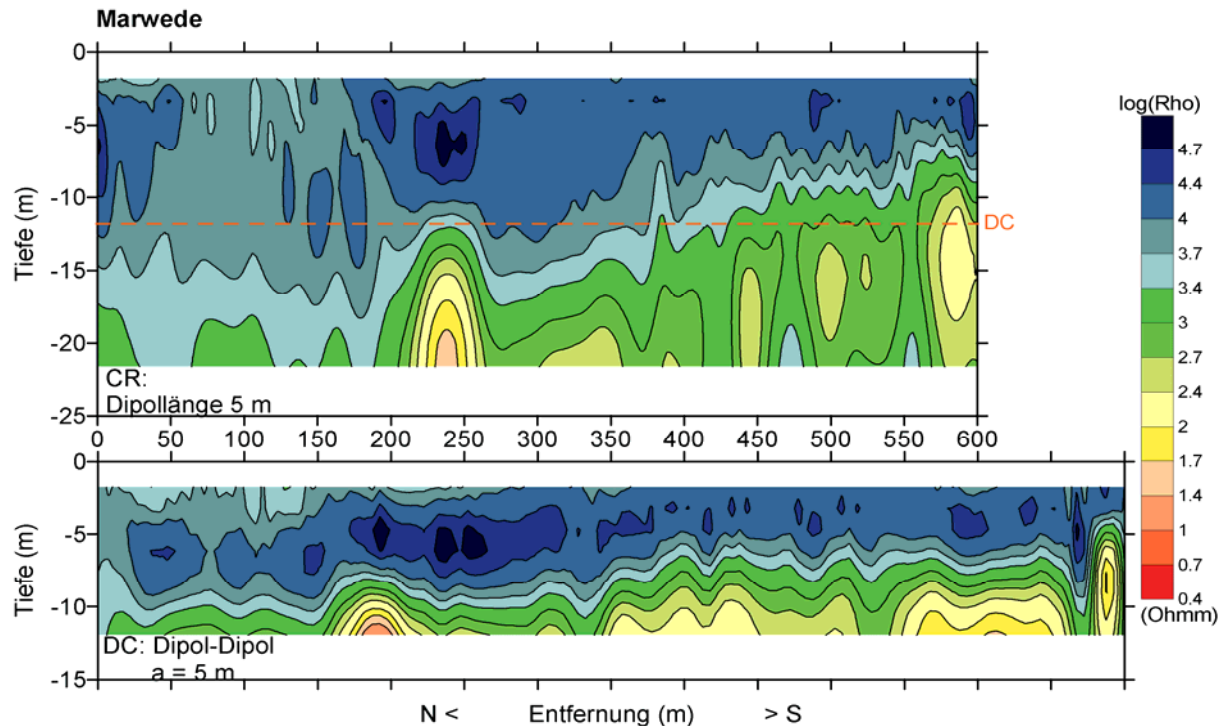


Abb. 8: Inversionsergebnisse von CR-Messungen (oben) und DC-Messungen (unten)

Bei der Benutzung der Inversionsprogramme und der Beurteilung der Ergebnisse muss bedacht werden, dass diese CR-Messungen zum einen mit einem bewegten System durchgeführt werden und zum anderen ein Wechselstrom von 16,5 kHz benutzt wird, so dass auch Induktionseffekte auftreten.

Signalverhalten

Durch eine Langzeitregistrierung (Abb. 9) sollte das Verhalten des Signals über mehrere Stunden ermittelt werden und zugleich festgestellt werden, nach welcher Zeit ein batteriebedingter Spannungsabfall eintritt. Die Datenaufnahme erfolgte mit 1 Hz. Nach fast achtestündiger Registrierung sackte die Spannung innerhalb von vier Sekunden auf den fast Null ab. Während der gesamten Registrierdauer zeigte das Signal eine beachtliche Stabilität. Der geringfügige Anstieg nach ca. 1¼ Stunden ist derzeit noch nicht zu erklären. Möglicherweise beruht er auf Fremdeinwirkung. Über die gesamte Zeit betrachtet ergibt sich der Mittelwert zu 302 Ohmm, Minimum und Maximum betragen 280 Ohmm und 312 Ohmm. Die gesamte Registrierung ist frei von Störsignalen, Peaks o. ä. sind nicht erkennbar.

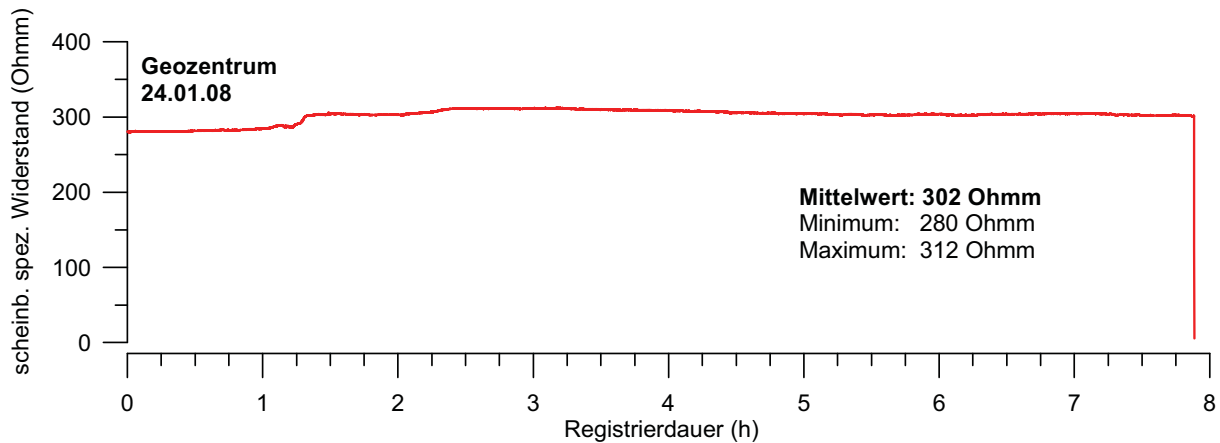


Abb. 9: Langzeitregistrierung: Datenaufnahme im Abstand von einer Sekunde

Beurteilung des OhmMappers™

- Die Datengewinnung ist ein Vielfaches schneller als mit der konventionellen DC-Methode. Somit sind großflächige Messungen in einem zeitlich vertretbaren Rahmen möglich, mit denen eine schnelle Übersicht in einem Gebiet gewonnen werden kann und DC-Messungen ggf. gezielter angesetzt werden können.
- Durch Anpassung der Fahrgeschwindigkeit ist mit diesem bewegten System eine dichte Datenaufnahme möglich.
- Das System ist sehr robust und leicht zu handhaben und mit Ausnahme von kleinen Flächen bzw. kurzen Profilen nahezu überall einsetzbar.
- Die Erkundungstiefe ist auf ca. 10 bis 20 m begrenzt und entspricht damit dem Tiefenbereich, der i. a. auch mit dem Georadar erfasst werden kann.
- Durch die Möglichkeit der Erzeugung von kontinuierlichen Tiefensektionen des spezifischen Widerstandes stellt der OhmMapper™ eine wertvolle Ergänzung und Interpretationshilfe für Georadaruntersuchungen dar.
- Wesentliche Nachteile liegen in der begrenzten Erkundungstiefe gegenüber dem Multielektrodensystem und der nur auf hochohmige Gebiete beschränkten Einsatzmöglichkeit.
- Das System gestattet nur die Kartierung mit der Dipol-Dipol-Anordnung.
- Die Anwendbarkeit von DC-Interpretationsprogrammen (Inversion) ist möglich, wirft derzeit aber noch zahlreiche Fragen auf, wie z. B. die Wahl der Parameter für die Inversionsrechnung (HAUCK & KNEISEL 2006).
- Die Modifikation der Ausgabedatei durch den Hersteller wäre im Hinblick auf zusätzliche Informationen wünschenswert.

Ausblick

Neben einigen weiteren systematischen Untersuchungen sollen in naher Zukunft die Einsatzmöglichkeiten des Systems für spezielle Fragestellungen (z. B. geologische Kartierung, Grundwassererkundung, Lagerstättenerkundung u. ä.) durch Testmessungen betrachtet und beurteilt werden.

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Radiomagnetotellurics: A case study for geomorphological questions

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Abstract

The major task of the presented study is to answer geomorphologic issues concerning the evolutionary history of the terraces of the river Rhine in the Lower Rhine Bay. More precisely the task is to map the two boundaries between tertiary sands, fluvial gravel and loess. The second task is the realisation of the first measurements in Germany, using the new Radiomagnetotelluric-device of the Institute. The main technical improvement of this device is the enlarged frequency range from 10 kHz to 1MHz.

Geology of the survey area

The survey area is part of the terraces of the river Rhine, which are arranged in steps. More precisely, the area is characterized by the transition from the Niederterrasse (lowest terrace) to the Mittelterrasse (middle terrace), which is covered with loess. The location of this area is accentuated in Fig.1. Two main causes formed today's geological situation, namely the terraces arranged in steps and a complex system of valleys:

1. During the pleistocene there was an

alternation of Ice Ages and Interglacial periods. In geomorphological terms, Ice Ages represent glacial erosion. In the interglacial periods accumulation of fluvial sediments took place.

2. Since the Miozän (Savische Bruchbildung) the whole Lower Rhine Bay descends.

In Fig.1 a section of the middle terraces at the Lower Rhine north of Cologne is presented. As it can be easily seen, gravel and Loess cover the tertiary terraces. Therefore the original shape of the terrace can not

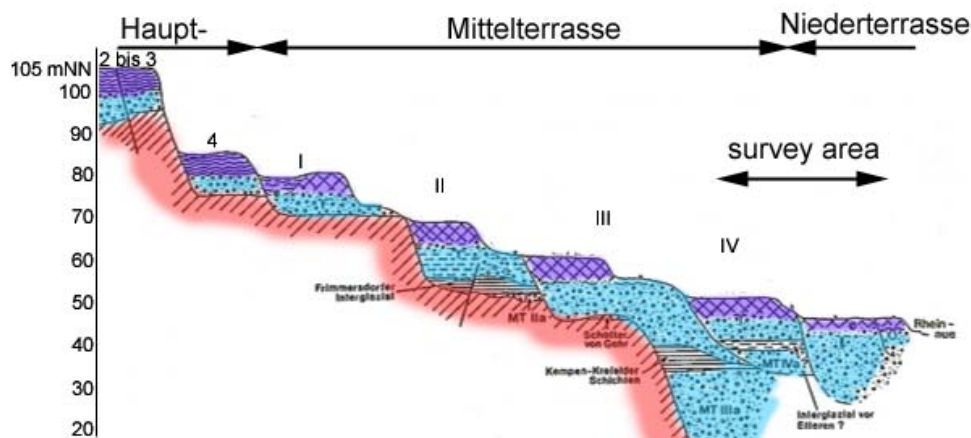


Figure 1:
Section of the Middleterraces north of Cologne [Brunnacker et al., 1978]. Colorindex:
Red = Tertiary sands. Blue = Fluvial gravel from Rhine. Violet = Loess.

be recognized. The tertiary topography is not traceable from today's topography. The Geomorphological questions concern the location of the tertiary terraces and the spreading of the loess.

today's topography. The fact that today's top ground surface is not at the same place as the tertiary top ground surface can be seen in Fig.2C: it's shifted to the left.

Geomorphologic questions

There are two main geomorphologic questions examined with RMT. See Fig.4 for the location of the two profiles given as examples below. The two tasks are:

1. The evolutionary history of a valley. First question is, whether the valley emerged before (Fig.2A) or after (Fig.2B) the covering with sediments.

2. Fluvial gravel and Loess cover the tertiary sands and pretend a wrong shape of the original terraces (cp. Fig.2C): The tertiary topography is not traceable from

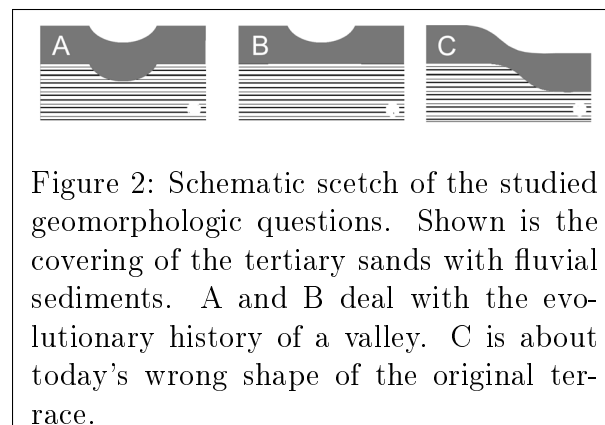


Figure 2: Schematic scetch of the studied geomorphologic questions. Shown is the covering of the tertiary sands with fluvial sediments. A and B deal with the evolutionary history of a valley. C is about today's wrong shape of the original terrace.

Physical background

The theory of radiomagnetotellurics (RMT) is similar to the well-known MT (Cagniard [1953]). Radiotransmitters

located far away are used as transmitters. Planewave approximation is valid and displacement currents can be ignored for used frequencies and resistivities situated in the survey area. The relation of the incoming electric and magnetic fields is specified by the complex impedance tensor Z (eq.1). In the case of two-dimensional distribution of conductivity, the Maxwell equations can be divided into the two modi tangential electric and tangential magnetic. Two physical parameters can now be derived from Z for each mode. Eq.2 and eq.3 show the formulas for apparent resistivity ρ_a and phase ϕ for one mode.

$$\begin{bmatrix} E_x(\omega) \\ E_y(\omega) \end{bmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \cdot \begin{bmatrix} H_x(\omega) \\ H_y(\omega) \end{bmatrix} \quad (1)$$

$$\rho_a = \frac{1}{\mu_0 \omega} \cdot |Z_{xy}(\omega)|^2 \quad (2)$$

$$\phi_{xy} = \arctan \left(\frac{\Im(Z_{xy}(\omega))}{\Re(Z_{xy}(\omega))} \right) \quad (3)$$

Measurement device

The 4-channel measurement device RMT-F (Fig.3) records time series of all four horizontal field components (E_x, E_y, H_x, H_y) simultaneously. The processing software, appendant to the device, calculates the transfer functions from the recorded time series by using spectral analysis. One can select the correct transmitters for each mode via the selected transmitter-azimuth and the qualitativly best transmitter-signals via the coherence level between electric and magnetic fields in x and y direction.

One technical improvement of the new measurement device of the institute is the

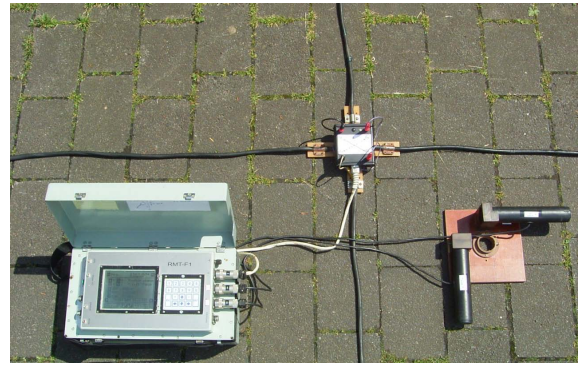
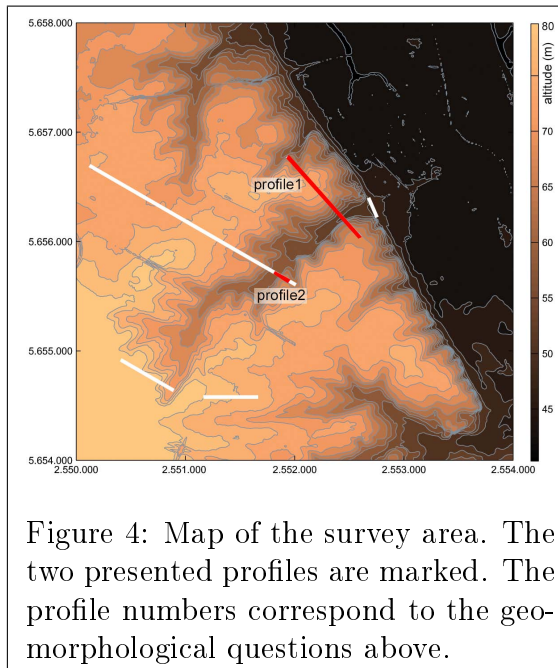


Figure 3: 4-channel measurement device RMT-F

enlarged frequency range up to 1MHz. From the higher frequencies one gets exclusive information about the upper layers of the earth. The effects on the inversion of this additional information are examined by *Wiebe [2007]*. Apparent resistivities (Eq.2) and Phases (Eq.3) as a function of frequency are shown in Fig. 9 as an example to demonstrate the data quality of the new RMT device.

Survey area

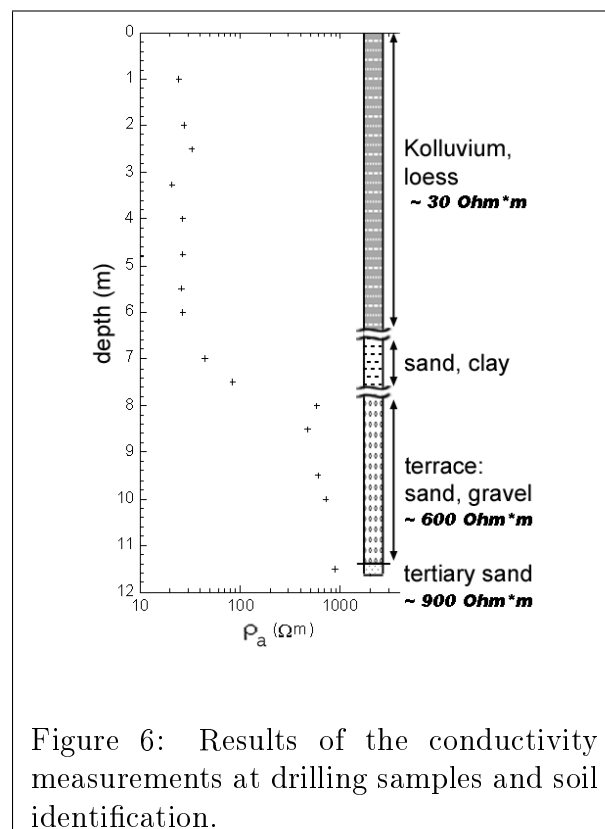
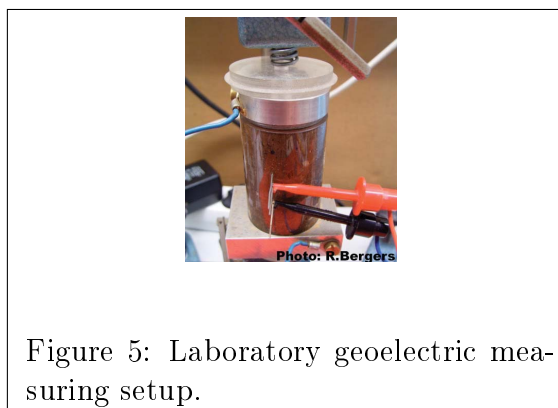
The survey area is located north-west of Cologne near the town Stommeln. The measured profiles are shown in Fig.4. For each geomorphologic question, given above, one example profile is picked and accentuated in the map.



The drilling compresses the drilling core differently, depending on the position in the core. The core was sampled every $\sim 0.7\text{m}$ and the arithmetic mean of the measured ρ_a was calculated out of differently compressed samples and of different electrode positions in the measuring setup. The result is presented in Fig.6. This study provides an informative basis of the resistivities situated in the survey area. There is a strong contrast in resistivity between the near surface sediments ($30\Omega m$) and the deeper fluvial gravel ($600\Omega m$). The resistivity contrast between tertiary sands ($900\Omega m$) and the upper gravel is not distinct.

Further studies

As a part of the presented study conductivity measurements at drilling samples from the survey area were done by using a laboratory geoelectric measuring setup (Fig.5). Current electrodes are at top and bottom of the sample. Four potential electrodes in the middle allow several configurations.



Inversion results

For the inversion of the RMT data the 2D-inversioncode by *Mackie & Rodi [2001]* was used. It uses a non-linear-conjugate-gradient algorithm with Tikhonov-regularization and a finite differences forward calculation. To get the optimal regularization parameter τ , every dataset was inverted separately for several τ , and after that the optimal τ was chosen via a log-log L-Curve according to *Hansen & O'Leary [1993]*. This procedure was very time-consuming for the big amount of stations (230) and the big amount of inverted frequencies (up to 20 in some datasets). The inversion was done for the TE and the TM mode separately and a joint inversion for both modes. As a quality control for the results normalised sensitivity values are calculated for every cell. A comparison with the maximal depth of investigation according to *Spies [1989]* shows that both methods give similar depths for reliable results.

Model calculations showed that the effect of topography can be ignored.

As one can see in the inversion-results (e.g. Fig.7b) the boundary between the two resistive layers tertiary sands and gravel from the terrace is not very well reproduced: the boundary blurs to a region with lateral increasing ρ values. Whereas the two layers resistive gravel and covering conductive sediments are well separated. This fact was also observed in synthetical modeling with typical transmitter frequencies and resistivities situated in the survey area.

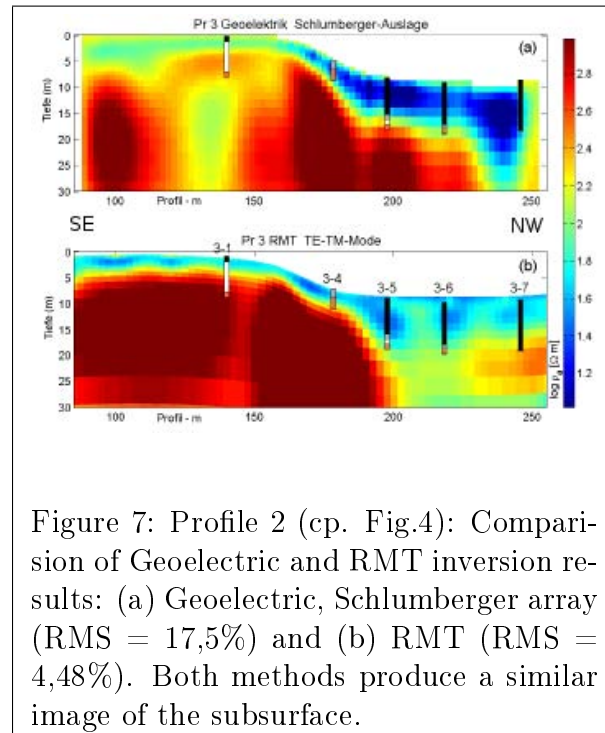


Figure 7: Profile 2 (cp. Fig.4): Comparison of Geoelectric and RMT inversion results: (a) Geoelectric, Schlumberger array (RMS = 17,5%) and (b) RMT (RMS = 4,48%). Both methods produce a similar image of the subsurface.

The RMT inversion results were compared with inversion results of geoelectric data. Both methods produce comparable images of the subsurface (cp. Fig.7). Also in Fig.7, one can compare the drilling and laboratory results by *P. Fischer [2003]* done in this area. Not every pedologic result can be reproduced by this two geophysical methods, but the main features, like tertiary sands and covering sediments are principally solved. Tertiary sands are presented in red in the drilling core, the covering sediments are black (Loess) and white (Kolluvium).

As an example in Fig.9 two stations are shown. Their locations are marked in Fig.8. The solid lines are the results from the inversion, which fit the measured data well.

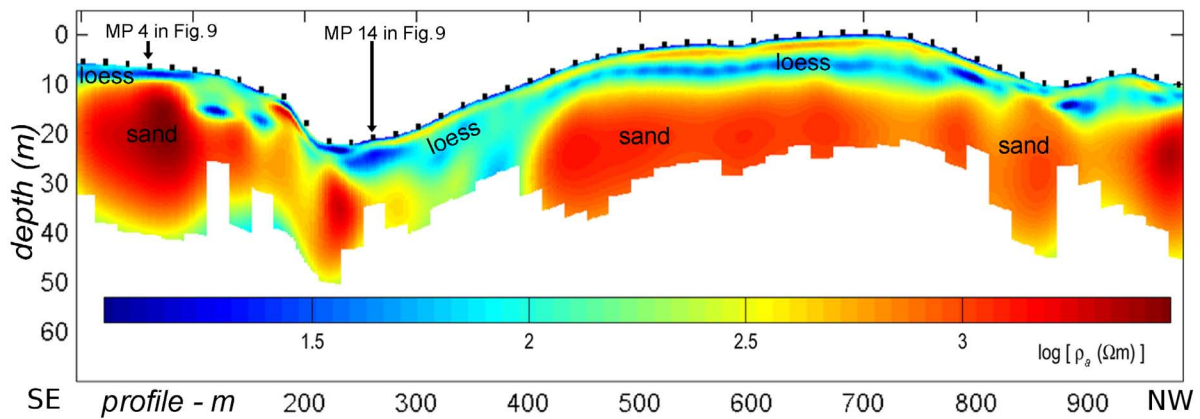


Figure 8:

Profile 1 (cp. Fig.4): RMT joint-inversion of both modes. $RMS = 1,81\%$. Areas below the maximal depth of investigation [Spies [1989]] are whitened.

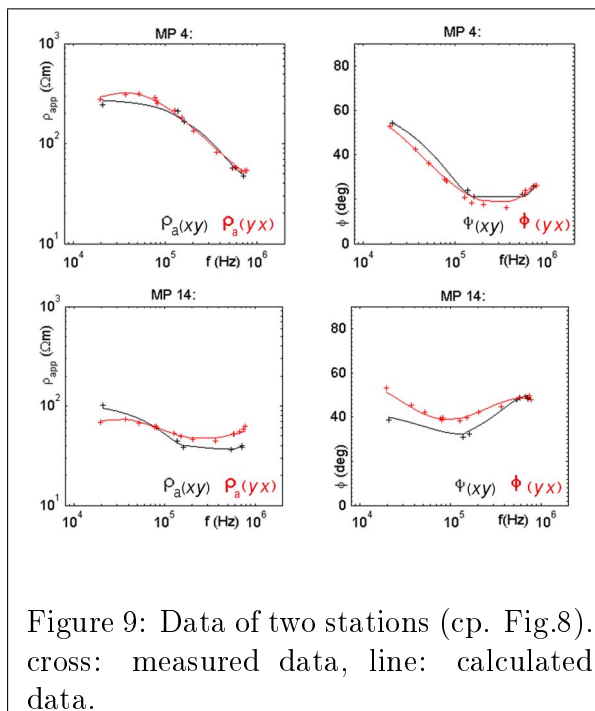


Figure 9: Data of two stations (cp. Fig.8). cross: measured data, line: calculated data.

the asked geomorphologic questions:

1. The evolutionary history of a valley can be revealed. In the RMT section presented in Fig.8 one can see that in this case the valley existed before the covering with sediments: one can see the original valley carved in the resistive tertiary sands. Furthermore one can see the asymmetric shape of today's valley, which differs from the tertiary valley. This asymmetric shape is typical for valleys emerged in glacial times (cp. e.g. Fischer [2003]).

2. The tertiary shape of the terraces can be mapped, although covered with sediments. An example therefor is the profile in Fig.7, where today's top ground surface doesn't correspond to the tertiary one.

Geomorphologic questions answered with Radiomagnetotellurics

By mapping the upper border of the resistive tertiary sands, RMT can answer

Summary

The geophysical methods RMT and Geo-electric can be applied for some geomorphological issues. Concerning the mapping of

the terraces, the results are comparable to drilling results. Of course, RMT can not substitute percussion drilling together with pedological laboratory studies. But RMT and Geoelectric give an adequate geological overview and can perfectly assist the choice of new drilling spots. The advantage is the measuring speed and the nondestructive measurement.

von Geoelektrik und Radiomagnetotellurik.,
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